Introduction

Migration based on conventional velocity analysis produces imaging error in complex media (Isaac and Lawton, 1999) as can be seen in the pre-stack migrated image (Figure 1) of the Foothills Thrust Sheet Synthetic Dataset. Generated by elastic finite-difference modeling, The model is a weak TI thrust sheet composed of four blocks dipping at 0, 30, 50 and 60 degrees as in figure 1, and seismic data are generated using finite differences (obtained courtesy of bp). For P-wave data, the thrust sheet is defined by anisotropy parameters α_0, ε and δ, and the slow direction is everywhere normal to the top of the sheet. The sheet embedded in an isotropic medium. Assuming isotropy ($\varepsilon=0$, $\delta=0$), as can be seen in figure 1, the flat base of the model appears pulled up.

To determine ε and δ, analysis based on Common Focus Point (CFP) technology (Berkhout, 1997; Berkhout and Verschuur, 2001) is promising in that error analysis allows easier model updating (Kabir, 1997;). In Kabir (1997), CFP is used to estimate lateral velocity variation for heterogeneous, isotropic media. Here we use CFP analysis for homogeneous, anisotropic media. For the Foothills data, we use differential time shifts (DTS)s, a product of CFP analysis, to invert for ε and δ. We find inversion estimates for ε and δ to converge within 6 percent error of the actual values. Migration using newly obtained anisotropic parameters results in improved imaging (Figure 6).

In future, we expect to relax the a priori requirements with acquisition of S-wave data and still obtain convergence for the parameters using joint inversion of P- and S-wave data.

Theory

For the physical model data, isotropic velocity analysis and subsequent migration results in de-focused and miss-positioned image below the highly dipping thrust sheet (Figure 1). It is intuitive that the poor image quality results from incorrect elastic parameter estimation for the thrust sheet, or simply negligence of anisotropy in the thrust sheet. We perform velocity analysis in CFP domain to obtain anisotropic parameter that focuses the reflection energy to the correct position of the reflection point or focus point.

To begin with, we assume the thrust sheet is isotropic and we compute the focusing operator (Figure 2a) using one-way traveltime from the given gridpoint to all receiver locations and place the source wavelet at the corresponding time. In the first focusing step (focusing in detection), each shot record is transformed into a single trace by the focusing operator (Berkhout and Verschuur, 2001). Together all transformed traces constitute CFP gather (Figure 2b), each trace being positioned at the source location of its corresponding shot record (Kabir and Verschuur, 2000). One event in CFP gather (shown by arrow in Figure 2b) is the focus point response (Kabir 1997). For correct model, the focusing operator and CFP gather should be identical and result in zero differential time shift (DTS) for all offsets under cross correlation (Kabir, 1997).

In this anisotropic example, the model is assumed isotropic, so non-flat DTS result after cross correlation as can be seen in Figure 3. To achieve zero DTS, we need to estimate the correct anisotropic parameters. Let f_k represent the DTSs measured for M offsets where $1 \le k \le M$, and let m_j represent the model for N anisotropic-parameters where $1 \le j \le N$. Here, we seek an updated model $\mathbf{m}+\delta\mathbf{m}$, where $\mathbf{m}$ is an initial guess, so that DTSs $f_k(\mathbf{m}+\delta\mathbf{m})$ are zero. Expand the desired DTSs $f_k(\mathbf{m}+\delta\mathbf{m})$) about the measured DTSs $f_k(\mathbf{m})$ to get:

$$f_k(\mathbf{m}+\delta\mathbf{m}) - f_k(\mathbf{m}) = \sum_{j=1}^{N} \frac{\partial f_k}{\partial m_j}\,\delta m_j + O_k(\delta\mathbf{m}^2). \qquad (1)$$

where $\partial f_k / \partial m_j$ are first differentials of measured $f_k(\mathbf{m})$ with respect to the model parameter updates. Setting $G_{kj} = \partial f_k / \partial m_j$, $f_k(\mathbf{m}+\delta\mathbf{m})$ to zero and neglecting terms of order $\delta\mathbf{m}^2$ and higher, we obtain a set of overdetermined linear equations for the variable updates $\delta\mathbf{m}$. The solution for the model updates is

$$\delta\mathbf{m} = -\left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{f}\,, \qquad (2)$$

Here $\delta\mathbf{m}$ is computed by minimizing the L_2 norm of objective function $\left\|\mathbf{G}\delta\mathbf{m}+\mathbf{f}\right\|^2$ (see, for example Menke, 1989).

We compute the derivatives of $f_k(\mathbf{m})$ directly from the minimum traveltime $\mathbf{t}(\mathbf{m})$ from the focus point to surface locations in perturbed media following Kabir (1997). The derivatives thus obtained are multiplied by two as DTS is given in two-way time, but error in the perturbed operator is given in one-way time (Kabir, 1997). For this model, we assume that the dips of each block of the thrust sheet are known, as well as minimum P-wave velocity α_0, depth, thickness, and that the thrust is a weak, transverse-isotropic medium (Thomsen, 1986). We assume also that ε and δ are homogeneous within the sheet.

Brute force mapping (Faria and Stoffa, 1994) is used to compute the traveltimes required in forming CFP gathers and focusing operators. Following Kumar et al (2004), minimum arrival traveltimes are computed as a function of group angle using weak TI slowness. As traveltimes are marched forward, slowness local to a grid point in the medium is computed as a function of a reduced set of group angles using a truncated, Fourier cosine-series following Byun et al. (1989). Because we assume the medium is homogeneous, series coefficients are computed only once following Kumar et al. (2004). When blocks are tilted, group velocity is modified by tilt angle (Kumar et al., 2004).

Result

In this section we show the result of CFP domain velocity analysis and inversion for the thrust sheet model. We start with an assumption of isotropic thrust sheet (ε = δ = 0). Known P-wave velocity α_0 along the slow direction is 2.925 km/s, and the known tilt angles of the thrust sheet are 0, 30, 50 and 60 degrees respectively from left to right (Figure 1). The CFP operator in x-t domain is generated by placing the source wavelet at the minimum traveltime from CFP to each surface location (Fig. 2a). The operator is applied to each shot gather to obtain a trace containing one-way response of focus point. Together, they constitute CFP response (Fig. 2b) that represents one-way traveltime from the focus point to the shot location. For an assumed model, the difference in the CFP operator and the CFP response represents the two-way traveltime error, called DTS. We perform trace by trace cross-correlation of the operator and the response to obtain DTS panel (Fig. 3) and then use a graphical interface to pick the values of DTSs or $f_k(\mathbf{m})$ to be used in equation (2). The partial derivatives of DTSs w.r.t. model parameters ($G_{kj} = \partial f_k / \partial m_j$) are computed numerically (Kabir, 1997). Then, equation (2) is used to obtain model update $\delta\mathbf{m}$. After a few iterations, we find that the CFP operator and response are much more similar (Figure 4). This results in a flat DTS panel (Figure 5). The migration using the newly obtained anisotropic parameters, given in table 1, give a better focused and correctly positioned image of the reflector below the highly dipping thrust sheet (Figure 6).

Parameters	Real value	Initial guess	After inversion	%age error
ε	0.150	0.0	0.156	4.0
δ	0.081	0.0	0.075	-7.4

Table 1. Model parameters

Conclusions

The isotropic model based pre-stack migration of the synthetic data of the foothills thrust model produced erroneous image below the thrust sheet. In CFP domain velocity analysis, the error in imaging can be seen as non-zero differential time shifts (DTS) between CFP operator and CFP response in the poorly imaged region. An estimate of anisotropic parameters for the thrust sheet ε and δ has been obtained that makes the DTSs zero. The obtained parameter values are very close to actual parameters. The migration performed after incorporating the newly obtained anisotropic parameters into the model, gives a better focused and correctly positioned reflector below the highly dipping thrust sheet.

Acknowledgement

The authors wish to thank bp for providing Foothills Structural Dataset, and the sponsors of EDGER forum for supporting this research.

References

[1] Berkhout, A. J., 1997, Pushing the limits of seismic imaging, Part I: Prestack migration in terms of double dynamic focusing: Geophysics, Vol. 62, p. 937-953.

[2] Berkhout, A. J. and Verschuur, D. J., 2001, Seismic imaging beyond depth migration: Geophysics, Vol. 66, p. 1895-1912.

[3] Byun, B. S., Corrigan, D., and Gaiser, J., 1989, Anisotropic velocity analysis for lithology discrimination: Geophysics, Vol. 54, p. 1564-1574.

[4] Faria, E. L., and Stoffa, P. L., 1994, Traveltime computation in transversely isotropic media: Geophysics, Vol. 59, p. 272-281.

[5] Issac, J. H., and Lawton, D. C., 1999, Image mispositioning due to dipping TI media: A physical seismic modeling study: Geophysics, Vol. 64, p. 1230-1238.

[6] Kabir, M. M. N., 1997, Velocity estimation of the complex subsurface using the common focus point technology: Ph.D. thesis, Delft Univ. Tech.

[7] Kumar, D., Sen, M. K. and Ferguson, R. J., 2004, Traveltime calculation and prestack depth migration in tilted transversely isotropic media: Geophysics, 69, 37-44.

[8] Menke, W., 1989: Geophysical Data Analysis: Discrete Inverse Theory. Academic Press, 289 pp.

[9] Thomsen, L. 1986. Weak elastic anisotropy, Geophysics, 51, 1954-1966.

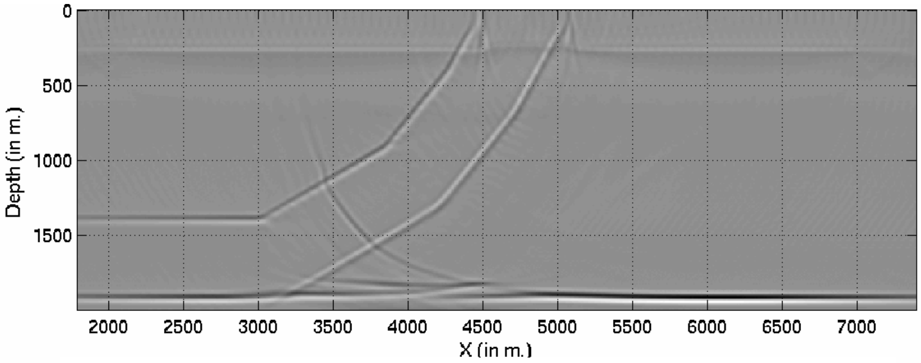

Fig. 1: Migrated image using isotropic velocity model.

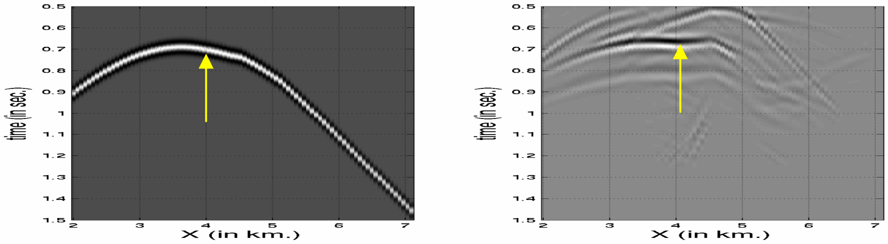

Fig. 2(a): CFP operator; (b) CFP response, for initial model

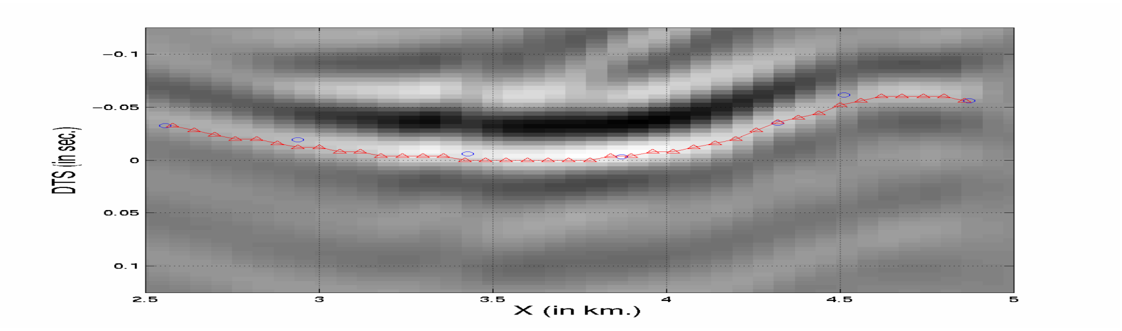

Fig. 3: The DTS panel for initial guess model.

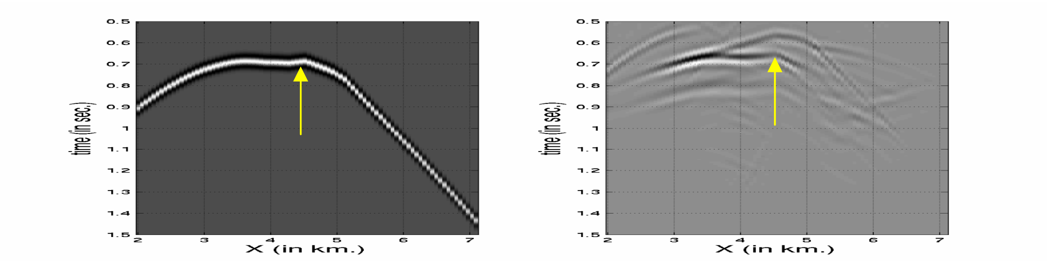

Fig. 4(a): CFP operator; (b) CFP response, after inversion

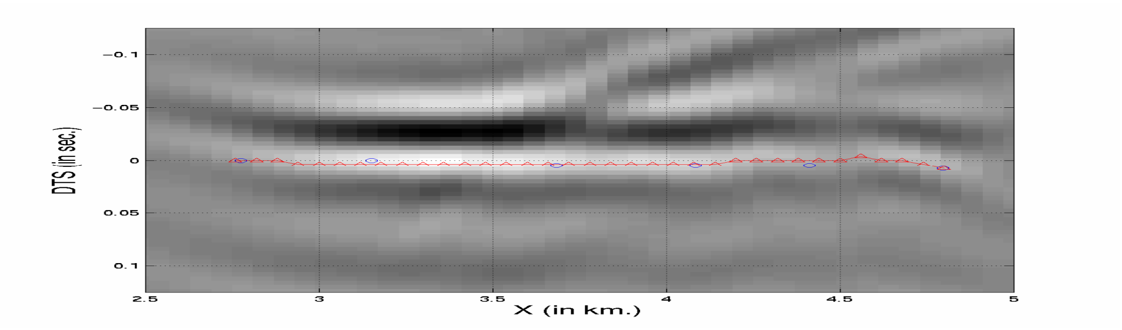

Fig. 5: The DTS panel after inversion.

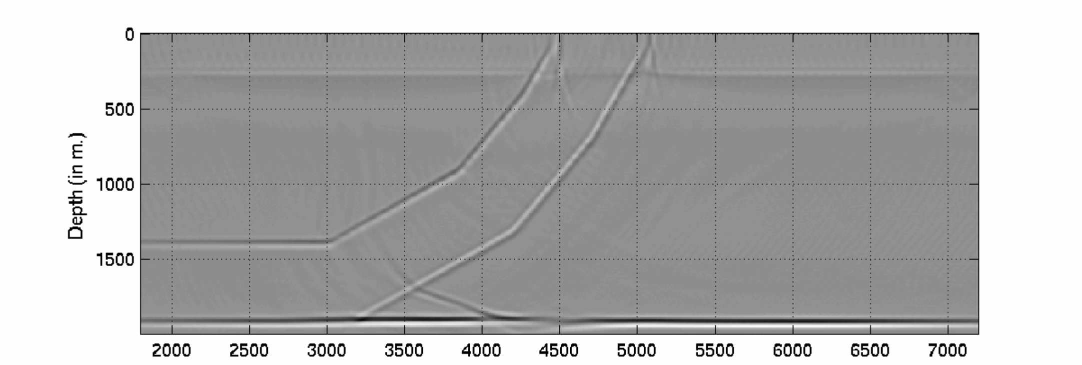

Fig. 6: Migrated image using anisotropic velocity model obtained from inversion..