# Traveltime Sensitivity Analysis for Parameter Estimation in TI media

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## Abstract

Limitations in Elastic parameter estimation using reflection seismic-data explored using sensitivity analysis of traveltime variation with variation in Elastic parameters. Analysis helps us quantify the limits of parameter estimation, and it suggests ways to recover the parameters meaningfully. We perform sensitivity analysis for transverse isotropy (TI) under different experimental settings. We do not impose the condition of weak anisotropy, and we perform the analysis in plane wave domain where, for every ray parameter, vertical traveltime is expressed as an exact analytical relation between ray parameter and anisotropic parameters. This analysis shows that joint inversion of P-wave data and converted SV-wave data can constrain anisotropic parameters as well as reflector depth. Moreover, we find increased offset-to-depth ratio and increased tilt angle of TI symmetry axis increases accuracy. For tilted transverse isotropy, we find that joint inversion can simultaneously provide a good estimate of subsurface anisotropy and depth as well as magnitude of tilt angle of the symmetry axis.

## Introduction

In seismology a medium is said to be anisotropic if seismic velocity varies with direction (Crampin, 1984). Undisturbed sedimentary rocks often exhibit anisotropy, where velocity is symmetric about the axis normal to the bedding, and isotropy is limited to the horizontal (the transverse) plane. The resulting anisotropy is called transverse isotropy with a vertical axis of symmetry (VTI). VTI results from thin laminations, preferred orientation of minerals like clay, and horizontally aligned fractures (Tatham and McCormack, 1991, page 35).

When deformed by tectonic forces, originally horizontal strata may form dipping beds and thrust-sheets where the axis of symmetry is tilted. The result is referred to, commonly, as tilted transversely isotropic medium (TTI) (Issac and Lawton, 1999; Vestrum et. al., 1999). For proper depth imaging in structured media, estimation of TTI parameters is crucial, to ensure correct location of exploration targets (Issac and Lawton, 1999).

For a VTI medium, seismic velocity is defined by five independent elements of the elasticity tensor (Crampin, 1984). Then Thomsen (1986) rearranges the five parameters so that two of the five parameters are in units of velocity rather than elasticity. Thomsen's (1986) parameters are $\alpha_0$ (vertical P-wave velocity), $\beta_0$ (vertical S-wave velocity), $\varepsilon$ (a measure of P-wave anisotropy), ? (parameter controlling near vertical propagation) and $\gamma$ (a measure of S-wave anisotropy). By vertical incidence we measure incidence parallel to the axis of symmetry. P-wave and SV-wave propagation in the plane containing the symmetry axis does not depend on parameter $\gamma$, and depends, therefore, on parameters $\alpha_0$, $\beta_0$, $\varepsilon$ and ? alone (Thomsen, 1986). Further simplification for P-wave propagation follows from the assumption of 'weak' anisotropy where $\beta_0$ is negligible (Thomsen, 1986).

Time domain velocity analysis in isotropic layered media involves estimating the P-wave normal moveout velocity ($Vp_{NMO}$) that fits the short-spread hyperbolic moveout (Dix, 1955). For a weak VTI medium, short-spread moveout remains hyperbolic (Tsvankin and Thomsen, 1994) and the NMO velocity that fits the hyperbolic moveout is a function of $\alpha_0$ and ?. Because moveout velocity $Vp_{NMO}$ and $\alpha_0$ are not related directly, inversion of $Vp_{NMO}$ in TI media yields erroneous layer thicknesses and interval velocities, and an irresolvable velocity-depth ambiguity occurs (Tsvankin and Thomsen, 1994). The same observation holds for a layered sequence of weak VTI formations where root-mean-squared (rms) vertical velocity $Vp_{RMS}$ takes the form of $\alpha_0$, and average anisotropy takes the place of ?. (Tsvankin and Thomsen 1995). For acquisition of very long offset data (offset/depth >1), Tsvankin and Thomsen (1994) provide a quartic moveout coefficient to account for nonhyperbolic moveout for P and Sv waves. Nonhyperbolic P-wave moveout provides two parameters – $Vp_{NMO}$ ( a combination of $\alpha_0$ and ?) and $\eta$ (a combination of $\varepsilon$ and ?) called anellipticity (Tsvankin and Thomsen, 1994). In weak anisotropic media, we cannot obtain $\alpha_0$, $\varepsilon$ and ? separately as needed for depth imaging. To accomplish this, we need long offset P and Sv data jointly. But for the long spread Sv wave, the quartic moveout coefficient may fail at very large offsets. Though the time domain analysis is exhaustive in nature, it makes several approximations based on weak anisotropy assumption and the moveouts based on estimated coefficient deviate from the actual moveout at very large offsets (Tsvankin and Thomsen, 1995).

In this paper, we perform traveltime sensitivity analysis to investigate the feasibility of estimating elastic parameters, dip, and layer thickness for transversely isotropic subsurface using reflection seismic data. Our study separately takes into account the effect of different experimental conditions like offset to depth ratios, availability of multi-component data, and a priori knowledge of subsurface geometry. Sensitivity analysis is a useful tool for anisotropy estimation as it examines the impact of measurement errors in traveltime data to the estimates of subsurface model parameters. It also includes theoretical uncertainty (prediction error) caused by the physics of the forward problem (Sen and Stoffa, 1995). Knowing these, resources of data acquisition can be better applied to give least erroneous estimates possible if we design acquisition such that prediction error has a very sharp minimum in the vicinity of the estimated solution. The solution will be well determined and precise in the sense that it has small variance. Conversely, where prediction error has a broad minimum, the estimated solution has large variance, and the estimated parameters are highly uncertain.

The variance of an estimator is related to the curvature of prediction error at its minimum through the second derivative of prediction error. When a linear inverse problem is solved in a least square sense, the second derivative of prediction error can be computed directly from the Hessian. In reflection seismology, traveltime is not linearly related to anisotropic parameters, however, in the vicinity of a solution, the problem can be set up as a linear relationship between model parameter update and error in traveltime, e.g., using a generalized

Newton's method (Ferguson and Sen, 2004). Using this linearization, uncertainty in parameter updates, and hence the parameters, can be computed directly from the Hessian, and a limited number of iterations performed until the parameter updates are close to zero (Ferguson and Sen, 2004). For our analysis, we set up the forward problem in the plane wave (?-p) domain (Stoffa et. al. 1981; Sen and Mukherjee 2003; Ferguson and Sen 2004), where ? is intercept time and p is horizontal slowness. The plane wave domain is advantageous as exact analytical expressions for ? exist in terms of p and the desired anisotropy parameters (Daley and Hron, 1977). Extension to multi-layer problems is straightforward and requires only the algebraic sum of delay times for individual layers.

The results of our analysis are relevant in context of pre-stack domain parameter estimation using a layer-stripping approach. For example, in common focus point (CFP) technology (Berkhout, 1997), traveltime error is directly used for parameter estimation. For each layer, a focusing operator is computed using a model of elastic parameters with which a CFP gather can be constructed using seismic data. Assuming local homogeneity, the resulting differential time shifts (DTSs) represent error in the model due to anisotropy and error in thickness (Ferguson and Sen 2004). In (?-p) domain, the DTSs are intercept time errors (??) that connect error in layer thickness $z$, vertical slowness $q$ and ray parameter $p$. The vertical slowness on the other hand is a function of $p$ and anisotropic parameters.

Our analysis shows quantitatively the limitation of P-wave data in resolving anisotropic parameters when depth of reflector is unknown. When the reflector depth is known, anisotropic parameters can be reasonably constrained for large offset to depth ratio. When the tilt of symmetry axis is known and is large in magnitude, anisotropic parameters as well as depth can be reasonably constrained. The most important outcome of our analysis is that the anisotropic parameters as well as depth can be constrained if we use limited offset P-P (incident P, reflected P) and P-SV (incident P, reflected SV) data jointly. This holds true for joint inversion of P-P and SV-SV data as well, the later being better constrained than the former.

## THEORY

Sensitivity analysis helps us quantify uncertainty associated with an estimated model parameter. In reflection seismology, for an assumed subsurface model (**m**), the estimated arrival time of reflections at different offsets may not be the same as those in the recorded data. Error in trveltime (**e**) is a vector of the discrepancy between estimated and recorded traveltimes at different offsets. Prediction error for model **m** is defined as $E(\mathbf{m}) = \mathbf{e}^T\mathbf{e}$ and is a scalar. Figure 1 shows a plot of traveltime prediction error against estimated P-wave velocity for a flat reflector in a homogeneous isotropic medium when (i) depth is fixed, and (ii) when depth is allowed to vary within a certain range. In the later case, the value of minimum traveltime error within the given depth range is plotted. We can see that the predication error of this over-determined problem has a sharp minimum in the vicinity of estimated parameter (P-wave velocity) for case (i) indicating that the solution is well-determined in the sense that it has small variance. For case (ii) E (**m**) has a broad minimum indicating large variance of the estimated parameter and hence poorly determined solution (Menke, 1984). The curvature of the error function is a measure of the sharpness of its minimum and the variance of the solution is related to the curvature. Hence, an estimate of how well a solution is constrained can be obtained from the curvature of prediction error. The curvature of the prediction error can be measured by its second derivative. For the model space **m**, the sensitivity matrix is given by

$$\mathbf{S} = \frac{\partial^2 E}{\partial m_i \partial m_j} \tag{1}$$

, where the i, j partial derivatives are with respect to the i and j model parameter **m**.

In the plane wave domain, expressions for vertical slowness for quasi P-waves ($q_p$) and quasi SV-waves ($q_{Sv}$) in terms of Thomsen's parameters are given by,

$$q_p = \frac{1}{2}\sqrt{2\beta_0^{-2} + 2\alpha_0^{-2} - 4Sp^2 - 4R} \tag{2}$$

$$q_{sv} = \frac{1}{2}\sqrt{2\beta_0^{-2} + 2\alpha_0^{-2} - 4Sp^2 + 4R} \tag{3}$$

where,

$$S = \frac{1}{2}\frac{\alpha_0^2}{\beta_0^2}\left[\varepsilon - \delta^*\right] + \frac{1}{2}\varepsilon + 1, \tag{4}$$

and

$$R = \frac{1}{2}\sqrt{4p^4\left[S^2 - 2\varepsilon - 1\right] + 4\left(\frac{p}{\beta_0}\right)^2\left[2\varepsilon - S + 1\right] + 4\left(\frac{p}{\alpha_0}\right)^2\left[1 - S\right] + (\beta_0^{-2} - \alpha_0^{-2})^2} \quad . \tag{5}$$

Vertical traveltime ($\tau_P$) for P-wave (incident P, reflected P) is given by,

$$\tau_p = 2zq_p \quad , \tag{6}$$

and vertical traveltime ($\tau_{Sv}$) for Sv-wave (incident Sv, reflected Sv) is given by,

$$\tau_{Sv} = 2zq_{Sv} \,. \tag{7}$$

For a converted P-SV-wave, vertical traveltime ($\tau_{P-Sv}$) (incident P, reflected Sv) is given by,

$$\tau_{P-Sv} = z(q_P + q_{Sv}) \quad . \tag{8}$$

Exact analytical expressions for partial derivatives of vertical traveltime with respect to the model parameters exist, using equations 6, 7, 8 and equations 1 and 2, for use in a generalized Newton's equation (9) to obtain model updates.

$$\mathbf{G}\Delta\mathbf{m} = -\Delta\boldsymbol{\tau} \,, \tag{9}$$

where the columns of **G** are the partial derivatives of vertical traveltime (?$_p$)with respect to the elastic parameters for observation points 1 to n. The model parameter update vector is given by $\Delta\mathbf{m}$ and $\Delta\boldsymbol{\tau}$ is the vector of vertical traveltime errors. Error ($E$) for an estimated model parameter is given by

$$E = \Delta\boldsymbol{\tau}'\,\Delta\boldsymbol{\tau} \;, \tag{10}$$

where ' indicates transpose.

Without assuming weak TI, the model space for VTI medium is given by $\mathbf{m}=[\alpha,\beta,\varepsilon,\delta^*,z]$, and for tilted TI medium is $\mathbf{m}=[\alpha,\beta,\varepsilon,\delta^*,z,\theta]$, where ? is tilt angle of symmetry axis with respect to vertical.

It can be shown that **S** (equation 1) is approximately equal to twice $\mathbf{G}^T\mathbf{G}$ and the covariance matrix is given by (Menke, 1984),

$$[Cov\;(\mathbf{m})] = \sigma^2\left[\mathbf{G}^T\mathbf{G}\right]^{-1} = \sigma^2\left[\frac{1}{2}\mathbf{S}\right]^{-1} \tag{11}$$

where, ? is the standard deviation of data.

The uncertainty associated with the model parameter $m_i$ equals squared root of $i^{th}$ diagonal element of covariance matrix. In this way, the curvature of error surface is mapped to the uncertainty in model parameter estimate.

## RESULTS

Sensitivity analysis is performed for Mesaverde clayshale (Thomsen, 1986), whose anisotropic parameters are $\alpha_0$=3794 m/s, $\beta_0$=2074 m/s, $\varepsilon$=0.189, and ?=0.204. Table 1 lists the uncertainty in the estimates of the anisotropic parameters for different types of data. The only cases when anisotropic parameters can be estimated with a reasonably small uncertainty are P-P data with known depth, joint P-P and P-Sv data, and joint P-P and Sv-Sv data. It can be seen that for all these three cases, uncertainty in estimates decreases with an increase in offset to depth ratio (Figure 2, 3 and 4). The joint inversion of either P-P and Sv-Sv data or P-P and P-Sv data provides improved estimates of model parameters even when depth is not known and is considered one of the model parameters to be estimated.

| Data Type | %age uncertainty in elastic parameters | | | | |
|---|---|---|---|---|---|
| | $?_0$ | $?_0$ | ? | $?^*$ | z |
| PP (fix z, weak TI) | 0.12 | - | 16.54 | 34.6 | - |
| PP (fix z, strong TI) | 0.15 | $9.3*10^3$ | 103 | 7961 | - |
| PP (var z, weak TI) | 2431 | $1.8*10^4$ | $1.4*10^4$ | 2431 | - |
| PP (var z), strong TI) | $4.3*10^5$ | $1.2*10^6$ | $3.1*10^6$ | $3.4*10^6$ | $4.3*10^5$ |
| PSv (var z, strong TI) | $4.9*10^4$ | $1.1*10^4$ | $3.5*10^5$ | $5.5*10^5$ | $1.0*10^4$ |

| | | | | | |
|---|---|---|---|---|---|
| PP-SvSv (var z, strong TI) | 1.39 | 1.42 | 12.5 | 5.3 | 1.4 |
| PP-PSv (var z, strong TI) | 10.3 | 10.3 | 85.5 | 47.1 | 10.3 |

Table 1. Uncertainty in elastic parameter estimates for different types of data as calculated analytically. Maximum offset to depth ratio is 1.5 and standard deviation is 4 ms.

A similar analysis has been performed for tilted TI medium when angle of tilt of symmetry axis is known, say from geology of overlying formations. It can be seen (Table 2) that using only P-wave data it is feasible to estimate ?$_{0,}$ ?, ?$^{*}$ and z if the tilt angle is known and is large in magnitude. For a near zero tilt of symmetry axis, the uncertainty in the estimation of anisotropic parameters ?$_{0,}$ ?, ?$^{*}$ and z for limited offset involves very high uncertainty. We may have a TI medium, in which the angle of tilt of symmetry axis of the medium is laterally variable. In such a case, if the angle of tilt is known at more than one location, a joint inversion of P-wave data from these locations coupled with the knowledge of tilt angles constrain the elastic parameters well (table 2).

When tilt angle of the TI medium is not known, a joint inversion of PP and SvSv data may be used to estimate all the parameters, i.e. ?$_{0,}$ ?$_{0,}$ ?, ?$^{*}$, ? (tilt angle), and z (depth) simultaneously. However, this approach will yield fruitful result only when the TI medium has a reasonable tilt. For a near VTI medium, this approach will result in high uncertainty for certain parameters, when tilt angle is also a model parameter (table 3).

Table 3 shows that uncertainty of parameters decreases with increase in angle of tilt of the symmetry axis. This observation is attributed to the fact that with an increase in tilt angle, the slowness curve becomes more asymmetric (Figures 5). Thus for the same range of ray parameter values, the effects of much larger incidence angles are incorporated. The assumption here is that the layer interfaces are flat even though the symmetry axis of transverse isotropy is not vertical.

| Tilt angles | Uncertainty Estimates (%age) | | | |
|---|---|---|---|---|
| | $?_0$ | ? | $?^*$ | z |
| $1^0$ | 1631 | 1010 | 7875 | 1380 |
| $10^0$ | 41 | 265 | 212 | 33 |
| $20^0$ | 13.5 | 94 | 247 | 8.8 |
| $30^0$ | 1 | 11 | 42 | 1 |
| $40^0$ | 1.7 | 15.7 | 31 | 1.18 |
| $0^0,10^0$ | 4 | 25 | 24 | 3.3 |
| $0^0,20^0$ | 1 | 7 | 11 | 0.8 |
| $0^0,10^0,20^0$ | 1 | 7 | 11 | 0.8 |
| $0^0,30^0,50^0$ | .1 | 0.9 | 6.3 | 0.15 |

Table 2. Uncertainty in elastic parameter estimates for different tilt angles for a TTI medium using P-wave data only when tilt angle of the TI medium is known.

| Tilt Angle | Uncertainty in %age for | | | | | |
|---|---|---|---|---|---|---|
| | ?$_0$ | ?$_0$ | ? | ?$^*$ | Tilt angle | z |
| $1^0$ | 27 | 56 | 195 | 107 | 1431 | 26 |
| $10^0$ | 2 | 1 | 24 | 32 | 140 | 1 |
| $20^0$ | 2 | 5 | 28 | 77 | 19 | 2 |
| $30^0$ | 2 | .3 | 30 | 25 | 10 | .7 |
| $40^0$ | 1 | .4 | 19 | 12 | .2 | .6 |
| $50^0$ | 1 | .4 | 16 | 12 | .2 | .6 |
| $60^0$ | .6 | .3 | 12 | 7 | .1 | .9 |

Table 3: Uncertainty in elastic parameter estimates for different tilt angles for Joint Inversion of PP and SvSv data for offset to depth ratio of 1 and standard deviation of 4 ms. The angle of tilt of TI medium is also a model parameter..

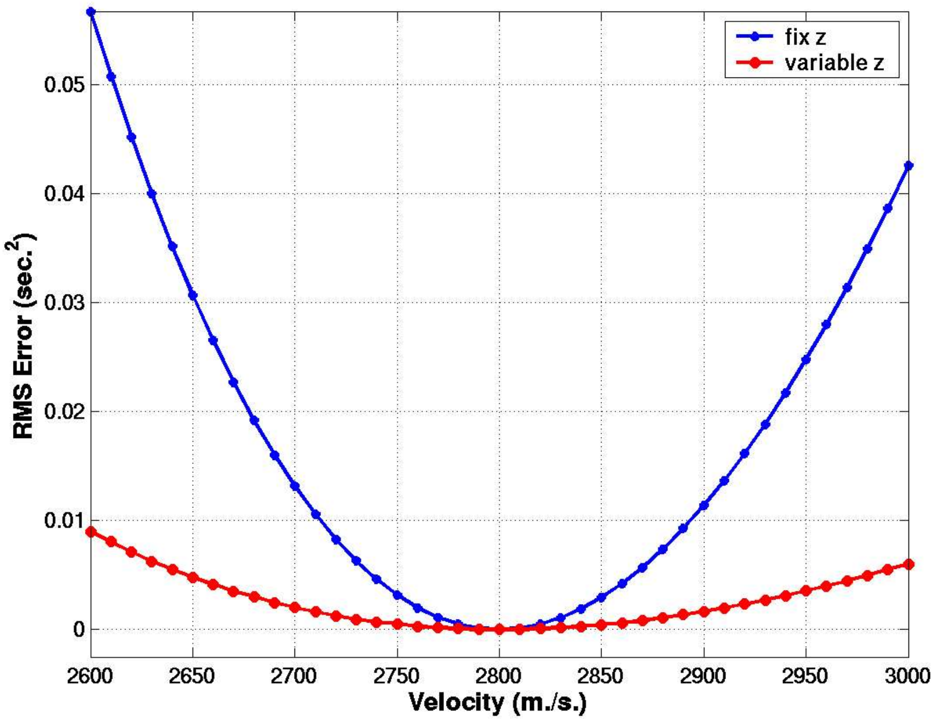


Figure 1. Plot of Vp versus error for fixed z and variable z. For fixed z, error has a sharp curvature at the minimum compared to that for variable z..

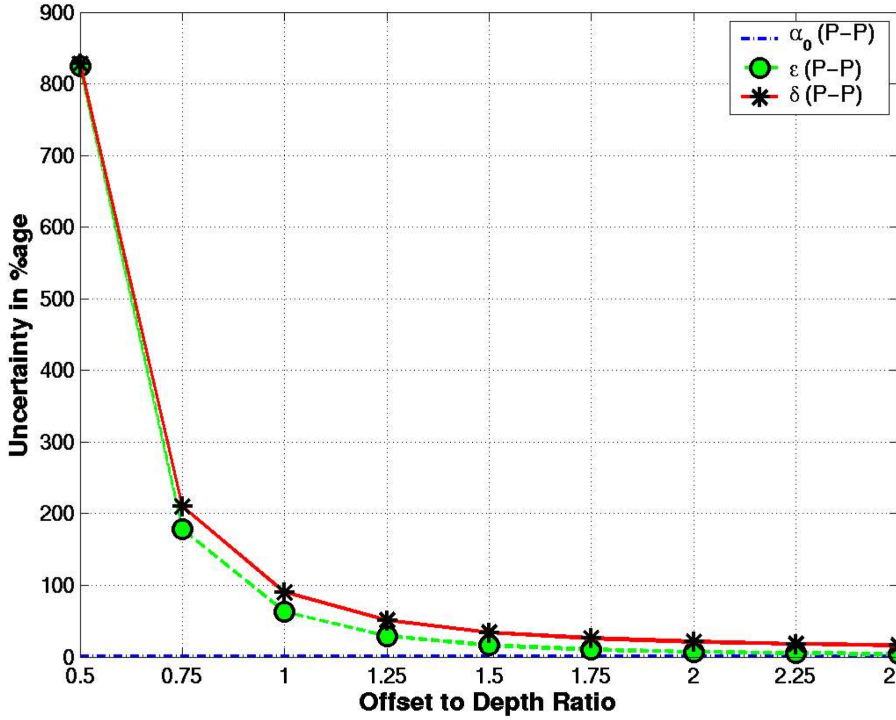


Figure 2: Plot of parameter uncertainty versus offset to depth ratio for P-P data, when depth is known. As the offset to depth ratio increases, parameter uncertainty for ε and ? decreases, while uncertainty for $\alpha_0$ remains low and nearly constant.

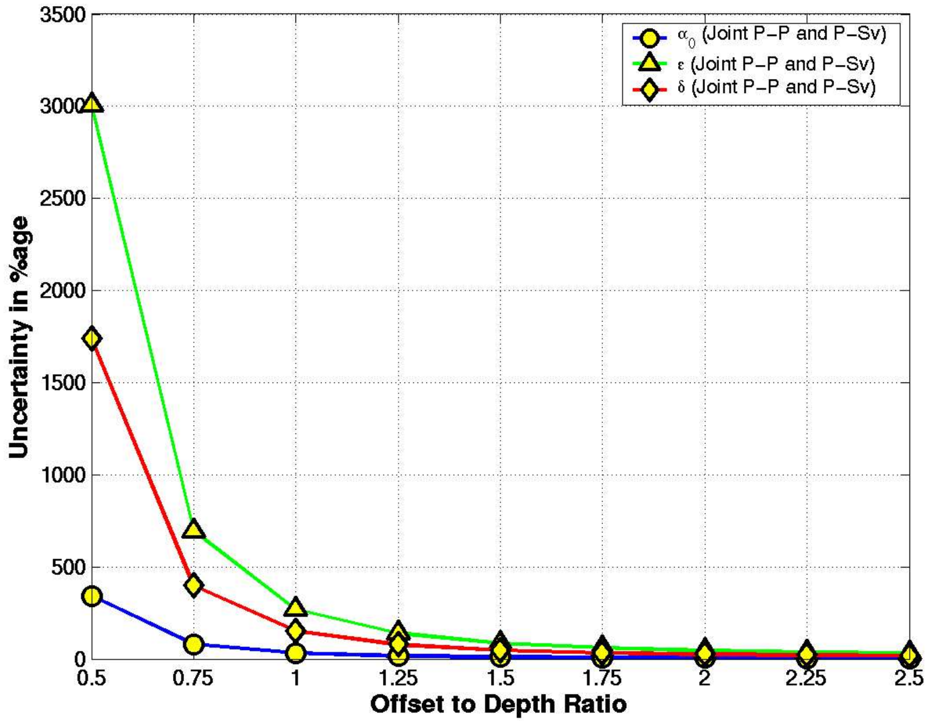


Figure 3: Plot of parameter uncertainty versus offset to depth ratio for joint inversion of P-P and P-Sv data. As the offset to depth ratio increases, parameter uncertainty decreases.

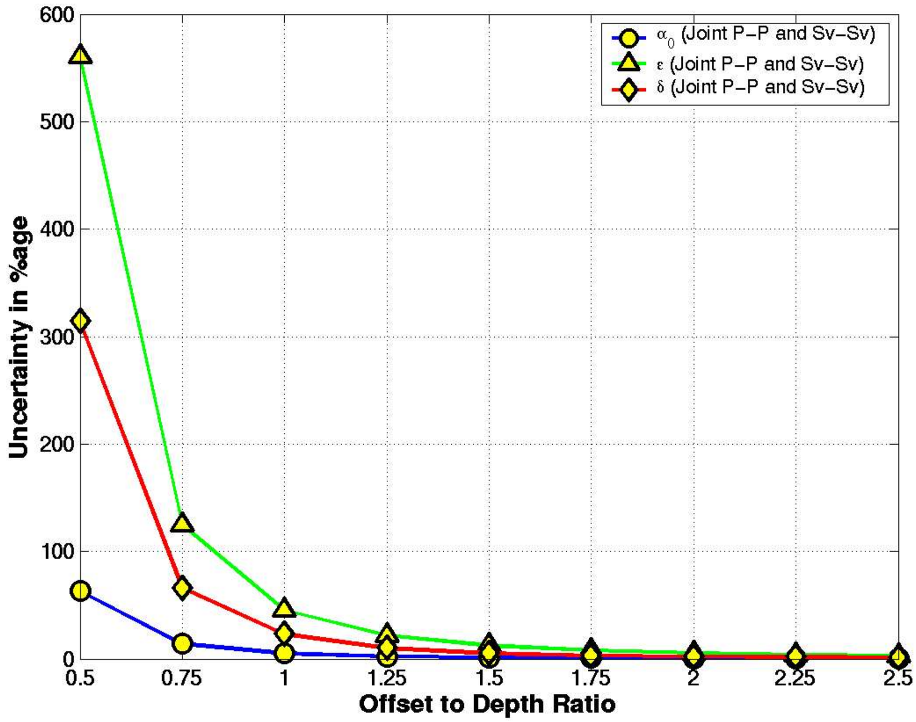


Figure 4: Plot of parameter uncertainty versus offset to depth ratio for joint inversion of P-P and Sv-Sv data. As the offset to depth ratio increases, parameter uncertainty decreases.

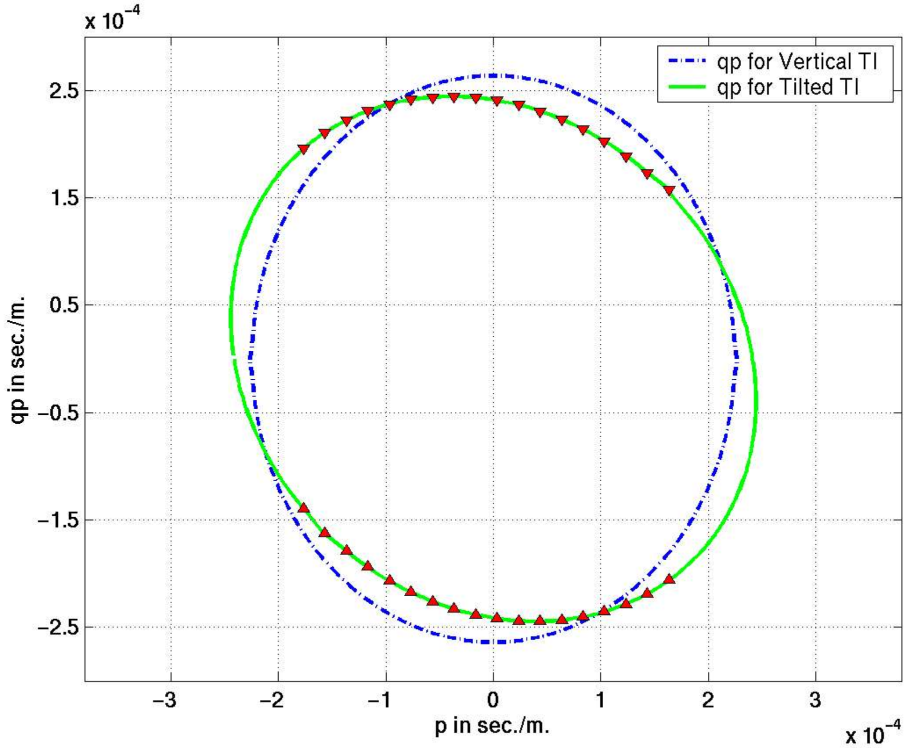


Figure 5. Slowness curves for Mesaverde clayshale. The dotted blue line is slowness curve when symmetry axis is vertical. The continuous green line is slowness curve when symmetry axis is tilted by $45^0$ w.r.t. vertical. The '$\nabla$'s are slownesses for downgoing waves and '$\Delta$'s for upgoing waves corresponding to a certain range of ray parameters.

## SUMMARY

The sensitivity analysis we have used in this paper is useful as it helps quantify the limitations of resolving anisotropic parameters for VTI media. We find that the estimation of anisotropic parameters as well as depth is feasible if we perform joint inversion of P-P and Sv-Sv data or P-P and P-Sv data. Though, for typical acquisition geometry, P-P or Sv-Sv or P-SV data alone cannot predict anisotropic parameters, an increase in offset to depth ratio or increase in known tilt angle of symmetry axis, adds confidence to the parameter estimates. For tilted TI medium overlying a flat reflector, joint inversion of P-P and Sv-Sv data, however, may help estimating depth, anisotropic parameters, and tilt angle simultaneously.


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## References


Berkhout, A. J., 1997a, Pushing the limits of seismic imaging, Part I: Prestack migration in terms of double dynamic focusing: Geophysics, Vol. 62, p. 937-953.

Crampin, S., 1984, Anisotropy in exploration geophysics: First Break, 2, No. 3, 19-21.

Daley, P. F., and Hron, F., 1977, Reflection and transmission coefficients for transversely isotropic media: Bull., Seis. Soc. Am., 67, 661-675.

Dix, C. H., 1955, Seismic velocities from surface measurements: Geophysics, 20, 68-86.

Ferguson, R. J., and Sen, M. K., 2004, Estimating the elastic parameters of anisotropic media using a joint inversion of P-wave and SV-wave traveltime error, Geophysical Prospecting, 52, 547-557.

Isaac, J. H., and Lawton, D. C., 1999, Image mispositioning due to dipping TI media: A physical model seismic modeling study: Geophysics, 64, 1230-1238.

Menke, W., 1984, Geophysical Data Analysis: Discrete Inverse Theory, Academic Press, Inc., Chapter 3, 58-60.

Sen, M. K., and Stoffa, P. L., 2005, Global optimization methods in geophysical inversion, Elsevier Science Publishing Company, The Netherlands.

Sen, M. K. & Mukherjee, A., 2003, ?-p analysis in transversely isotropic media: Geophys. J. Int., 154, 647-658.

Stoffa, P. L., Buhl, P., Diebold, J. B., and Wenzel, F., 1981, Direct mapping of seismic data to the domain of intercept time and ray parameter – A plane-wave decomposition, Geophysics, 46, 255-267.

Tatham, R., and McCormack, M. D., 1991, Multicomponent seismology in petroleum exploration, SEG, Tulsa, Oklahoma.

Thomsen, L. 1986, Weak elastic anisotropy, Geophysics, 51, 1954-1966.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media, Geophysics, 59, 1290-1304.

Tsvankin, I., and Thomsen, L., 1995, Inversion of reflection traveltimes for transverse isotropy, Geophysics, 60, 1096-1108.

Vestrum, R. W., Lawton, D. C., and Schmid, R., 1999, Imaging structures below dipping TI media: Geophysics, 64, 1239-1246.