0-000 WAVE PROPAGATION IN HETEROGENEOUS MEDIA BY A LEAST SQUARES ONE WAY OPERATOR

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Abstract

In seismic modeling and imaging, one-way operators are used to estimate wavefields in the subsurface by extrapolating recorded wavefields. The most general one-way operator for isotropic media takes the form of a pseudo-differential operator that is a function of vertical slowness q, and the vertical distance Δz between the recording location and the source location. It is assumed that q varies in the lateral coordinates x and y, but is invariant over Δz.

The general extrapolation operator is still only an approximation, and we use least squares to improve accuracy. Using a seismic modeling algorithm, we implement the least-squares operator and extrapolate a point source through a strongly heterogeneous medium. We compare the resulting wavefields to the expected wavefield obtained analytically. We find that the least-squares approach provides a superior result to the general operator but for much higher computational cost. We derive a series approximation to the least-squares operator that suggests for a controllable loss of accuracy, large increases in computational efficiency can be realized.

Introduction

In a seismic array, geophones are located relative to three orthogonal axes x_1, x_2 and z. Two of them, x_1 and x_2 represented in short by x, define a plane normal to the approximate direction of the Earth's gravity near the center of the geophone array. Orthogonal to x, axis z represents depth into the Earth.

We will represent the seismic waves recorded in the geophone array by $\psi\left(x,z,t\right)$, where t is an axis representing the passage of time since excitation of the source. Fourier transform of ψ is

$$\psi\left(x,z,\omega\right)=\frac{1}{2\pi}\int\psi\left(x,z,t\right)e^{i\omega t}dt, \tag{1}$$

where ω is temporal frequency, and the limits of integration are defined by the compact support of t. Subsequent operations on ψ are general in ω so the ω notation is omitted hereafter.

Extrapolation of ψ from z at the surface to $z+\Delta z$ in the subsurface can be computed using

$$\left[\mathbf{P}\left(\Delta z\right)\psi\left(y,z\right)\right]\left(x,z+\Delta z\right)=\frac{1}{\left(2\pi\right)^2}\int e^{-i\langle x-y,\xi\rangle}c\left(x,\xi,\Delta z\right)\psi\left(y,z\right)d\xi dy, \tag{2}$$

[2, 1] where $\mathbf{P}$ represents the pseudo-differential operator that transforms input $\psi\left(y\right)$ from input coordinates $\left(y,z\right)$ to output coordinates $\left(x,z+\Delta z\right)$, and

$$c\left(x,\xi,\Delta z\right)=e^{i\Delta z\omega q\left(x,\xi\right)}, \tag{3}$$

where q is vertical slowness representing the heterogeneity and anisotropy of seismic velocity in the medium spanned by Δz.

Important in the development of the next section is the adjoint $\mathbf{N}$ of the above operator:

$$[\mathbf{N}(-\Delta z)\psi(x,z)](y,z-\Delta z)=\frac{1}{(2\pi)^2}\int e^{i\langle x-y,\xi\rangle}c(x,\xi,-\Delta z)\psi(x,z)\,d\xi dy. \tag{4}$$

Equations (2) and (4) differ in the sign of the depth interval Δz, as well as in the sign of the exponent of the Fourier kernal, and on the spatial dependence of c.

For visual comparison, an input seismic record consisting of four impulses are extrapolated a large distance through a strongly variable velocity field using the operators given described above. The resulting impulse responses for $\mathbf{N}$ and $\mathbf{P}$ are given in Figures 1 (a) and (b) respectively. To aid comparison, the velocity function is chosen so that the analytic solution exists and so the exact impulse responses can be plotted (blue lines). Note the significant departures of the extrapolated results from the analytic results for both $\mathbf{N}$ and $\mathbf{P}$.

Implicit wavefield extrapolation operator

As demonstrated above, $\mathbf{P}$ and $\mathbf{N}$ can be inaccurate in extrapolating wavefields long distances through strong velocity variation. As a remedy, a least-squares operator can be derived beginning with the following equality:

$$\mathbf{P}(\Delta z/2)\psi(z)=\mathbf{P}(-\Delta z/2)\psi(z+\Delta z). \tag{5}$$

Multiply equation (5) by the adjoint of $\mathbf{P}(-\Delta z/2)$ and estimate $\psi(z+\Delta z)$ by inverting $\mathbf{N}(\Delta z/2)\mathbf{P}(-\Delta z/2)$

$$\psi(z+\Delta z)=[\mathbf{N}(\Delta z/2)\mathbf{P}(-\Delta z/2)]^{-1}\mathbf{N}(\Delta z/2)\mathbf{P}(\Delta z/2)\psi(z). \tag{6}$$

Matrix multiplication and inversion of extrapolator matrices, especially in 3D can be computationally prohibitive. So fast alternatives to the cascade of operators $\mathbf{N}$ and $\mathbf{P}$ are desirable.

Series representation for $\mathbf{NP}$

To achieve runtime improvements to equation (6), we seek a series expansion of the $\mathbf{NP}$ operator. Here, it is convenient to cast operator $\mathbf{NP}$ entirely in the Fourier domain

$$[\mathbf{N}(\Delta z/2)\mathbf{P}(\Delta z/2)\varphi(\eta,z)](\xi,z+\Delta z)=\frac{1}{(2\pi)^2}\int\varphi(\eta,z)\,c\left(x,\eta,\frac{\Delta z}{2}\right)c\left(x,\xi,\frac{\Delta z}{2}\right)e^{i\langle x,\xi-\eta\rangle}dxd\eta \tag{7}$$

where

$$\varphi(\eta,z)=\int\psi(y,z)\,e^{i\langle\eta,y\rangle}dy. \tag{8}$$

Symbol $c(x,\eta,\Delta z/2)$ can be written as a Taylor series in $c(x,\xi,\Delta z/2)$:

$$c(x,\eta,\Delta z/2)=\sum_{m=0}^{\infty}\frac{(\eta-\xi)^m}{m!}\partial_\xi^m c(x,\xi,\Delta z/2), \tag{9}$$

and equation (7) becomes

$$\begin{aligned}&[\mathbf{N}(\Delta z/2)\mathbf{P}(\Delta z/2)\varphi(\eta,z)](\xi,z+\Delta z)=\\&\sum_{m=0}^{\infty}\int\frac{1}{m!}\partial_\xi^m c(x,\xi,\Delta z/2)\,c(x,\xi,\Delta z/2)\times i^m\partial_x^m\left[\psi(x,z)\,e^{i\langle\xi,x\rangle}\right]dx.\end{aligned} \tag{10}$$

Integrating by parts, equation (10) becomes

$$[\mathbf{N}(\Delta z/2)\mathbf{P}(\Delta z/2)\psi(x,z)](\xi,z+\Delta z)=\int\psi(x,z)\,\beta(x,\xi,\Delta z)\,e^{i\langle\xi,x\rangle}dx, \tag{11}$$

where

$$\beta\left(x,\xi,\Delta z\right)=\sum_{m=0}^{\infty}\frac{i^{m}}{m!}\,\partial_{x}^{m}\left[c\left(x,\xi,\Delta z/2\right)\partial_{\xi}^{m}c\left(x,\xi,\Delta z/2\right)\right], \tag{12}$$

and

$$\psi\left(x,z\right)=\frac{1}{\left(2\pi\right)^{2}}\int\varphi\left(\eta,z\right)e^{-i\langle\eta,x\rangle}d\xi \tag{13}$$

Cast entirely in the space domain, equation (11) becomes

$$\left[\mathbf{N}\left(\Delta z/2\right)\mathbf{P}\left(\Delta z/2\right)\psi\left(x,z\right)\right]\left(y,z+\Delta z\right)=\frac{1}{\left(2\pi\right)^{2}}\int\psi\left(x,z\right)\beta\left(x,\xi,\Delta z\right)e^{i\langle\xi,x-y\rangle}d\xi dx. \tag{14}$$

The desired series representation of the effective $\mathbf{NP}$ operator is achieved with equation (14) for use in the least-squares operator of equation (6).

Efficient operators

The series for $\mathbf{NP}$ given by equation (14) allows direct computation of what otherwise is a cascade of two operators, and computational efficiency is achieved by truncating the series. As an example, for $\mathbf{N}\left(\Delta z/2\right)\mathbf{P}\left(\Delta z/2\right)$, to first order, β from equation (12) becomes

$$\beta\left(x,\xi,\Delta z\right)=\left[1-2\,\omega^{2}\Delta z^{2}\,\partial_{x}q(x,\xi)\,\partial_{\xi}q(x,\xi)+i\omega\Delta z\,\partial_{x}\partial_{\xi}q(x,\xi)\right]c(x,\xi,\Delta z). \tag{15}$$

Similarly, for $\mathbf{N}\left(\Delta z/2\right)\mathbf{P}\left(-\Delta z/2\right)$, β becomes

$$\beta\left(x,\xi,\Delta z\right)=1-i\omega\Delta z\,\partial_{x}\partial_{\xi}q(x,\xi). \tag{16}$$

The required ∂_{ξ} derivatives are available analytically, and the ∂_{x} derivatives can be determined using finite differences or Fourier transforms.

Figures 2 (a) and (b) demonstrate the effect of the two end members of the class of least-squares operators derived here. Figure 2 (a) uses the zeroth order operator for $\mathbf{N}\left(\Delta z/\,2\right)\mathbf{P}\left(-\Delta z/\,2\right)$ so that the inverse $\left[\mathbf{N}\left(\Delta z/\,2\right)\mathbf{P}\left(-\Delta z/\,2\right)\right]^{-1}=\mathbf{I}$. The resulting impulse response shows damping of the diffraction tails where they diverge from the analytic curve. One-way operators are inaccurate in strongly heterogeneous media and the approximate-least-squares-operator appears to respond by attempting to damp the error. Figure 2 (b) uses the full series $(0\leq m\leq\infty)$ in equation (12). The damping effect is even more pronounced here when compared to Figure 2. Correct selection of values for $0<m<\infty$ will ensure an optimum tradeoff between accuracy and computational cost.

Conclusions

We develop extrapolation operators that are numerically efficient and robust. In particular, we present a series approximation for wavefield extrapolation by least-squares that allows for optimization of runtime verses accuracy. These operators respond to the error inherent in one-way extrapolation in strongly heterogeneous media by attempting to damp out erroneous wave amplitudes. Representation of the least-squares operator as a series ensures that accuracy and runtime can be modulated by selecting an appropriate number of terms.

References

[1] J. H. Le Rousseau and M. V. de Hoop. Modeling and imaging with the scalar generalized-screen algorithms in isotropic media. *Geophysics*, 66:1551–1568, 2001.

[2] G. F. Margrave and R. J. Ferguson. Wavefield extrapolation by nonstationary phase shift. *Geophysics*, 64:1067–1078, 1999.

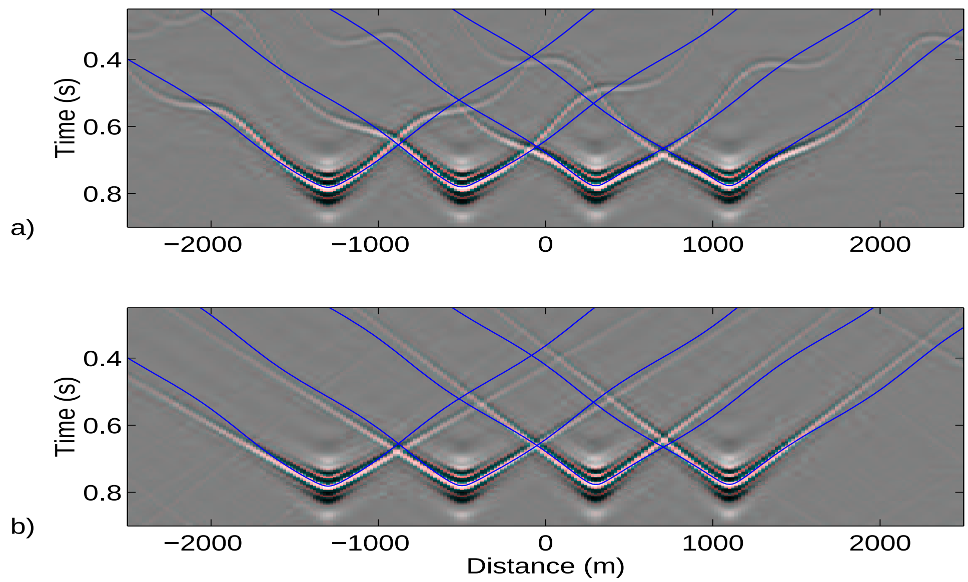

Figure 1: Impulse responses with analytic impulse response overpoltted. (a) $\mathbf{P}$, large error relative to analytic impulse response. (b) $\mathbf{N}$, large error relative to analytic impulse response. The velocity used is a sinusoid varying laterally between 1000 and 3000 m/s. The extrapolation interval is 300m.

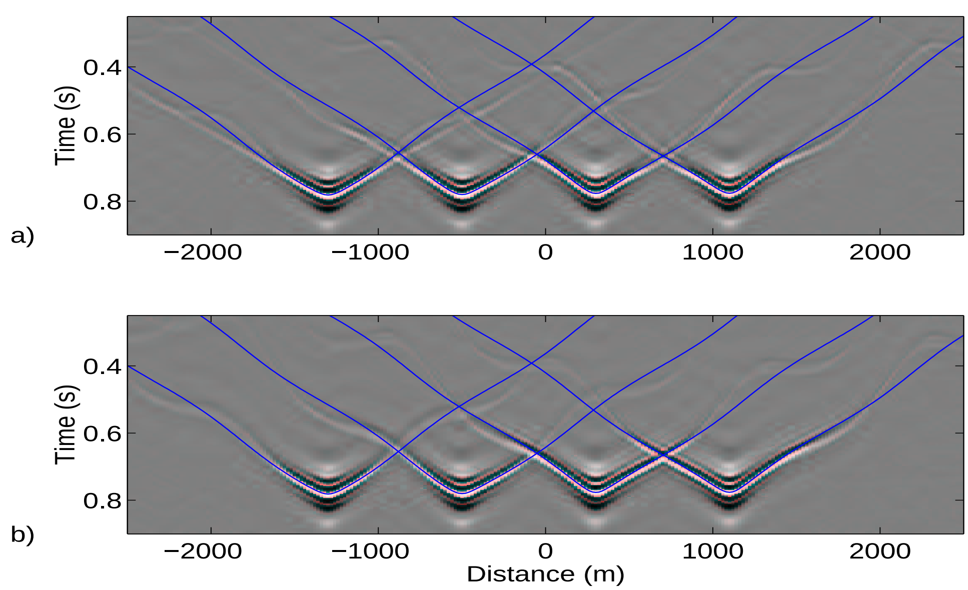

Figure 2: Impulse responses of the least-squares operator with analytic impulse response overpoltted. (a) For a fast operator, assume $[\mathbf{N}^{+}\mathbf{P}^{-}]^{-1} = \mathbf{I}$. The operator still damps the erroneous amplitudes compared to Figures 1a and 1b. (b) Full computation of $[\mathbf{N}^{+}\mathbf{P}^{-}]^{-1}$, provides improved damping when compared to (a) but is much more expensive.