$\tau - p$ domain estimation of elastic parameters in VTI media

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SUMMARY

A method is presented to compute the elastic parameters for a homogeneous medium that is transversely isotropic. The method requires an initial estimate of the parameters and, using a process similar to common focus point (CFP) analysis, arrives at a set of differential time shifts (DTS). That is, the recorded wave£eld, and models of the sources, are propagated to a CFP using extrapolation operators parameterized through q with the initial elastic parameters. Cross correlation of the two wave£elds resolves an angle dependant re¤ectivity estimate (from which the data along zero lag are used in imaging). When the initial parameters are not correct, re¤ection data corresponding to the CFP will not perfectly align at zero lag and, if the medium is homogeneous, the DTS will appear as a symmetric curve over negative and positive offsets. In the $\tau - p$ (Radon) domain, the DTS relate error in q to horizontal wave number p — traveltime error as a function of phase angle. Expansion of $q(p)$ is used to linearize $q(p)$ with respect to error Δ in the elastic parameters α, ε and δ. Least-squares inversion yields $\Delta\alpha$, $\Delta\varepsilon$ and $\Delta\delta$ for use in updating the initial estimate. Comparison of the linear expression with the exact expression for an anisotropic shale shows that error is most sensitive to error in the initial α, followed by δ and is least sensitive to error in ε.

INTRODUCTION

Seismic imaging using the Common Focal Point (CFP) technology (Berkhout and Vershuur, 2001) is a useful tool in exploration seismology. Analysis points are selected in the subsurface that correspond to re¤ectors of interest. Wave-equation operators from depth migration are then used to back propagate the re¤ection data to each CFP (focusing in detection (Berkhout and Vershuur, 2001)). There they are cross correlated with focusing operators computed by forward propagating a model of the seismic source (focusing in emission (Berkhout and Vershuur, 2001)). For each CFP, a CFP gather results where each trace represents the transformation of one common-source gather. If the wave-equation operators exactly mimic wave propagation from the source and receiver locations to the CFP, then energy corresponding to the specular re¤ection at the CFP aligns at zero-lag on the CFP gather. The amplitude of this energy is a measure of the re¤ectivity at the CFP location, and can be analyzed as such, as well summed into an average re¤ectivity for imaging (de Bruin et al., 1990). Misalignment is assumed to be the result of the heterogeneity and anisotropy of the medium above. In strict adherence to the CFP method, misalignment - the differential time shift (DTS) on the CFP is measured and used to update vertical slowness q such that, upon subsequent analysis, alignment is achieved and optimal re¤ectivity should be obtained. Note how the CFP process is independent of having estimated the heterogeneity or anisotropy of the medium above.

Kabir and Vershuur (2000) present a departure from the CFP method by which they use DTS measurements to estimate velocity heterogeneity. They assume an isotropic medium where velocity increases linearly with depth, and use a constrained parametric inversion to deduce lateral velocity variation. Though this method may not improve the resulting image relative to CFP imaging, it has value in providing estimates of velocity intermediate to imaging, for example, to provide constraints for amplitude variation with offset (AVO) inversion.

By assuming a homogeneous medium, a departure from the CFP method can be used to estimate the anisotropy of the medium above the CFP. DTS are picked on the CFP gathers, Radon transformed, and repicked in Radon space. The resulting traveltime error relates variation in q to horizontal wavenumber p. The analytic expression for q in a weak VTI medium (Thomsen, 1986) is then expanded in anisotropic parameters α (the slow-direction P-wave velocity), ε and δ to provide a linear relationship between the DTS and errors Δ in the TI parameters.

METHODOLOGY

Initial values of α, δ and ε can be estimated from re¤ection analysis to provide the starting point for this analysis. Alternatively, an isotropic best £t might be selected, and δ and ε simply set to zero. Then, for each source gather $\psi_R(z=0)$ (subscript R stands for re¤ections) at datum $z=0$ above a given CFP, an estimate of the source $\psi_S(z=0)$ is constructed, and both ψ_R and ψ_S are propagated to the CFP for analysis. Propagation can be achieved with wave-equation operators having the following form ((1999) and (2001a)):

$$\psi_S(x,t,\Delta z) = \frac{1}{2\pi}\int \varphi_S(p,\omega,0)\, e^{i(-\Delta z)\omega q(p)} e^{i\omega[p-t]}\, dp\, d\omega \qquad (1)$$

for the source, where φ_S is the space and temporal Fourier transform of ψ_S at datum $z=0$ above the CFP

$$\varphi_S(p,\omega,0) = \frac{1}{2\pi}\int \psi_S(x,t,0)\, e^{i\omega[p-t]}\, dx\, dt, \qquad (2)$$

ω is temporal frequency, and p is horizontal wavenumber. The expression equivalent to equation (1) for ψ_R is:

$$\psi_R(x,t,\Delta z) = \int \varphi_R(p,\omega,0)\, e^{i\Delta z\omega q(p)} e^{i\omega[p-t]}\, dp\, d\omega, \qquad (3)$$

with φ_R computed using the same transform given in equation (2). In equations (1) and (3), the sign difference on Δz distinguishes focusing in emission ($-\Delta z$) and focusing in detection ($+\Delta z$) (Berkhout and Vershuur, 2001).

q is vertical slowness and contains all of the elastic information of the medium above the CFP. For a weak TI medium (Thomsen, 1986)

$$q(p) = \sqrt{\frac{1-\alpha^2 p^2 (1+2\varepsilon)}{\alpha^2 (1+2\alpha^2 p^2 (\delta-\varepsilon))}}. \qquad (4)$$

Note that this expression for q eliminates the need to estimate S-wave velocity β, and is a fairly simple expression to manipulate (**??**).

For each CFP, a CFP gather is formed by cross correlating $\psi_S(\Delta z)$ and $\psi_R(\Delta z)$:

$$\psi_r(x,t,\Delta z) = \psi_R(x,t,\Delta z) \otimes \psi_S(x,t,\Delta z), \qquad (5)$$

and the resulting traces are arranged by offset. In Figure 1, the results are shown for the above procedure given a single CFP lying at the boundary between two homogeneous anisotropic-halfspaces, with weak TI parameters for the upper halfspace being $\alpha = 3377$, $\varepsilon = 0.2$ and $\delta = -0.282$. Here, q is parameterized with the correct values and crosscorrelation of ψ_S in Figure 1a with ψ_R in Figure 1b yields a CFP gather with re¤ection energy lined up at zero lag Figure (1c). Radon transform of Figure 1c yields an impulse at $\tau = 0$ and $p = 0$ as expected.

If, however, for the same data as above, we parameterize q with the wrong elastic parameters to get $\tilde{q}$, erroneous wave£elds $\tilde{\psi}_S$ (Figure 2a) and $\tilde{\psi}_R$ (Figure 2b) result. In the example given in Figure 2, $\alpha = 4000$

and $\varepsilon = \delta = 0$. Cross correlation of $\tilde{\psi}_S$ (Figure 2a) and $\tilde{\psi}_R$ (Figure 2b) places the re¤ection event at non zero-lag for all offsets on the CFP gather (Figure 2c) and Radon transform returns a curve rather than an impulse (Figure 2d). The DTS between zero lag and the center of the re¤ection are the result of error in the downward component of propagation (the source side), and post-re¤ection propagation (receiver side), so for P-P recordings, the DTS are picked, then halved for analysis (Kabir and Vershuur, 2000) (Figure 2f). Radon transform of the resulting $\frac{1}{2}$ picks converts the DTS times $t(x)$ that vary with distance, to DTS times $\tau(p)$ that vary with ray parameter p. DTS picked from the $\tau - p$ image Figure (2f) provide $\tau(p)$ related to q by:

$$\frac{\tau(p)}{\Delta z} = \left[\tilde{q}(p) - q(p)\right]. \tag{6}$$

Expansion of q by Taylor series provides a useful approximation for inversion. Expanded in α, ε and δ, q is:

$$q(p) = \tilde{q}(p) + \frac{\partial \tilde{q}(p)}{\partial \alpha}\Delta\alpha + \frac{\partial \tilde{q}(p)}{\partial \varepsilon}\Delta\varepsilon + \frac{\partial \tilde{q}(p)}{\partial \delta}\Delta\delta + \cdots. \tag{7}$$

To £rst order in α, ε and δ, q, substitution of (7) into (6) gives

$$\frac{\tau(p)}{\Delta z} \sim \frac{\partial \tilde{q}(p)}{\partial \alpha}\Delta\alpha + \frac{\partial \tilde{q}(p)}{\partial \varepsilon}\Delta\varepsilon + \frac{\partial \tilde{q}(p)}{\partial \delta}\Delta\delta, \tag{8}$$

where $\partial_{\alpha,\delta,\varepsilon} q(p)$ are given, for example, by (Rousseau and de Hoop, 2001b)

The error associated with equation (8) (itself a measure of error) is examined in Figure 3. Here, for an anisotropic shale (again, $\alpha = 3377\frac{m}{s}, \varepsilon = 0.2$ and $\delta = -0.282$), the elastic parameters are individually perturbed +- 50 % from their correct value with the two remaining parameters £xed at their correct values. For example, in Figure 3a, α is varied between $1688\frac{m}{s}$ and $5065\frac{m}{s}$ while δ and ε are £xed at 0.2 and -0.282 respectively. Comparing Figures 3a, b and c show that, for this material, equation (8) is a good approximation when error in the anisotropic parameters lies within $\sim 20\%$ of the true values. This evaluation should be repeated as different materials are introduced.

Equation (8) is a linear relationship between $\tau(p)$, and vector form can be written:

$$\tau = \mathbf{Ab} \tag{9}$$

Where $\mathbf{A}$ is $\Delta z \times$ an $m \times 3$ matrix of the partial derivatives of q evaluated using estimates of α, ε and δ, and $\mathbf{b}$ is a 3 element vector containing the errors $\Delta\alpha, \Delta\varepsilon$, and $\Delta\delta$ in the corresponding estimate. Least-squares inversion of equation (9)

$$\mathbf{b} = \left[\mathbf{A}^T\mathbf{A}\right]^{-1}\mathbf{A}^T\tau \tag{10}$$

yields correction terms $\mathbf{b}$ suitable for updating the values of the initial estimates.

CONCLUSIONS

Inversion of differential time shifts (DTS) that have been Radon transformed can be used to estimate the elastic parameters of homogeneous VTI media. The Radon transform is used to relate error in vertical wavenumber q to horizontal wavenumber p, and a series expansion of q is used to derive a linear relationship between q and the errors Δ in the elastic parameters α, ε and δ. This relationship can be inverted by least-squares to deduce the desired parameters from traveltime errors repicked in the Radon domain. The linear approximation is compared to the analytical description and, for an anisotropic shale, is found to be accurate for errors of +- 20 % in the elastic parameters. Errors in α are found to have the greatest deleterious effect on the validity of the expansion, followed by δ with errors in ε having the least impact.

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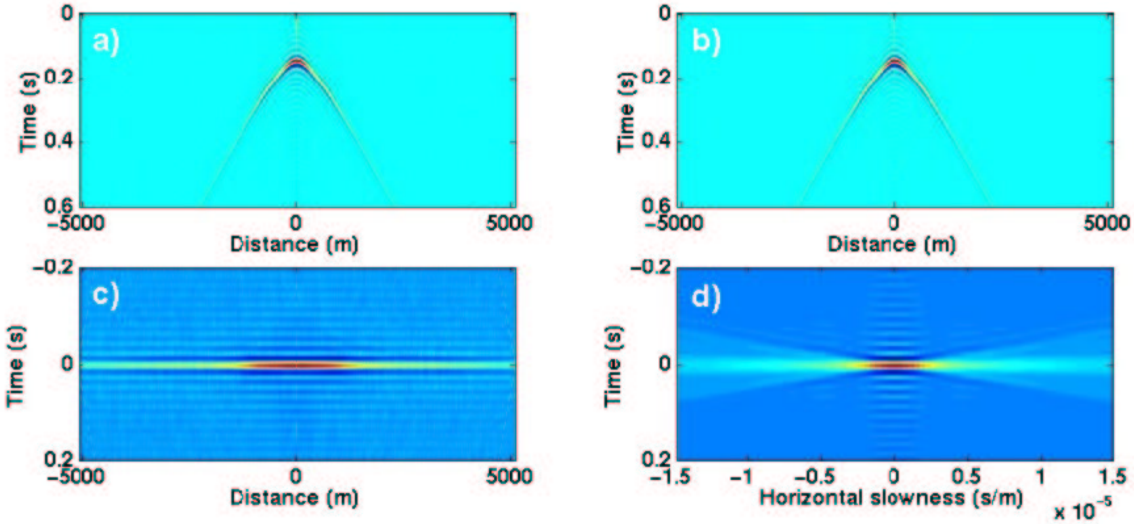

Figure 1: a) Focusing operator computed with correct elastic parameters $\alpha = 3377\,\frac{m}{s}$, $\varepsilon = 0.2$ and $\delta = -0.282$ for a single CFP. b) Back propagated traces corresponding to the same CFP and elastic parameters as in a). c) Cross correlation of a) and b) — note alignment of re¤ection amplitudes at zero lag. d) Radon transform of c) — a ¤at event at zero lag transforms to an impulse centered on $\tau = 0$, $p = 0$.

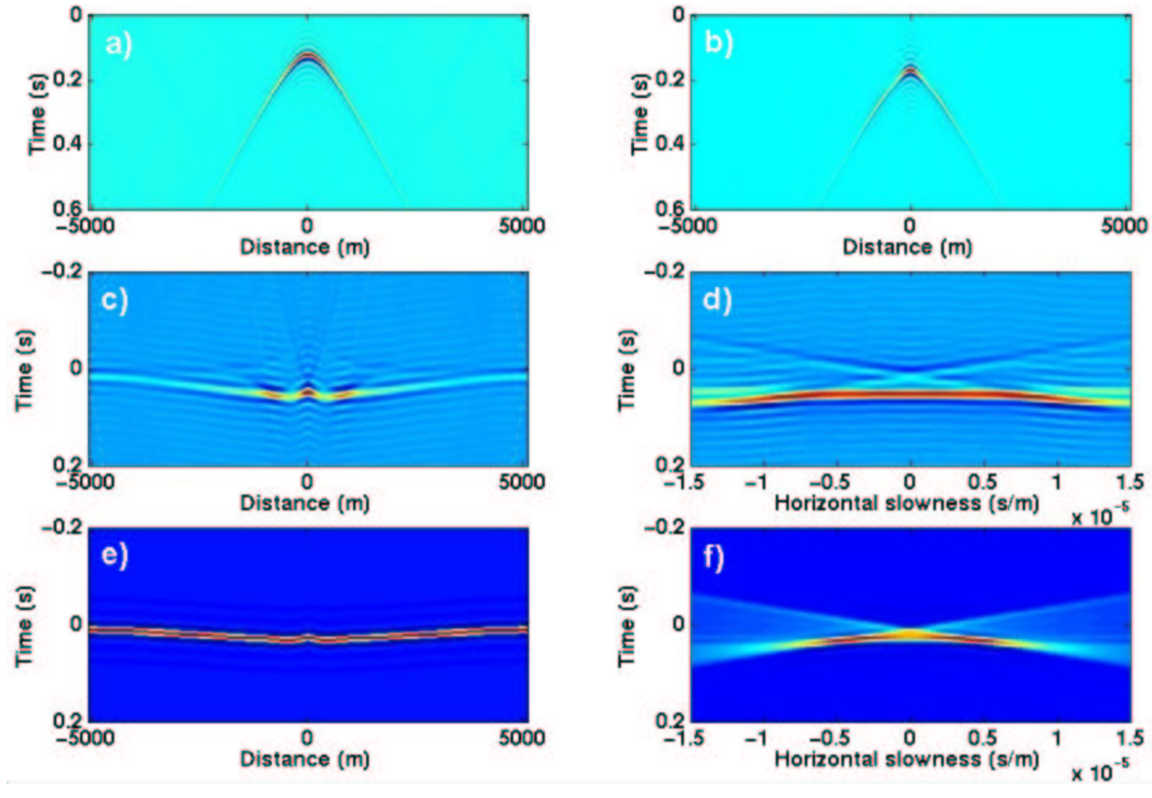

Figure 2: a) Focusing operator computed with incorrect elastic parameters $\alpha = 4000\,\frac{m}{s}$ and $\varepsilon = 0 = \delta = 0$ for a single CFP. b) Back propagated traces. c) Cross correlation of a) and b). Due to the error in elastic parameters, re¤ection amplitudes misaligned at zero lag. d) $\tau - p$ transform of c). e) DTS picked on c) divided by 2. f) $\tau - p$ transform of e). DTS picked on e) can be related to a linearized expression for $\tau(p)$ that can be inverted to estimate the errors Δ in α, ε and δ.

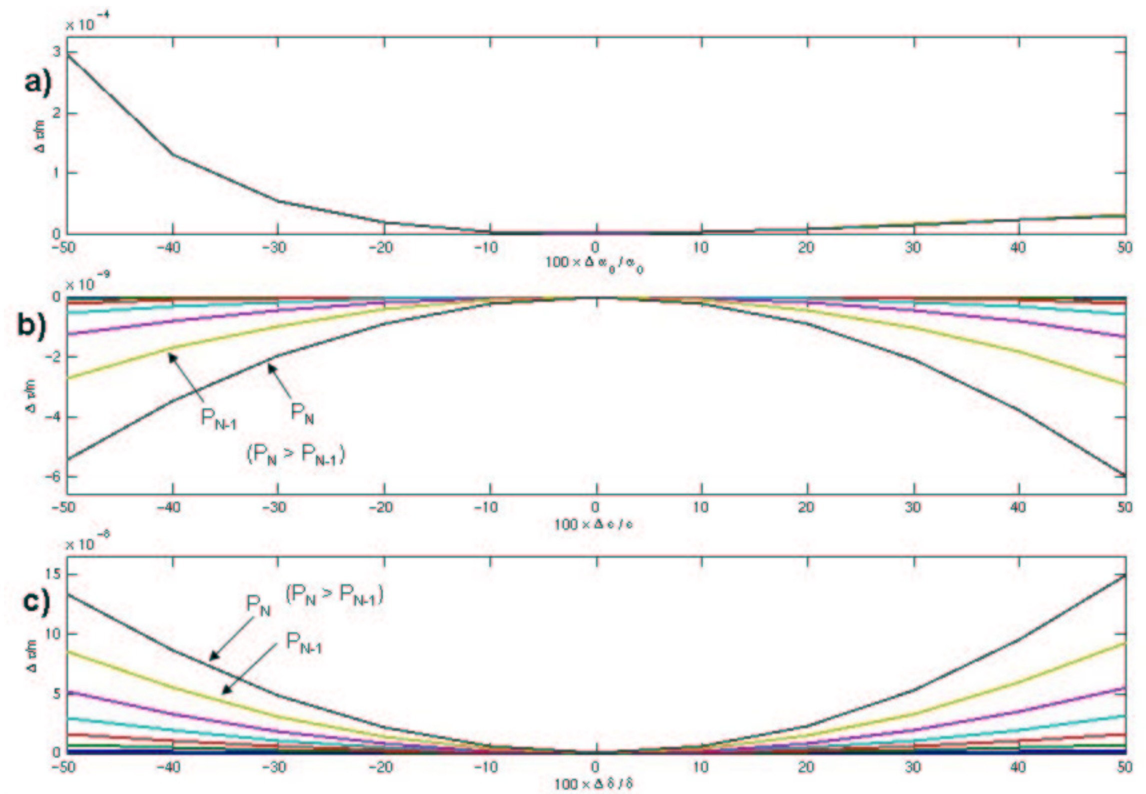

Figure 3: Error due to the linear approximation equation (8) of equation (6) computed by individually perturbing the elastic parameters α, ε and δ +- 50 % from their correct values (in this example, $\alpha = 3377$, $\varepsilon = 0.2$ and $\delta = -0.282$). a) α varies between 1688 $\frac{m}{s}$ and 5065 $\frac{m}{s}$. b) ε varies between 0.1 and 0.3. c) δ varies between -0.141 and -0.423. For this material, equation (8) is a good approximation for parameters within $\sim$ 20 % of their true values. Error sensitivity is greatest for α, less so for δ and least sensitive for ε. Multiple values for p are represented in these images, with $|\Delta|$ increasing with p for ε and δ. α is insensative to p.