Reflection Travel Time Sensitivity Analysis for VTI media

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Summary

We perform reflection traveltime sensitivity analysis for a VTI medium for P-P data, P-Sv data, Joint P-P and P-Sv and Joint P-P and Sv-Sv data to quantify the limitations in resolving the medium anisotropic parameters. Our analysis is performed in the plane-wave domain using exact analytical expressions for slowness as a function of anisotropic parameters. We find, in general that an increase in offset to depth ratio improves our ability to resolve anisotropic parameters. For a typical value of offset to depth ratio, the P-P data or P-Sv data alone do not provide unique estimates of anisotropic parameters and depth. Joint analysis, however, of P-P and Sv-Sv data, and of P-P and P-Sv data provide highly resolved estimates. If the angle of tilt of a tilted TI medium is known and is significantly large, P-P data alone can provide unique estimates of anisotropic parameters as well as depth. Joint inversion of P-P and Sv-Sv data, however, may help estimating depth, anisotropic parameters, and tilt angle as well. We support our observations using error plots based on a global search in model space.

Introduction

Sedimentary rocks often exhibit vertical transverse isotropy (VTI) (Helbig 1994),so an anisotropic velocity model is required for proper depth imaging. For VTI media, the absence of depth information leads to non-uniqueness and uncertainty in estimation of elastic parameters using P-wave data alone (Tsvankin and Thomsen 1994, Tsvankin and Thomsen 1995). This paper investigates the feasibility of using S-wave data in conjunction with P-wave data for resolving the non-uniqueness and quantifies the parameter uncertainties. Sensitivity analysis is done in intercept time - plane wave (τ-p) domain (Stoffa et. al. 1981; Sen and Mukherjee 2003; Ferguson and Sen 2004), and it uses exact analytical expressions for phase velocities (Kennett, 1983) in terms of Thomsen's parameters (Thomsen, 1986).

Theory

Sensitivity analysis helps us in quantifying the uncertainty associated with an estimated model parameter. For example, Figure 1 shows the plot of traveltime error against P-wave velocity for a flat reflector in a homogeneous isotropic medium (i) when depth is fixed and (ii) when depth is allowed to vary within a certain range. In the later case, the value of minimum traveltime error within the given depth range is plotted. We can see that the predication error $E(\mathbf{m}) = \mathbf{e}^T\mathbf{e}$ of this over-determined problem has a sharp minimum in the vicinity of estimated parameter (P-wave velocity) for case (i) indicating that the solution is well-determined in the sense that it has small variance. For case (ii) E(**m**) has a broad minimum indicating large variance of the estimated parameter and hence poorly determined solution (Menke, 1984). This demonstrates the well known velocity-depth ambiguity problem for insufficient offset. The curvature of the error function is a measure of the sharpness of its minimum and the variance of the solution is related to the curvature. The curvature of the prediction error can be measured by its second derivative. For the model space **m**, the sensitivity matrixs is given by

$$\mathbf{S} = \frac{\partial^2 E}{\partial m_i \partial m_j} \tag{1}$$

, where the i, j partial derivatives are with respect to the I and j model parameter **m**.

The traveltime error calculation is based on common focus point (CFP) technology (Berkhout, 1997). For each layer, a focusing operator is computed using a model of elastic parameters with which a CFP gather can be constructed using seismic data. Assuming local homogeneity, the resulting differential time shifts (DTSs) represent error in the model due to anisotropy and error in thickness (Ferguson and Sen 2004). In (τ-p) domain, the DTSs are intercept time errors (Δτ) that connect error in layer thickness *z*, vertical slowness *q* and ray parameter *p*.

Model parameters to be estimated for a VTI medium are $\alpha, \beta, \varepsilon, \delta$ (Thomsen's parameters; Thomsen, 1986) and depth, *z*. In the plane wave domain, expressions for vertical slowness for P-waves (q_p) and Sv-waves (q_{Sv}) in terms of Thomsen's parameters are given by,

$$q_p = \frac{1}{2}\sqrt{2\beta_0^{-2} + 2\alpha_0^{-2} - 4Sp^2 - 4R} \tag{2}$$

$$q_{sv} = \frac{1}{2}\sqrt{2\beta_0^{-2} + 2\alpha_0^{-2} - 4Sp^2 + 4R} \tag{3}$$

where,

$$S = \frac{1}{2}\frac{\alpha_0^{2}}{\beta_0^{2}}\left[\varepsilon - \delta^*\right] + \frac{1}{2}\varepsilon + 1, \tag{4}$$

and

$$R = \frac{1}{2}\sqrt{\begin{array}{l}4p^4\left[S^2 - 2\varepsilon - 1\right] + 4\left(\frac{p}{\beta_0}\right)^2\left[2\varepsilon - S + 1\right] + 4\left(\frac{p}{\alpha_0}\right)^2\left[1 - S\right] + \\ (\beta_0^{-2} - \alpha_0^{-2})^2\end{array}} \tag{5}$$

Vertical traveltime (τ_P) for P-wave (incident P, reflected P) is given by,

$$\tau_p = 2zq_p \quad , \tag{6}$$

and vertical traveltime (τ_{Sv}) for Sv-wave (incident Sv, reflected Sv) is given by,

$$\tau_{Sv} = 2zq_{Sv} \tag{7}$$

For P-Sv-wave, vertical traveltime (τ_{P-Sv}) (incident P, reflected Sv) is given by,

$$\tau_{P-Sv} = z(q_P + q_{Sv}) \quad . \tag{8}$$

Thus exact analytical expressions for partial derivatives of vertical traveltime with respect to the model parameters exist, and can be used in a generalized Newton's equation (9) to obtain model updates.

$$\mathbf{G}\Delta\mathbf{m} = -\Delta\boldsymbol{\tau} , \tag{9}$$

where the columns of $\mathbf{G}$ are the partial derivatives of vertical traveltime (τ_p)with respect to the elastic parameters for observation points 1 to n. Δ is the model parameter update vector and $\Delta\boldsymbol{\tau}$ is the vector of vertical traveltime errors. Error (*E*) for an estimated model parameter is given by

$$E = \Delta\boldsymbol{\tau} \ \Delta\boldsymbol{\tau} \quad , \tag{10}$$

where ' indicates transpose.

The model space for VTI medium is given by **m**=$[\alpha,\beta,\varepsilon,\delta^*,z]$, and for tilted TI medium is **m**=$[\alpha,\beta,\varepsilon,\delta^*,z,\theta]$, where θ is tilt angle of symmetry axis with respect to vertical.

It can be shown that **S** (equation 1) is approximately equal to twice $\mathbf{G}^T\mathbf{G}$ and the covariance matrix is given by (Menke, 1984),

$$[Cov(\mathbf{m})] = \sigma^2[\mathbf{G}^T\mathbf{G}]^{-1} = \sigma^2\left[\frac{1}{2}\mathbf{S}\right]^{-1} \tag{11}$$

where, σ is the standard deviation of data.

The correlation matrix can be obtained from the covariance matrix as,

$$Corr(\mathbf{m})_{i,j} = \frac{Cov(\mathbf{m})_{i,j}}{\sqrt{Cov(\mathbf{m})_{i,i}Cov(\mathbf{m})_{j,j}}} \tag{12}$$

The uncertainty associated with the model parameter m_i equals squared root of i^{th} diagonal element of covariance matrix. In this way, the curvature of error surface is mapped to the uncertainty in model parameter estimate.

Results

Sensitivity analysis is performed for Mesaverde clayshale (Thomsen, 1984). Table 1 lists the uncertainty in the estimates of the anisotropic parameters for different types of data. It can be seen that uncertainty in estimates improves with an increase in offset to depth ratio and that standard deviation of data decreases. In the absence of depth knowledge, the joint inversion of either P-P and Sv-Sv data or P-P and P-Sv data provides improved estimates of model parameters. Figure 2 shows the plot of α versus minimum error in model space **m** for P-P and joint P-P and Sv-Sv data. The minimum error curve for the case of joint P-P and Sv-Sv data has sharper curvature and hence less variance and less uncertainty compared to that for P-P data alone.

Data Type	%age uncertainty in elastic parameters				
	α_0	β_0	ε	δ^*	z
PP (fix z, weak TI)	0.12	-	16.54	34.6	-
PP (fix z, strong TI)	0.15	$9.3*10^3$	103	7961	-
PP (var z, weak TI)	2431	$1.8*10^4$	$1.4*10^4$	2431	-
PP (var z), strong TI)	$4.3*10^5$	$1.2*10^6$	$3.1*10^6$	$3.4*10^6$	$4.3*10^5$
PSv (var z, strong TI)	$4.9*10^4$	$1.1*10^4$	$3.5*10^5$	$5.5*10^5$	$1.0*10^4$
PP-SvSv (var z, strong TI)	1.39	1.42	12.5	5.3	1.4
PP-PSv (var z, strong TI)	10.3	10.3	85.5	47.1	10.3

Table 1: Uncertainty in elastic parameter estimates for different types of data as calculated analytically. Maximum offset to depth ratio is 1.5 and standard deviation is 4 ms.

A similar analysis has been performed for tilted TI medium when more than one tilt angle is known for the similar subsurface lithology. It can be seen (Table 2) that using only P-wave data to estimate α_0, ε, δ^* and z is feasible if more than one tilt angles are known. For a VTI medium, the uncertainty in estimation of anisotropic parameters α_0, ε, δ^* and z for limited offset involves very high uncertainty. We may have a TI medium, in which the angle of tilt of symmetry axis of the medium is laterally variable. In such a case, if the angle of tilt is known at more than one location, a joint inversion of P-wave data from these locations coupled with the knowledge of tilt angles constrain the elastic parameters well. Table 2 shows the result of sensitivity analysis for some TTI media.

When tilt angle of the TI medium is not known, a joint inversion of PP and SvSv data may be used to estimate all the parameters, i.e. $\alpha_{0,}$ $\beta_{0,}$ ε, δ^*, θ (tilt angle), and z (depth) simultaneously. However, this approach will yield fruitful result only when the TI medium has a reasonable tilt. For a VTI medium, this approach will result in high uncertainty for certain parameters, when tilt angle is also a model parameter (table 3).

Table 3 shows that uncertainty of parameters decreases with increase in angle of tilt of symmetry axis. This observation is attributed to the fact that with an increase in tilt angle, the slowness curve becomes more asymmetric (Figures 3 and 4). Thus for the same range of ray parameter values, the effects of much larger incidence angles are incorporated. The assumption here is that the layer interfaces are flat even though the symmetry axis of transverse isotropy is not vertical.

Tilt angles	Uncertainty Estimates (%age)			
	α_0	ε	δ^*	z
0	1631	1010	7875	1380
10	41	265	212	33
20	13.5	94	247	8.8
30	1	11	42	1
40	1.7	15.7	31	1.18
0,10	4	25	24	3.3
0,20	1	7	11	0.8
0,10,20	1	7	11	0.8
0,30,50	.1	0.9	6.3	0.15

Table 2: Uncertainty in elastic parameter estimates for different tilt angles for a TTI medium using P-wave data only when tilt angle of the TI medium is known.

Tilt Angle	Uncertainty in %age for					
	α_0	β_0	ε	δ^*	Tilt angle	z
1^0	27	56	195	107	1431	26
10^0	2	1	24	32	140	1
20^0	2	5	28	77	19	2
30^0	2	.3	30	25	10	.7
40^0	1	.4	19	12	.2	.6
50^0	1	.4	16	12	.2	.6
60^0	.6	.3	12	7	.1	.9

Table 3: Uncertainty in elastic parameter estimates for Joint Inversion of PP and SvSv data for offset to depth ratio of 1 and standard deviation of 4 ms. The angle of tilt of TI medium is also a model parameter.

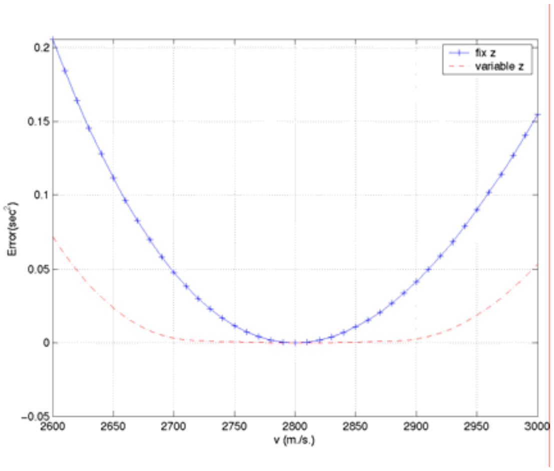

Figure 1: Plot of Vp versus error for fixed z and variable z. For fixed z, error has a sharp curvature at the minimum compared to that for variable z.

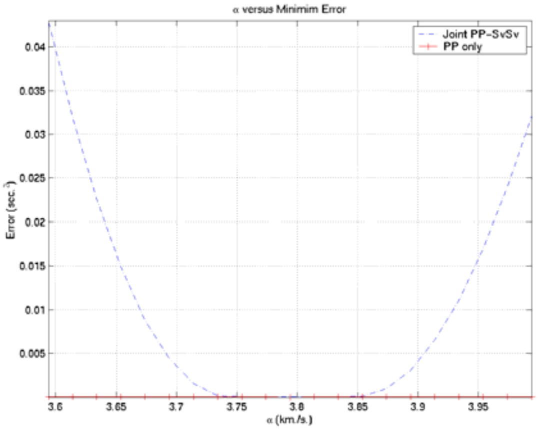

Figure 2: Plot of Vp versus minimum squared error in the model space m for P-P data (red line) and Joint P-P and Sv-Sv data (blue line). The error curvature is sharper for the later case.

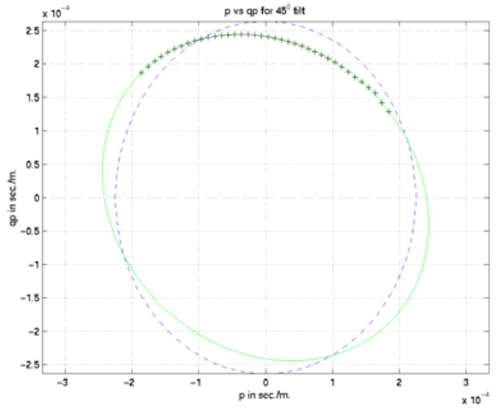

Figure 3: Slowness curves for Mesaverde clayshale. The dotted blue line is slowness curve when symmetry axis is vertical. The continuous green line is slowness curve when symmetry axis is tilted by 45^0 w.r.t. vertical. The '+'s are slownesses for downgoing waves corresponding to a certain range of ray parameters.

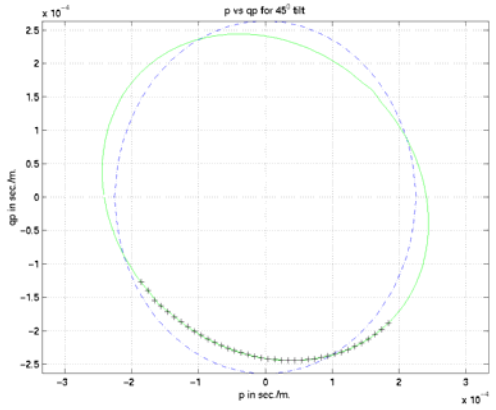

Figure 4: Slowness curves for Mesaverde clayshale. The dotted blue line is slowness curve when symmetry axis is vertical. The continuous green line is slowness curve when symmetry axis is tilted by 45^0 w.r.t. vertical. The '+'s are slownesses for upgoing waves corresponding to a certain range of ray parameters.

Conclusions

Sensitivity analysis presented here helps us in quantifying the limitations of resolving anisotropic parameters for a VTI media. This paper assesses the feasibility of Joint inversion of P-P and Sv-Sv data and P-P and P-Sv data in estimating anisotropic parameters as well as depth. Though P-P data or P-Sv data alone do not provide unique estimates of anisotropic parameters and depth, the Joint inversion provide highly resolved estimates. For tilted TI medium overlying a flat reflector, Joint inversion of P-P and Sv-Sv data, however, may help estimating depth, anisotropic parameters, and tilt angle as well.

References

Ferguson, R. J., and Sen, M. K., 2004, Estimating the elastic parameters of anisotropic media using a joint inversion of P-wave and SV-wave traveltime error, Geophysical Prospecting, 52, 547-557.

Kennett, B. L. N., 1983, Seismic wave propagation in stratified media: Cambridge University Press.

Menke, W., 1984, Geophysical Data Analysis: Discrete Inverse Theory, Academic Press, Inc., Chapter 3, 58-60.

Sen, M. K. & Mukherjee, A., 2003, τ-p analysis in transversely isotropic media: Geophys. J. Int., 154, 647-658.

Stoffa, P. L., Buhl, P., Diebold, J. B., and Wenzel, F., 1981, Direct mapping of seismic data to the domain of intercept time and ray parameter – A plane-wave decomposition, Geophysics, 46, 255-267.

Thomsen, L. 1986, Weak elastic anisotropy, Geophysics, 51, 1954-1966.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media, Geophysics, 59, 1290-1304.

Tsvankin, I., and Thomsen, L., 1995, Inversion of reflection traveltimes for transverse isotropy, Geophysics, 60, 1096-1108.

EDITED REFERENCES

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References

Ferguson, R. J., and M. K. Sen, 2004, Estimating the elastic parameters of anisotropic media using a joint inversion of P-wave and SV-wave traveltime error: Geophysical Prospecting, **52**, 547–557.

Kennett, B. L. N., 1983, Seismic wave propagation in stratified media: Cambridge University Press.

Menke, W., 1984, Geophysical data analysis: Discrete inverse theory: Academic Press, 58–60.

Sen, M. K., and A. Mukherjee, 2003, Tau-p analysis in transversely isotropic media: Geophysical Journal International, **154**, 647–658.

Stoffa, P. L., P. Buhl, J. B. Diebold, and F. Wenzel, 1981, Direct mapping of seismic data to the domain of intercept time and ray parameter – A plane-wave decomposition: Geophysics, **46**, 255–267.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.

Tsvankin, I., and L. Thomsen, 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, **59**, 1290–1304.

—, 1995, Inversion of reflection traveltimes for transverse isotropy: Geophysics, **60**, 1096–1108.