Introduction

A seismic method is developed to correct irregular trace spacing and near-surface complexity in one inversion. The required extrapolation operators are implemented as spatially variable phase-shifts applied within a Fourier integral operator. The associated Hessian is extremely costly to compute, so we use the method of conjugate gradients (CG) to avoid direct computation of the Hessian. Our CG approach reduces the total number of operations from $\mathcal{O}\left(n^3\right)$ for direction computation of the Hessian, to $\mathcal{O}\left(n^2\right)$ for the CG method, where n is the number of trace locations.

We use a synthetic data example plus a real data example to demonstrate the this simultaneous inversion. The synthetic data are the result of an exploding reflector model where the traces are generated by finite differences. Our setup simulates an irregular, horizontal recording aperture above a point source in which a series of point diffractors are embedded - the design of the sources results in a flat reflection event and a series of steep diffractions with conflicting dips. The near-surface correction aspect of our CG inversion removes the lateral velocity effects in the synthetic data, and the trace interpolation aspect reconstructs the missing traces.

Our real data example comes from the Alberta Foothills of the Canadian Rocky Mountains acquired by Husky Oil Ltd. Shot spacing for these data is very irregular, and common receiver gathers suffer from incomplete trace coverage as a result. Further, the near-surface is highly heterogeneous due to significant topographic variation and lateral velocity variation, and reflector continuity is compromised as a result. CG inversion of these data successfully reconstructs the data, with some remaining artifacts due to aliasing, and lateral continuity of the reflectors is improved. As a side benefit, because our extrapolation operator is implemented in the temporal and spatial Fourier domain, ground roll is suppressed.

Least-squares inversion

For the problem of irregular trace spacing, the square of the l_2 norm $\mathbf{e}$ is

$$E_d = \parallel \mathbf{e} \parallel^2 = \parallel \mathbf{W}_e \left[\psi_z - \mathbf{U}_{-\Delta z}\psi_{z+\Delta z}\right] \parallel^2, \tag{1}$$

where $\mathbf{W}_e$ is a diagonal matrix (non-zero elements on the main diagonal only) that gives unit weight to live traces and zero weight to null traces. (In our approach, the desired, regular trace-spacing is achieved through insertion of null traces [Ferguson, 2006].) Wavefield ψ_z is known at depth z, and $\psi_{z+\Delta z}$ is the desired wavefield at reference depth $z+\Delta z$. Operator $\mathbf{U}_{-\Delta z}$ is a wavefield extrapolation operator that carries wavefields $-\Delta z$ through the medium.

The number of traces estimated in this problem exceed the number of actual traces, so this problem is underdetermined. Damping, therefore, is required to establish solution uniqueness. Here, damping takes the form of a model norm according to

$$E_m = \parallel \mathbf{W}_m \, \psi_{z+\Delta z} \parallel, \tag{2}$$

where $\mathbf{W}_m$ is 2nd order spatial derivative used to select the smoothest model. Total cost function E to be minimized, then, is

$$E = E_d + \varepsilon^2 E_m, \tag{3}$$

where ε is a user-defined scalar that is determined by trial and error. The minimum of cost function E with respect to $\psi_{z+\Delta z}$ is [Ferguson, 2006]

$$\psi_{z+\Delta z} = \left[\mathbf{U}^A_{-\Delta z}\mathbf{W}_e\mathbf{U}_{-\Delta z} + \epsilon^2\mathbf{W}_m\right]^{-1}\mathbf{U}_{-\Delta z}\mathbf{W}_e\psi. \tag{4}$$

Extrapolation operators

The required extrapolation operators $U^A_{-\Delta z}$ and $U_{-\Delta z}$ [Margrave and Ferguson, 1999,] in our approach are nonstationary in lateral velocity and monochromatic according to

$$\psi\left(x,y,z-\Delta z,\omega\right) = \frac{1}{\left(2\pi\right)^2}\int \varphi\left(k_x,k_y,z,\omega\right) e^{-i\,\Delta z\sqrt{\left(\frac{\omega}{v(x,y)}\right)^2 - k_x^2 - k_y^2}} e^{-i\left[k_x\,x + k_y\,y\right]} dk_x\, dk_y. \tag{5}$$

for example, where the x, y, and ω are spatial and temporal frequency coordinates, $\varphi(k_x, k_y, z, \omega)$ is the Fourier spectrum of wavefield ψ (limits $-\infty$ to $+\infty$ are omitted for brevity), and k_x and k_y are wavenumbers that correspond to x and y respectively [Ferguson, 2006]. $\mathbf{U}^A_{-\Delta z}$ is simply the complex conjugate of $\mathbf{U}_{-\Delta z}$. The $\omega \rightarrow t$ transform completes extrapolation. Manipulation of the sign on Δz in the complex exponential in equation 5 ensures exponential decay where the square root is complex.

Conjugate Gradients

Because $\mathbf{U}^A_{-\Delta z}\mathbf{W}_e\mathbf{U}_{-\Delta z}$ is a positive definite matrix, the solution is the minimum of a quadratic form, and the residuals are the direction of steepest descent and are orthogonal to each other. Based on the method of conjugate gradients (CG), a solution to the minimization problem is found through a search in the direction of the conjugation of these residuals. The true residuals are $\mathbf{r} = \psi_z - \mathbf{U}_{-\Delta z}\psi_{z+\Delta z}$ whereas in the conjugate gradient method the residual polynomials are constructed. A line search is then conducted to find the step length γ by minimizing the quadratic form in a straight line along the search direction. If the start point is $\mathbf{x}_o$, which can be completely arbitrary, each update thereafter to the model is

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \gamma_i \mathbf{p}_i, \tag{6}$$

where $\mathbf{p}_o = \mathbf{r}_o$. The step length γ is

$$\gamma_i = \frac{\mathbf{r}^t\mathbf{r}_i}{\mathbf{p}_i^t\mathbf{A}\mathbf{p}_i} \tag{7}$$

as derived by [Shewchuk, 1994] and $\mathbf{A}$ is the augmented matrix $\begin{bmatrix} \mathbf{W}_d\mathbf{U}_{\Delta z} \\ \epsilon\mathbf{W}_m \end{bmatrix}$. The new residual is constructed as

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \gamma\mathbf{A}\mathbf{p}_i. \tag{8}$$

The new direction is made as a conjugation of this new residual being

$$\mathbf{p}_{i+1} = \mathbf{r}_{i+1} + \beta_{i+1}\mathbf{p}_i, \tag{9}$$

where β_{i+1} is a scalar multiplier derived from the conjugate Gram-Schmidt process. It is defined as

$$\beta_{i+1} = \frac{\mathbf{r}_{i+1}\mathbf{r}_{i+1}}{\mathbf{r}_i^t\mathbf{r}_i}, \tag{10}$$

taken from [Shewchuk, 1994]. The final model update $\mathbf{x}_k$ is the regularized and redatumed spectrum for a single frequency $\psi_{z+\Delta z}$. The solution should converge in no more than n iterations. If the eigenvalues of the operators are spread over a small scale, or there are clusters of eigenvalues, or even eigenvalues with multiplicity > 1 then the solution should converge in iterations $< n$. The method can be made to terminate at a given tolerance and the final model is the solution.

Synthetic Example

The model setup (Figure 1a) is that of an exploding reflector model with a flat line of weak sources and five strong point sources at a depth of 300m. The receiver array is irregularly spaced at a depth of 100m so that the propagation distance is large (200 m).

Finite difference data for this model are found in Figure (1b). The Least-Squares Conjugate Gradient algorithm used in this work was developed and implemented according to the algorithm provided by [van den Eshof and Sleijpen, 2004]. Following inversion, phase-shift extrapolation is used extrapolate the result back to the receiver level for direct comparison with the input. The smoothing parameter, ϵ is found by trial and error with $\epsilon = 0.5$ found to reduce noise without oversmoothing. The tolerance level was 1%. The exact and costly solution from the Newton's method is displayed for comparison (Figure 1d). An approximation can be applied to make the Hessian more efficient as done by but this imposes a dip limitation onto the outcome [Ferguson, 2006]. Computation using the Newton's method is $\mathcal{O}(n^3)$ in cost. Cost for the conjugate gradient method for each iteration is $\mathcal{O}(n^2)$ which translates into a computational cost for a 3D survey to be $\mathcal{O}(10^{15})$ operations. The data regularized with the conjugate gradients fill in the missing amplitudes and removes the velocity effects by flattening the line and point sources. Compared with Newton's method, there is no significant difference.

Canadian Foothills Example

The real data example is from the Canadian foothills in Alberta. It was acquired by Husky Oil Ltd. and it was recorded in an overthrust belt region. The region is characterized by overthrust structures of various geometric complexity and stratigraphy units ranging from carbonates, shales as well as other clastics [Stork et al., 1995,]. The velocity model for this area was derived by turning-wave tomography (Figure 2).

A common source gather (Figure 3a) was used to demonstrate the effectiveness of the method described in this paper. The receiver array is regularly spaced for the most part so the gather was taken and decimated randomly (Figure 3c)from 300 traces to 56 traces. The conjugate gradient based method was applied to the two gathers following [Reshef, 1991] and using the velocities in Figure 2. The redatumed results in Figure 3b show improved lateral continuity in the reflections. The regularization and redatumed gather in Figure 3d shows the robustness of the method in that it successfully reconstructs the data and improves the lateral continuity of the reflectors.

Conclusions

A method for regularization and redatuming is presented that uses conjugate gradients to optimize weighted damped least-squares. Conjugate gradients are shown to improve efficiency of computing the Hessian by reducing the computation of the Hessian from matrix-matrix multiplication to vector-matrix multiplication. This cost is reduced by 10 in 2D and we expect to see a greater cost savings in 3D. This method successfully regularizes and removes the lateral velocity effects of a synthetic model. The line source is continuous and the steep dips are restored. The method also successfully reconstructs the real data with some artifacts and also removes the phase distortions caused by the near-surface which is apparent in the improved lateral continuity of the reflectors.

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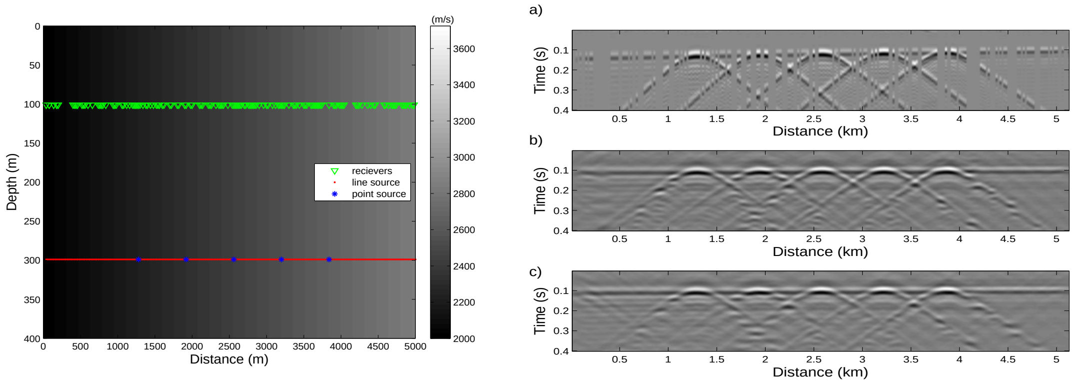

Figure 1: Velocity model a) with a line source, five point sources, and an irregular receiver array. b) The corresponding data. c) The solution using the conjugate gradient scheme, d) The Newton's least-square solution.

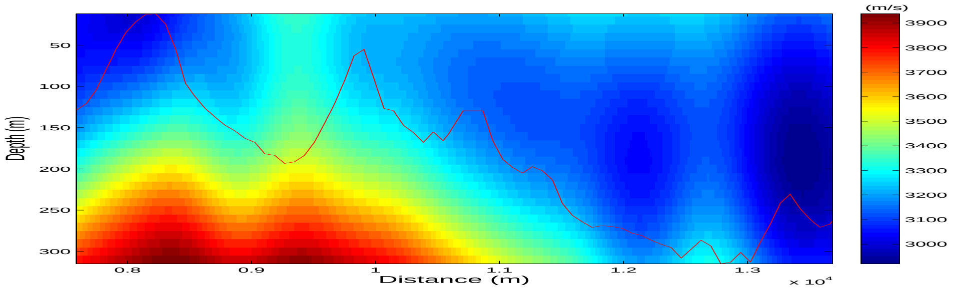

Figure 2: Velocity model with topography (red line).

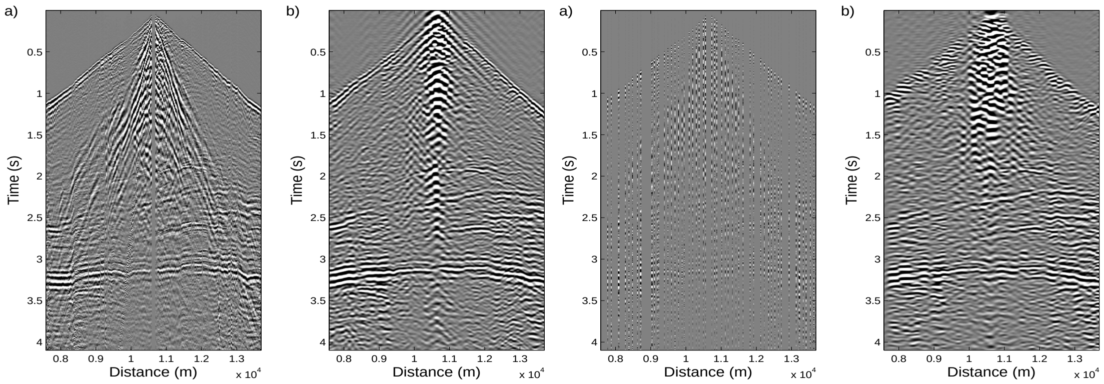

Figure 3: Input data a) Shot record, b) Redatumed data using CG method. c) Decimated record, d) Regularized and redatumed data using CG method.