Elastic corrections to acoustic finite-difference simulations

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SUMMARY

Acoustic finite-difference modeling is playing an increasingly important role in seismic imaging (e.g. in reverse time migration) but the additional cost of elastic finite-difference modeling restricts its use in commercial imaging technology. The cost of full elastic finite-difference modeling can exceed the cost of acoustic modeling in the same velocity model by two orders of magnitude or more. A technique is described that corrects an acoustic finite-difference simulation for elastic effects. It is based on calculating the errors in the elastic wave equation using the acoustic simulation as an approximate solution. The errors are used to generate an effective source field for an additional acoustic simulation that calculates a correction to the wavefield produced in the original acoustic simulation. The cost of this approach is greater than that of an acoustic simulation but much less than that of a full elastic simulation.

INTRODUCTION

For many applications in imaging and reservoir characterization (reverse time migration, waveform inversion, etc.), we require accurate simulations of seismic wave propagation. To realistically model the Earth, these are needed for elastic, anisotropic and anelastic models. The finite-difference method is widely used in this context as it is robust, simple to implement, and offers a good balance between accuracy and efficiency. However, it is still a computational challenge to perform elastic, anisotropic finite-difference simulations in three dimensions, so approximate calculations are often performed in an equivalent acoustic model. Even for *P* waves, the amplitudes of the first arrivals in the acoustic medium differ from those in the elastic medium. The objective of this paper is to describe a scheme whereby the acoustic wavefield can be partially corrected for elastic effects without incurring the cost of the full elastic computation.

The computational costs (memory and CPU) of acoustic and elastic finite-difference simulations differ for four reasons:

1. The number of model parameters is smaller in an acoustic model. Only two parameters (velocity and density) are required to describe an acoustic medium. An isotropic elastic medium requires three (the *P*- and *S*-wave velocities plus density), and an anisotropic elastic medium requires up to 21 elastic parameters plus density;
2. The number of field variables that must be computed and stored during acoustic modeling is significantly lower. For example, in the velocity-stress formulation for acoustic media we have four, (three components of particle velocity and pressure), whereas for elastic media we have nine (the particle velocity and the six components of the symmetric stress tensor);
3. The computational cost of solving the equation of motion and constitutive equations is significantly less in acoustic media than in elastic media;
4. The cell size and temporal interval must be chosen carefully in order to avoid numerical dispersion and maintain stability. When comparing acoustic and elastic modeling for a given model, this results in the number of times the wave equation must be solved being increased by the ratio of the minimum *P*-wave velocity to the minimum *S*-wave velocity in the elastic model, raised to the fourth power.

Normally the final item above is the most significant. Even if the elastic medium is a Poisson solid ($V_P = \sqrt{3}V_S$), the increase in cost is a factor of nine, and for more realistic models with low shear velocity sediments (e.g. near the ocean floor), the ratio is often between one and two orders of magnitude. The combination of all four effects makes elastic modeling a real challenge, often raising the cost by between two and three orders of magnitude and introducing memory limitations.

Consider two models, one acoustic and the other elastic, designed so that the density and acoustic/*P*-wave velocity fields match. For a pressure source, only *P* waves will be excited, so the solutions in the acoustic medium and for *P* waves in the elastic medium are expected to be very similar, at least in a limited time window around the first arrivals. The most significant differences will occur in the amplitudes of reflected and transmitted *P* waves from interfaces (or pseudointerfaces where properties vary rapidly). In the regions away from interfaces, properties are either homogeneous or varying slowly and smoothly and the coupling between *P* and *S* waves is insignificant. The objective is to correct the acoustic solution for elastic effects at interfaces, without incurring the cost of the full elastic solution.

This paper describes a method to correct acoustic simulations for some of the effects of elasticity. We hope to correct the amplitudes of the *P*-wave arrivals for the effects of elasticity, particularly those caused by reflection and transmission coefficients at interfaces, at a cost considerably less than the cost of full elastic simulations. We do not expect to simulate the shear waves generated at interfaces. If necessary, the process can be applied iteratively to improve the accuracy of the correction. We assume that the coupling between *P* and *S* waves is small, and that it is only significant in a small fraction of the total model, e.g. at interfaces or pseudointerfaces. Typically we would expect these to occupy at most 10% of the grid points in the model, and in many cases a significantly lower proportion.

THEORY

Following the notation of Chapman (2004), we consider the equation of motion

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \frac{\partial \mathbf{t}_j}{\partial x_j} + \mathbf{f}, \qquad (1)$$

and the constitutive equations

$$\frac{\partial \mathbf{t}_k}{\partial t} = \mathbf{c}_{kj}\frac{\partial \mathbf{v}}{\partial x_j}, \qquad (2)$$

where $\mathbf{v}$ is the particle velocity, ρ is the density of the medium, $\mathbf{t}_j$ the traction on the surface normal to the $\hat{\mathbf{e}}_j$ axis and $\mathbf{f}$ is the body force per unit volume. The notation $\mathbf{c}_{kj}$ for 3×3 matrices of elastic parameters follows Woodhouse (1974).

In acoustic media, the tractions reduce to minus the hydrostatic pressure, i.e. $(\mathbf{t}_k)_j = -P\delta_{jk}$, with zero shear stress. The equations simplify significantly, so equation 1 becomes

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \mathbf{f}, \qquad (3)$$

while 3 of the components in equation 2 become

$$-\frac{\partial P}{\partial t} = \kappa\nabla\cdot\mathbf{v} \qquad (4)$$

(P is the pressure and κ is the bulk modulus).

Elastic correction theory

We denote the acoustic model and field variables by a superscript $^{\mathrm{A}}$, e.g. $\mathbf{v}^{\mathrm{A}}$, and those in the elastic model by a superscript $^{\mathrm{E}}$, e.g. $\mathbf{v}^{\mathrm{E}}$. Although we expect the acoustic and elastic particle displacement and velocity fields for P waves to be similar, the differences in the elastic parameters ($\mathbf{c}^{\mathrm{A}}_{kj}$ and $\mathbf{c}^{\mathrm{E}}_{kj}$) are not small. We expect

$$\left|\mathbf{v}^{\mathrm{A}} - \mathbf{v}^{\mathrm{E}}\right| \sim O(\varepsilon), \qquad (5)$$

where ε is a small parameter, at least for some limited time, whereas $\mathbf{t}^{\mathrm{A}}_k$ and $\mathbf{t}^{\mathrm{E}}_k$ will differ significantly. For example, the shear stresses are zero in the acoustic medium because the shear modulus, μ, is zero. However, they will be non-zero in the elastic medium because the corresponding shear strains are non-zero.

Although the stiffness matrices, $\mathbf{c}^{\mathrm{A}}_{kj}$ and $\mathbf{c}^{\mathrm{E}}_{kj}$ may differ significantly, the densities in the two models must be identical. If they were to differ then the amplitudes of the acoustic and elastic solutions would differ significantly due to the impedance differences. It is also important that the acoustic and P-wave velocities are identical to ensure consistent kinematic behavior and good convergence of iterative corrections.

We do not discuss here how anisotropic kinematic properties can be introduced into the acoustic equation (e.g. Fowler et al., 2010), although this is a logical extension of the method. The method described here is a variant of error Born scattering theory. The difference is that the Green's function is calculated in the acoustic model, whereas the error is calculated in the elastic equation of motion. It also has some features of the finite-difference injection method (Robertsson and Chapman, 2000).

Suppose we have the solution of the acoustic wave equation, i.e. $\mathbf{v}^{\mathrm{A}}$ satisfying equations 3 and 4,

$$\begin{aligned} \rho\frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial t} &= -\nabla P^{\mathrm{A}} + \mathbf{f}, && (6)\\ -\frac{\partial P^{\mathrm{A}}}{\partial t} &= \kappa^{\mathrm{A}}\nabla\cdot\mathbf{v}^{\mathrm{A}}. && (7) \end{aligned}$$

Using the acoustic solution $\mathbf{v}^{\mathrm{A}}$ as an approximation for the elastic solution, we can calculate the traction solutions (equation 2) for the acoustic wavefield in the elastic medium

$$\frac{\partial \mathbf{t}^{\mathrm{A}}_k}{\partial t} = \mathbf{c}^{\mathrm{E}}_{kj}\frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial x_j}. \qquad (8)$$

The acoustic solution will not exactly satisfy equation of motion 1 and we can calculate the error

$$\mathscr{E} = \rho\frac{\partial \mathbf{v}^{\mathrm{A}}}{\partial t} - \frac{\partial \mathbf{t}^{\mathrm{A}}_j}{\partial x_j} - \mathbf{f}. \qquad (9)$$

An effective source $\mathscr{E}$ in the acoustic equation

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \mathbf{f} - \mathscr{E}, \qquad (10)$$

will modify the acoustic solution by scattering new waves from each point where the acoustic solution does not satisfy the elastic equation. As the acoustic equation does not support shear waves, it is tempting to believe that we have satisfied our objective: the error in elastic equation of motion 9 need only to be calculated at the small number of points where it is significant, and the acoustic waves will be corrected for these errors (equation 10). In fact we have done more than that. The error in equation 9 is

$$\mathscr{E} = \rho\frac{\partial}{\partial t}\left(\mathbf{v}^A - \mathbf{v}^E\right), \qquad (11)$$

and we have exactly reproduced the elastic solution with the modified acoustic equation. If the error term were only included in a small region then artifact reflections would be generated from the boundaries of this region due to the change from elastic to acoustic media.

However, an alternative iterative scheme achieves our objective. The error term (equation 9), computed from the acoustic simulation, is introduced as an 'effective' source field in a second, independent solution of the acoustic equation. This solution provides a correction to the wavefield obtained in the original, zeroth-iteration solution of the acoustic equation. Thus the solution (equation 6) is called the zeroth iteration. This solution ($\mathbf{v}^{(0)}$) is used to calculate the error in the elastic equation ($\mathscr{E}^{(0)}$) from equation 9, which defines effective sources that are used to compute the first iteration correction wavefield ($\delta\mathbf{v}$). The corrected solution is then given by

$$\mathbf{v} \approx \mathbf{v}^{(1)} = \mathbf{v}^{(0)} + \delta\mathbf{v}. \qquad (12)$$

We note that the point-force Green's functions for a P wave in a homogeneous elastic medium and in an equivalent acoustic medium are identical. The elastic Green's function depends on the elastic parameters via the P-wave velocity, and is independent of the S-wave velocity. Thus the error from the elastic equation is the appropriate source term in the acoustic equation. The iterative procedure described above can be continued if a further correction is required.

In addition to the cost of two independent acoustic solutions, the equivalent elastic tractions (equation 8) and error terms (equation 9) need to be calculated, but only on the coarser acoustic grid and at the small percentage of points where the P–S coupling is significant. In addition, the derivatives of the equivalent elastic tractions in equations 8 and 9 are with respect to different variables, and when calculating the errors (equation 9), particle accelerations are needed that are not normally saved. Nevertheless the cost (memory and CPU) of computing and storing these extra field variables is not great as they are only needed in regions where P–S coupling is important. For this algorithm to be efficient, it is important to identify these regions (the interfaces and pseudointerfaces in the model) *a priori*, either during model building or by other means (e.g. from a single full elastic simulation).

EXAMPLE

We implemented the method described above to correct reflected and transmitted P-wave amplitudes for elastic effects at an interface using a finite-difference method. The finite-difference implementation is based on a staggered grid formulation of equations of motion 1 and 3 and constitutive relations 2 and 4 for wave propagation in heterogeneous media, as described by Virieux (1986) and Levander (1988). The method is fourth-order accurate in space and second-order accurate in time. For the moment we have only considered isotropic elastic media. Simple periodic boundary conditions were applied at the edges.

We implemented three versions of the code,

1. acoustic wave propagation, equations 3 and 4 (propagating P waves only),
2. elastic wave propagation, equations 1 and 2,
3. the new method, equations 6 to 12, which will be referred to as the *hybrid method*.

We tested the method on a simple example in which the model consisted of two homogeneous half spaces separated by an interface, running the method for a single iteration. The seismic parameters for the medium containing the source were V_P = 1500 m/s, V_S = 500 m/s, density = 1000 kg/m^3, and for the medium beyond the interface they were V_P = 2000 m/s, V_S = 800 m/s, density = 1500 kg/m^3.

Figure 1 contains snapshots of the acoustic, correction, hybrid and elastic wavefields showing arrivals that have transmitted and reflected at the interface (as well as converted S-wave arrivals in the elastic simulation). The simulation with the new source terms generate a reflected and a transmitted event that modulates the amplitudes of the original acoustic simulation.

Figure 2 shows the recorded horizontal particle velocity at a point between the source and the interface. It shows the direct (P-wave) arrival, followed by reflected P-wave and S-wave arrivals. In the direct arrival the acoustic, hybrid and elastic results all agree perfectly as expected. The reflected P-wave arrival demonstrates the correction achieved using the hybrid method, which has come close to predicting the elastic waveform correctly. The S-wave reflection appears only in the elastic result as expected.

CONCLUSIONS

A technique has been described that corrects an acoustic finite-difference simulation for elastic effects on the amplitude of P waves. It has been demonstrated on a simple synthetic model that this method can produce corrected acoustic waveforms that come close to predicting the behavior of elastic modeling at significantly lower cost than that of running a full elastic simulation.

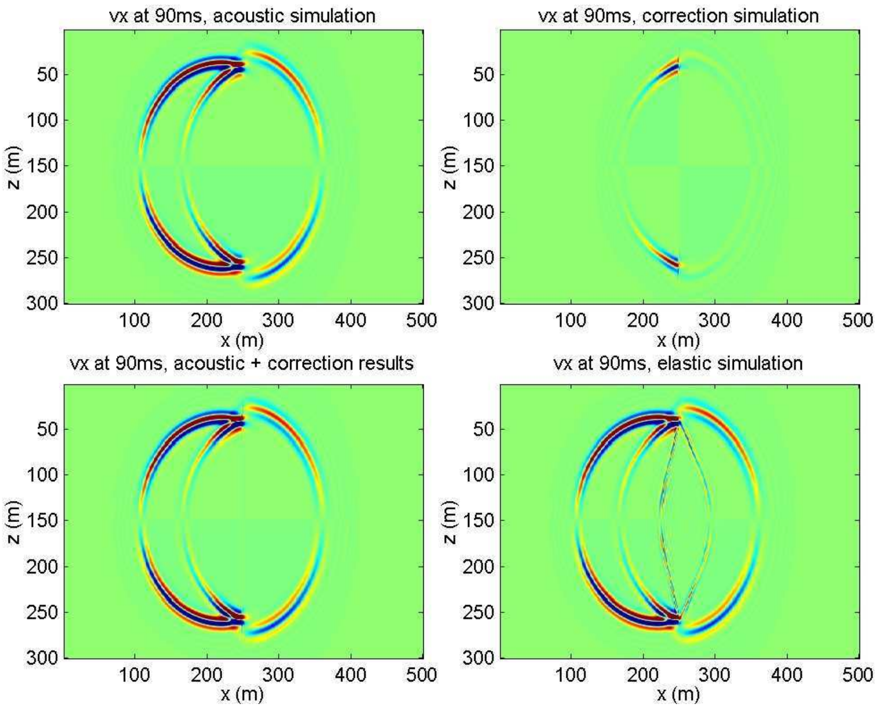

Figure 1: Snapshots of the horizontal particle velocity wavefield. The upper-left panel corresponds to the acoustic simulation, the upper-right panel to the simulation where the effective sources are excited, the lower-left to the hybrid simulation, and the lower-right to the reference elastic solution.

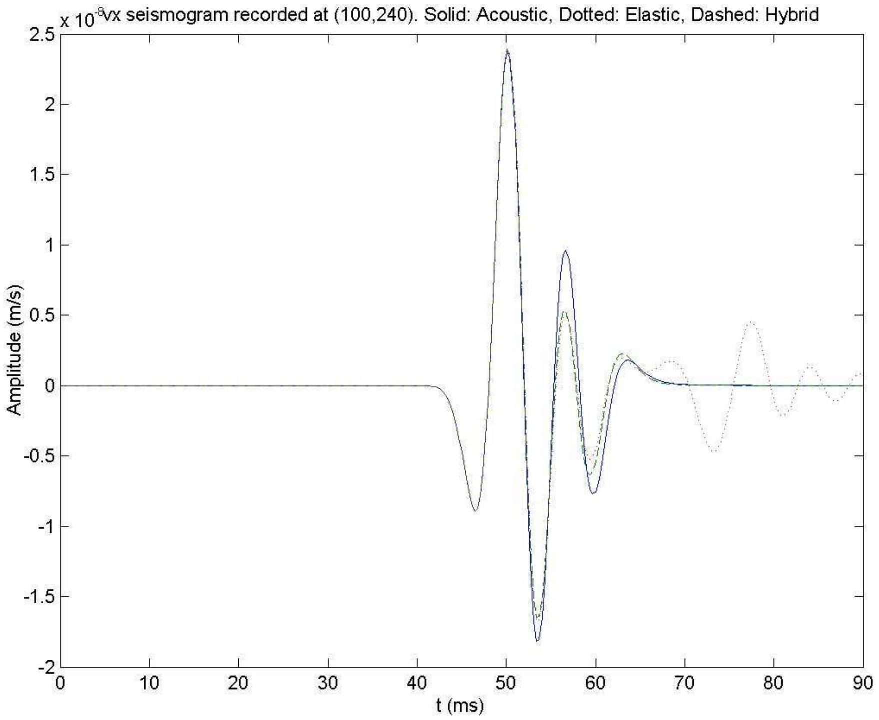

Figure 2: Recorded horizontal particle velocity at a point between the source and reflector. The solid blue curve is from the acoustic simulation, the dashed green curve is from the hybrid simulation, and the dotted red curve is from the elastic simulation.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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