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APPROXIMATE FOURIER INTEGRAL WAVEFIELD EXTRAPOLATORS FOR HETEROGENEOUS, ANISOTROPIC MEDIA

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ABSTRACT. Seismic imaging uses wavefield data recorded on the earth's surface to construct images of the internal structure. A key part of this process is the extrapolation of wavefield data into the earth's interior. An alternative to the commonly used ray theory approximation is to perform a plane-wave decomposition of the recorded data and extrapolate each plane wave independently. For homogeneous media, the Fourier transform can be used for the plane-wave decomposition and phase shifts propagate the plane waves.

We explore an approximate extension of this concept to heterogeneous media that uses pseudodifferential operator theory. A derivation of a Fourier integral operator is presented, which implements the appropriate plane-wave mixing. We show the transpose of this operator is also a viable Fourier integral wavefield extrapolator with a first order error that opposes the original operator. The symmetric average of these two extrapolators is shown to be more accurate than either of the original two. We present both numerical experiments and theoretical arguments to characterize our results and discuss their possible extensions.

1 Introduction. Seismic imaging requires as inputs (1) the recordings of seismometers placed at regular positions on the earth's surface and (2) a velocity model. Let a cartesian coordinate system (x_1, x_2, x_3) be positioned with origin at the location of an artificial seismic source with depth $z = x_3$ increasing downward and $x = (x_1, x_2)$ describing lat-

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eral position. The seismic recordings, that we represent as $\Psi(x, z = 0, t)$, are a discrete sampling of the earth's response to the seismic source. They are a measurement of the seismic wavefield on a portion of the earth's boundary. The velocity model is an a priori estimate of the seismic wavespeed as a function of position in the subsurface ($z > 0$).

The desired output from seismic imaging is the earth's reflectivity $r(x, z)$, $z > 0$. (Physically, we expect the reflectivity to also depend upon direction of the incident wave though we ignore that complication here.) The estimation of $r(x, z)$ is commonly done with an assumption called an *imaging condition*. For a specific depth, a common imaging condition is that $r(x, z)$ is the ratio of the wavefield scattered from that depth to the wavefield incident upon that depth. The incident wavefield can be estimated by constructing a simple mathematical model of the source and propagating it down to the depth of interest. Similarly, the scattered wavefield is estimated by reverse-propagation of the recorded data to the desired depth.

Both the forward propagation of the source wavefield and the reverse-propagation of the recorded wavefield can be accomplished by a *wavefield extrapolator*. The construction of wavefield extrapolators that can propagate a wavefield through an arbitrary heterogeneous and anisotropic medium is a major research topic. In this paper we accommodate velocity variation with z by partitioning the problem into a large number of vertical steps, called Δz steps. Within any particular step we allow velocity to depend arbitrarily upon the lateral coordinate and possibly the angle of propagation but not upon z. We then develop wavefield extrapolators for this reduced problem and apply them recursively in a computer program. Here, recursive means that the input wavefield for a particular extrapolation step is the output from the previous step. For the first step, the input is the recorded data.

Thus we pose the elemental wavefield extrapolation problem. Given an acoustic wavefield on a fixed depth plane $z = z_0$, $\Psi(x, z = z_0, t)$, we desire a formula for the extrapolated wavefield, $\Psi(x, z = z_0 + \Delta z)$. We further assume that Ψ is a solution to the source-free scalar wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} = 0 \tag{1}$$

where the velocity, v, may depend upon x but is independent of z. Let $\psi(x, z, \omega)$ be the Fourier transform of $\Psi(x, z, t)$ (e.g. $\psi(x, z, \omega) = \int_{-\infty}^{\infty} \Psi(x, z, t) e^{2\pi i \omega t}\, dt$) with respect to t that therefore satisfies the

Helmholtz equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{4\pi^2\omega^2}{v^2}\psi = 0. \tag{2}$$

In the special case that v is constant, then the exact solution of equation (2) can be developed using Fourier transforms. For spatial variables, we use a convention in the Fourier transform that is opposite to that of the time variable (e.g. $\phi(k_x, z, \omega) = \int_{-\infty}^{\infty}\psi(x, z, \omega)e^{-2\pi i k_x x}\,dx$). The solution to equation (2) is then

$$\begin{aligned}\psi(x, z, \omega) = &\int_{-\infty}^{\infty} A(k_x, \omega)e^{2\pi i(k_x x - k_z z)}\,dk_x \\ &+ \int_{-\infty}^{\infty} B(k_x, \omega)e^{2\pi i(k_x x + k_z z)}\,dk_x\end{aligned} \tag{3}$$

where

$$k_z = \sqrt{\frac{\omega^2}{v^2} - {k_x}^2} \tag{4}$$

and $A(k_x, \omega)$ and $B(k_x, \omega)$ are arbitrary functions.

This development makes it apparent that the extrapolation problem is ill-posed in that only a single boundary condition is available to determine the two arbitrary functions. This circumstance could be resolved by measuring the vertical derivative of the wavefield on the recording surface but this is rarely done in practice. Instead, it is common to make an assumption of *one-way wave propagation*. For example, if recorded data is to be extrapolated downward, it is commonly assumed to consist entirely of upward traveling waves. Since the z axis increases downward, this corresponds to setting $B(k_x, \omega) = 0$. Then, it follows that $A(k_x, \omega) = \int_{-\infty}^{\infty}\psi(x, z = z_0, \omega)e^{2\pi i k_x x}\,dx = \phi(k_x, z_0, \omega)$, that is it is the Fourier transform (over x) of the wavefield at the $z = z_0$ depth plane. Thus, under the assumption of one-way waves, the extrapolated wavefield is written

$$\psi(x, z = z_0 + \Delta z, \omega) = \int_{-\infty}^{\infty} \phi(k_x, z_0, \omega)e^{2\pi i(k_x x - k_z \Delta z)}\,dk_x. \tag{5}$$

Equation (5) is a prescription for an explicit one-way wavefield extrapolator formulated as a phase-shift operator in the Fourier domain.

Gazdag [**4**] first applied the phase-shift method to the vertical extrapolation of seismic waves. Berkhout [**1**] showed that the equation (5) can be derived as an infinite order Taylor series extrapolation.

Gazdag and Squazzero [**5**] extended phase shift to lateral velocity variation through the PSPI (*phase shift plus interpolation*) algorithm. Stoffa et al. [**12**] proposed the spilt-step Fourier technique that applied the vertical phase delay (thin-lens term) exactly in the (x, ω) domain and used a single reference velocity for the angle-dependent delay (focusing term) in the (k_x, ω) domain. Better approximations to the above method were developed by Wu [**14**] and Wu and Wu [**15**] as the phase-screen method that extended the split-step method by improving focusing. Margrave and Ferguson ([**8**] and [**9**]) used nonstationary filter theory (Margrave [**7**]) to derive two alternative extensions of phase-shift to lateral velocity variations. They gave explicit, analytic integral forms for these extensions that can be identified as Fourier integral operators (Stein [**11**]). One form was shown to be a generalization of PSPI in the limiting case of very rapid lateral velocity variations. The other, called NSPS (*nonstationary phase shift*), has been shown to be the (x, ω) domain transpose of PSPI (Margrave and Ferguson [**10**]).

In related developments, Fishman and McCoy [**3**] gave a general algorithm for factoring the Helmholtz operator, for arbitrary lateral velocity variations, into upgoing and downgoing operators and derived several approximate factorizations. One of their approximations is equivalent to the generalized PSPI expression of Margrave and Ferguson [**9**]. Grimbergen et al. [**6**] used eigenvalue decomposition to numerically factor the Helmholtz equation. For arbitrary lateral velocity variations, they approximated the lateral derivatives with finite difference operators and achieved a high-quality though computationally intensive factorization. Yao and Margrave [**16**] demonstrate that a similar eigenvalue factorization may be performed in the Fourier domain that affords a better lateral derivative.

In this paper, we show that a Taylor series approach can be used to derive the PSPI and NSPS integral operators from the Helmholtz equation for laterally varying media. The derivation shows precisely how both operators are approximate but in complementary ways. We show that first order error terms from both operators tend to be similar but opposite in sign. This suggests that using them alternately in a multi-step extrapolation will lead to higher accuracy.

2 Exact second derivatives from the Helmholtz equation. A seismic wavefield $\psi(x, z, \omega)$ at depth z is predictable from a wavefield $\psi(x, 0, \omega)$ recorded at $z = 0$ by Taylor series (Berkhout [**1**]). All orders of the depth derivatives of ψ must be known at $z = 0$. However, from the Helmholtz equation (equation (2)), only the second depth derivative is easily found. Two equivalent forms of the second derivative can be derived from the Helmholtz equation and are classifiable as pseudodifferential operators or nonstationary filters. The two equivalent second derivatives yield two approximate forms for the n-th depth derivative, and thus to two elemental, but not equivalent, extrapolators. (These forms are elemental in that they are simple, complementary and can be combined to get higher-order extrapolators.) The Taylor series expansion of a monochromatic wavefield, $\psi(x, z, \omega)$, in the z coordinate, gives an expression for the wavefield at depth z in terms of the wavefield at the reference depth 0, and is

$$\psi(x,z) = \psi(x,0) + \sum_{j=1}^{\infty} \frac{z^j}{j!} \left[\frac{\partial^j \psi(x,z)}{\partial z^j} \right]_{z=0} \tag{6}$$

where the ω dependence is suppressed. In the constant velocity case, Berkhout [**1**] showed that all orders of derivatives can be obtained exactly from the wave equation (in the (k_x, ω) domain) and the resulting series can be summed to give the phase-shift operator.

An expression for the second depth derivative of the wavefield follows from equation (2)

$$\frac{\partial^2 \psi(x,z)}{\partial z^2} = -\left[\frac{\partial^2 \psi(x,z)}{\partial x^2} + \frac{4\pi^2 \omega^2}{v(x)^2} \right] \psi(x,z) \tag{7}$$

where the x dependence of velocity v is explicitly indicated. Fourier transformation of this expression over x gives

$$\frac{\partial^2 \phi(k_x,z)}{\partial z^2} = -4\pi^2 \int_{-\infty}^{\infty} k_z^2(x,k_x) \psi(x,z) e^{2\pi i k_x x} \, dx \tag{8}$$

where the wavenumber spectrum ϕ of ψ is

$$\phi(k_x,z) = \int_{-\infty}^{\infty} \psi(x,z) e^{2\pi i k_x x} \, dx \tag{9}$$

and the vertical wavenumber k_z is defined by equation (4).

Equation (8) is an exact prescription for the second z derivative of ψ and is an *dual-form* pseudodifferential operator that maps a wavefield ψ to the second depth derivative of a spectrum ϕ, and whose symbol is $-{k_z}^2$. It is also a nonstationary convolution filter. As a nonstationary filter, equation (3) is classified as a mixed-domain filter: the input is a wavefield and the output is a spectrum (Margrave [**7**]).

An alternative exact prescription for the second depth derivative is found by substituting for ψ on the right-hand side of equation (2) with the inverse Fourier transform of ϕ (e.g. $\psi(x,z) = \int_{-\infty}^{\infty} \phi(k_x, z) e^{2\pi i k_x x}\, dx$). The operator contained by the square brackets in equation (7) can be moved inside the Fourier integral with the result

$$\frac{\partial^2 \psi(x,z)}{\partial z^2} = -4\pi^2 \int_{-\infty}^{\infty} k_z^2(x, k_x) \phi(k_x, z) e^{2\pi i k_x x}\, dk_x. \tag{10}$$

Equation (10) is a pseudodifferential operator that maps a spectrum ϕ to the second depth derivative of a wavefield ϕ, and whose symbol is also $-k_z^2$. It is also a nonstationary combination filter (Margrave, [**7**]). Like the convolution filter in equation (3) the combination filter is a mixed domain filter; the input and output are in different Fourier domains. These two expressions for the second derivative are exact, and therefore, equivalent.

3 Estimation of all depth derivatives. By inspection, equation (8) suggests that the n-th depth derivative is approximately

$$\frac{\partial^n \phi(k_x,z)}{\partial z^n} \approx [D_+^n \psi](k_x, z) \equiv \int_{-\infty}^{\infty} [\pm 2\pi i k_z]^n \psi(x,z) e^{-2\pi i k_x x}\, dx \tag{11}$$

where the subscript '+' in the operator D_+^n indicates the operator applies a forward Fourier integral.

Similarly, equation (10) suggests the generalization

$$\frac{\partial^n \psi(x,z)}{\partial z^n} \approx [D_-^n \phi](x, z) \equiv \int_{-\infty}^{\infty} [\pm 2\pi i k_z]^n \phi(k_x,z) e^{2\pi i k_x x}\, dk_x \tag{12}$$

where the subscript '−' in the operator D_-^n indicates the operator applies an inverse Fourier integral.

The operators D_-^n and D_+^n provide the exact second derivatives but are only approximate for all other orders. In the limit of constant velocity they become exact for all n. When D_-^n and D_+^n are used in the

Taylor series expansion, the results are the two elemental extrapolation methods PSPI and NSPS.

Returning to equation (6), the required n-th depth derivatives can be replaced by D_-^n to give

$$\psi(x,z) \approx \psi(x,0) + \sum_{j=1}^{\infty} \frac{z^j}{j!}[D_-^j \phi](x,0) \tag{13}$$

or, using equation (12), and interchanging the order of summation and integration leads to

$$\psi(x,z) \approx \int_{-\infty}^{\infty} \left[\sum_{j=0}^{\infty} \frac{\pm 2\pi i z k_z}{j!}\right] \phi(k_x,0) e^{2\pi i k_x x}\, dk_x \tag{14}$$

where the first term of equation (13) is included through the 0-th term in the summation. Recognizing the series expansion for the exponential function allows this to be written as

$$\psi(x,z) \approx \int_{-\infty}^{\infty} \alpha(x,k_x,\pm z)\phi(k_x,0) e^{2\pi i k_x x}\, dk_x \tag{15}$$

where the symbol of this pseudodifferential operator is called the *non-stationary phase shift operator* and is given by

$$\alpha(x,k_x,\pm z) = e^{\pm 2\pi i z k_z(x,k_x)}. \tag{16}$$

Equation (15) is the nonstationary wavefield extrapolator identified as the limiting form of PSPI by Margrave and Ferguson [**9**]. Fishman and McCoy [**3**] developed the same expression as an approximate factorization of the Helmholtz equation when v depends only on the transverse coordinates. They characterize it as a high frequency approximation.

The development of a second expression for wavefield extrapolation using D_+^n (equation (11)) follows in a similar fashion but starts with the Fourier transform of the Taylor series in equation (6)

$$\phi(k_x,z) = \phi(k_x,0) + \sum_{j=1}^{\infty} \frac{z^j}{j!}\left[\frac{\partial^j \phi(k_x,z)}{\partial z^j}\right]_{z=0}. \tag{17}$$

After replacing the derivatives with the approximation D_+^n and performing similar manipulations as before, this becomes

$$\phi(k_x,z) \approx \int_{-\infty}^{\infty} \alpha(x,k_x,\pm z)\psi(x,0) e^{-2\pi i k_x x}\, dx \tag{18}$$

with $\alpha(x,k_z,\pm z)$ given by equation (16). This result is the NSPS extrapolator identified by Margrave and Ferguson [**9**].

4 Assessing the accuracy of the approximations. The wavefield extrapolators given by equations (15) and (18) have been well described in Margrave and Ferguson [**9**]. They are equivalent to ordinary phase shift when velocity is constant but can give dramatically different results with rapid lateral gradients. In the (x, ω) domain, they can be shown to be the transpose of one another (Margrave and Ferguson [**10**]). Here we investigate the accuracy of the approximations made in the preceding derivation. For this purpose, we compare the exact second derivatives (equations (8) and (10)) to the result of two applications of the approximate first derivatives D_+^1 and D_-^1. This comparison reveals error terms in both approximations that are complex valued, and have opposing trends.

Beginning with D_+^1, the approximate second derivative is

$$\frac{\partial^2 \phi(k_x, z)}{\partial z^2} \approx [D_+^1 D_+^1 \psi](k_x, z) = -\int_{-\infty}^{\infty} \gamma_z(y, k_x)\psi(y, z) e^{-2\pi i k_x y}\, dy \tag{19}$$

where the symbol γ_z is defined by

$$\gamma_z(y, k_x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_z(x, k_x) k_z(y, m) e^{2\pi i [m-k_x][y-x]}\, dx\, dm. \tag{20}$$

From D_-^1 the approximate second derivative is

$$\frac{\partial^2 \psi(x, z)}{\partial z^2} \approx [D_-^1 D_-^1 \phi](x, z) = -\int_{-\infty}^{\infty} \bar{\gamma_z}(y, k_x)\psi(y, z) e^{-2\pi i k_x y}\, dy \tag{21}$$

where $\bar{\gamma_z}$ is the complex conjugate of γ_z.

Equations (19) and (21) are pseudodifferential equations that map the wavefield ψ (equation (19)) or the spectrum ϕ (equation (21)) to their approximate second depth derivatives simultaneous with a change in Fourier domain. The symbol γ_z is the composition of symbols $k_z(x, k)$ and $k_z(y, m)$. A general theorem for a composition of symbols (Stein, [**11,** 237–238]) can be used to provide an asymptotic formula for γ_z with the result

$$\gamma_z(x, m) = k_z^2(x, m) - i\frac{\partial k_z(x, m)}{\partial m}\frac{\partial k_z(x, m)}{\partial x} - \frac{1}{2}\frac{\partial^2 k_z(x, m)}{\partial m^2}\frac{\partial^2 k_z(x, m)}{\partial x^2} + \cdots . \tag{22}$$

Since the even terms are real, $\bar{\gamma}_z$ will have the opposite sign on every other term. The first term in this asymptotic series reproduces the action of the exact second depth derivative. Higher order terms represent error, and the odd numbered terms are complex. Generation of complex terms by application of D_+^n or D_-^n may explain the instability of PSPI observed by Etgen [**2**] and demonstrated for NSPS by Margrave and Ferguson [**10**]. Uncontrolled complex values in the exponent k_z of α (equation (16)) can lead to instability during recursive application.

This result suggests, though it falls short of a proof, that the first-order errors in the PSPI and NSPS extrapolators oppose one another. NSPS uses D_+^1 while PSPI uses D_-^1 and these operators have been shown to have opposing errors. However, the extrapolators also use all other orders of the D operators so the complete story is much more complex than we present here.

The validity of these asymptotic series requires the existence of all orders of spatial and wavenumber derivatives of k_z. The wavenumber derivatives will exist to all orders except possibly at the evanescent boundary. The spatial derivatives impose a condition of smoothness upon $v(x)$. This condition is not necessarily required for the NSPS and PSPI extrapolators themselves, but it is needed for this form of error analysis.

These investigations, together with the fact that NSPS and PSPI are the (x, ω) domain transposes of one another, suggest that a symmetric combination of these elemental extrapolators may be more accurate. This does indeed seem to be the case as was reported by Margrave and Ferguson [**10**] but there are many possible symmetric forms. Also, either NSPS or PSPI is as accurate as the best explicit finite difference technique and accuracy increases as the extrapolation step size decreases.

As an illustration, consider an initial wavefield consisting of nine bandlimited impulses arranged symmetrically from -1500 m to $+1500$ at .52 seconds. Let a velocity model be defined such that $v = 5000$ m/s if $x < 0$ and $v = 2000$ m/s if $x > 0$. Figure 1 shows the result of the extrapolation of the initial wavefield 200 m upward using NSPS while Figure 2 shows the result from using PSPI. The PSPI result shows characteristic wavefield discontinuities where the velocity model is discontinuous. While the NSPS result looks more pleasing it is also incorrect since the hyperbolae should change slope where they cross the velocity boundary. (The PSPI result does change slope but is discontinuous.) Figure 3 shows the simple arithmetic average of the results in Figures 1 and 2. The result in Figure 3 is more physical than either of the others because it has minimal discontinuities and events tend to change slope at velocity

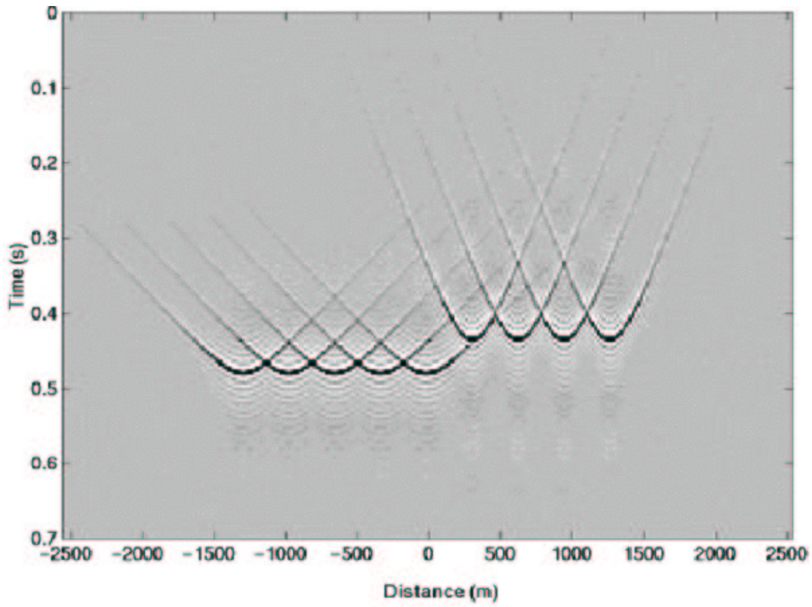

FIGURE 1: A horizontal alignment of 9 bandlimited impulses have been upward extrapolated 200 m using NSPS through a velocity model that is discontinuous at $x = 0$. On the left, the velocity was 5000 m/s and on the right it was 2000 m/s.

boundaries. The arithmetic average is a type of symmetric extrapolator. Another possibility is the alternating cascade of NSPS and PSPI. Even though this example violates the conventions of the derivation in that kz is discontinuous, the expected complementary behavior is still seen. This suggests that the result is more general than the boundaries of our derivation might indicate.

5 Conclusions. The NSPS and PSPI wavefield extrapolators can be derived from the Helmholtz equation for laterally variable media. The Taylor series approach relies on the development of approximate pseudodifferential operators for the n-th order wavefield derivatives taken in the extrapolation direction. Two alternative forms for such operators were developed from the Helmholtz equation. Both forms have the same kernel but one applies it with a change from the Fourier domain to the space domain and the other changes domains in the reverse direction.

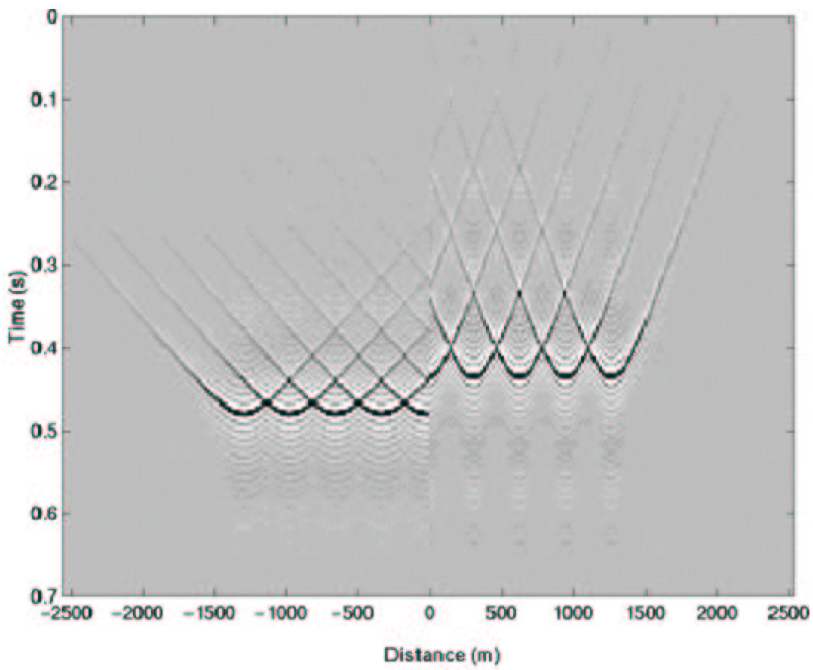

FIGURE 2: A similar result to Figure 1 except that the PSPI extrapolator has been used.

When these approximate derivatives are used in the Taylor series, the series can be summed to generate two alternative wavefield extrapolators of the familiar exponential form. The effectiveness of the approximate derivative operators was assessed using the composition theorem of pseudodifferential operators. The result suggests that the two elementary extrapolators are complementary in that their errors tend to oppose one another. A numerical example supports this by showing that the average of NSPS and PSPI extrapolations is superior to either alone.

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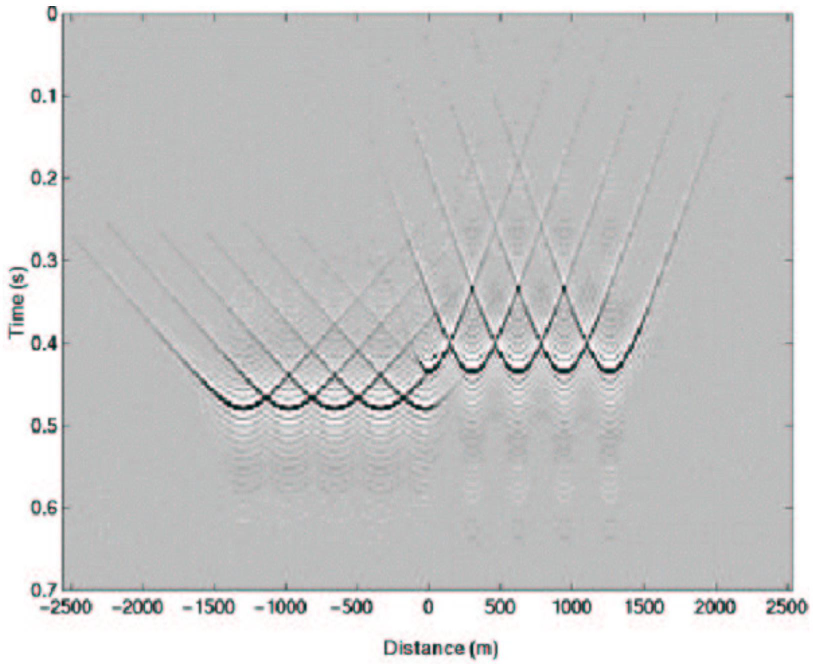

FIGURE 3: The wavefield of Figures 1 and 2 have been arithmetically averaged.

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