Differencing of time-lapse survey data using a projection onto convex sets algorithm

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Goal

Utilizing projection onto convex sets (POCS) Algorithm to obtain a reliable difference section from ireegulary sample baseline and monitor surveys.

Time-lapse survey

We suppose that $\mathbf{d}_b$ and $\mathbf{d}_m$ show the spatial samples of baseline and monitor surveys at a given frequency in the frequency-space (f-x) domain, respectively. Both baseline and monitor surveys have N samples with the same spatial locations. The observed baseline and monitor surveys, $\mathbf{d}_b^{obs}$ and $\mathbf{d}_m^{obs}$, have randomly missing samples. The ideal and observed surveys are related by

$$\begin{aligned} \mathbf{d}_b^{obs} &= \mathbf{T}_b \mathbf{d}_b, \\ \mathbf{d}_m^{obs} &= \mathbf{T}_m \mathbf{d}_m, \end{aligned} \qquad (1)$$

where $\mathbf{T}_b$ and $\mathbf{T}_m$ are the baseline and monitor sampling matrices, respectively. Notice that sampling matrices are diagonal matrices with size $N \times N$ with diagonal zero and one values for missing and available samples, respectively. The sampling matrix of subtracted section is obtained by

$$\mathbf{T}_s = \mathbf{T}_b \mathbf{T}_m. \qquad (2)$$

The observed subtracted section is equal to

$$\mathbf{d}_s^{obs} = \mathbf{T}_s \mathbf{d}_s. \qquad (3)$$

Projection onto convex sets (POCS)

The POCS (Abma and Kabir, 2003) algorithm is a Fourier based data reconstruction method for the signals with a sparse number of harmonics. Let's explain POCS method by implementing it to recover the baseline survey. The POCS algorithm for baseline survey reconstruction can be summarized as

$$\begin{aligned} &\text{Initialization} \\ &\quad \mathbf{d}_b^0 = \mathbf{d}_b^{obs} \\ &\text{For} k = 1, 2, 3 \ldots \\ &\quad\quad \mathbf{d}_b^k = \mathbf{d}_b^{obs} + (\mathbf{I} - \mathbf{T}_b)(\mathbf{F}^H \Gamma^k \mathbf{F} \mathbf{d}_b^{k-1}), \\ &\text{End} \end{aligned} \qquad (4)$$

where $\mathbf{F}$ and $\mathbf{F}^H$ are the forward and inverse Fourier transforms and Γ^k is the thresholding function which enforces samples with absolute values below the threshold values to be zero. Notice that the term $\mathbf{I} - \mathbf{T}$ as an operator which only keeps the values of the missing spatial samples. The thresholding function performs as follow

$$\Gamma(\mathbf{D}) = \begin{cases} D(i) & |D(i)| > \lambda_k, \\ 0 & |D(i)| \le \lambda_k, \end{cases} \qquad (5)$$

Where $\mathbf{D}$ represent the Fourier representation of of the data $\mathbf{D}$. It is recommended to choose higher threshold values at the start iterations and decrease its value for the last ones.

POCS time-lapse survey subtraction

The procedure is summarized as below

$$\begin{aligned} &\text{Initialization} \\ &\quad \mathbf{d}_b^0 = \mathbf{d}_b^{obs} + [(\mathbf{I} - \mathbf{T}_b)\mathbf{T}_m]\mathbf{d}_m^{obs}, \\ &\quad \mathbf{d}_m^0 = \mathbf{d}_m^{obs} + [(\mathbf{I} - \mathbf{T}_m)\mathbf{T}_b]\mathbf{d}_b^{obs}, \\ &\quad \mathbf{T}_s = \mathbf{T}_b \mathbf{T}_m, \\ &\quad \mathbf{d}_s^{obs} = \mathbf{T}_s(\mathbf{d}_m^{obs} - \mathbf{d}_b^{obs}), \\ &\quad \mathbf{d}_s^0 = \mathbf{d}_s^{obs}, \\ &\text{For} \quad k = 1, 2, 3 \ldots \\ &\quad\quad \mathbf{M}_b^k = \Upsilon_{\lambda_k}(\mathbf{F}\mathbf{d}_b^{k-1}), \\ &\quad\quad \mathbf{M}_m^k = \Upsilon_{\lambda_k}(\mathbf{F}\mathbf{d}_m^{k-1}), \\ &\quad\quad \mathbf{d}_b^k = \mathbf{d}_b^{obs} + (\mathbf{I} - \mathbf{T}_b)(\mathbf{F}^H \mathbf{M}_b^k \mathbf{d}_b^{k-1}), \\ &\quad\quad \mathbf{d}_m^k = \mathbf{d}_m^{obs} + (\mathbf{I} - \mathbf{T}_m)(\mathbf{F}^H \mathbf{M}_m^k \mathbf{d}_m^{k-1}), \\ &\quad\quad\quad \mathbf{M}_s^k = \Upsilon_0(\mathbf{M}_b^k + \mathbf{M}_m^k), \\ &\quad\quad\quad \mathbf{d}_s^{k-1} = \mathbf{d}_s^{k-1} + (\mathbf{I} - \mathbf{T}_s)(\mathbf{d}_m^k - \mathbf{d}_b^k), \\ &\quad\quad\quad \mathbf{d}_s^k = \mathbf{d}_s^{obs} + (\mathbf{I} - \mathbf{T}_s)(\mathbf{F}^H \mathbf{M}_s^k \mathbf{d}_s^{k-1}), \\ &\text{End} \end{aligned} \qquad (6)$$

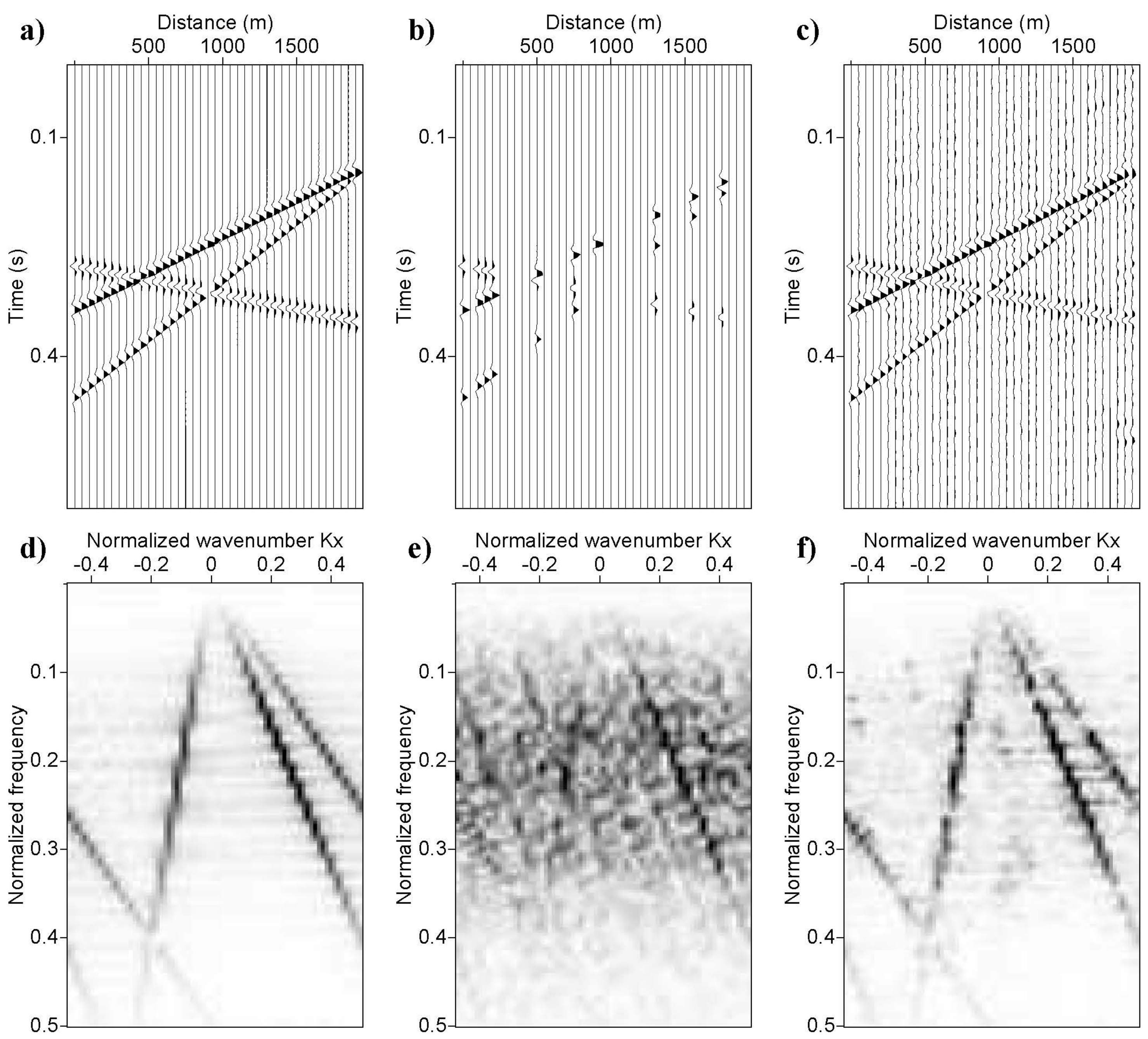
where Υ_{λ_k} is a mask building function defined as

$$\Upsilon(\mathbf{D}) = \begin{cases} 1 & |D(i)| > \lambda_k, \\ 0 & |D(i)| \le \lambda_k, \end{cases} \qquad (7)$$

The matrices $\mathbf{M}_b$, $\mathbf{M}_m$, and $\mathbf{M}_s$ are the mask function correspondent to baseline, monitor and difference data, respectively.

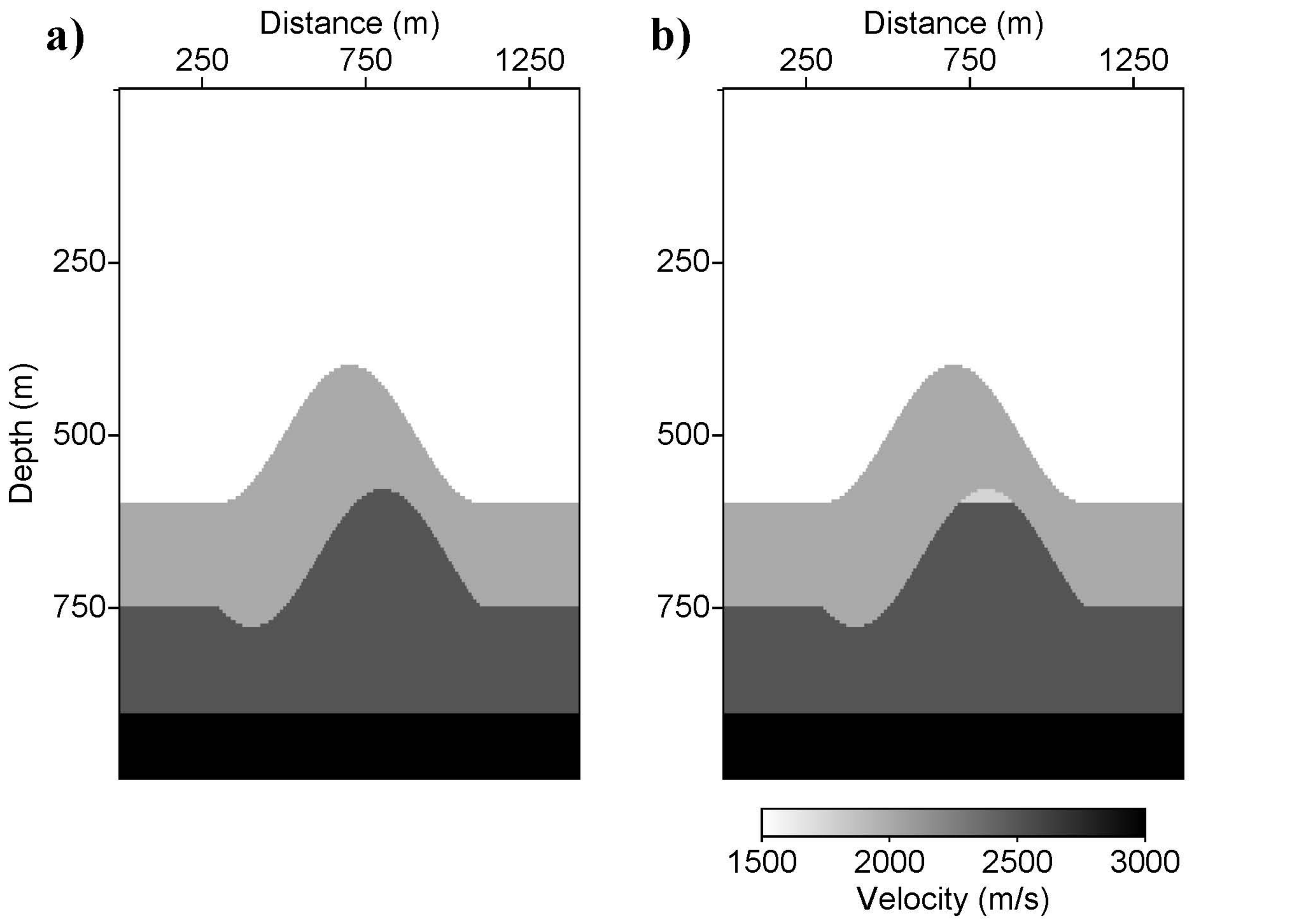
Synthetic example

a) Original difference data. b) Difference data with missing traces. c) Reconstructed difference data. d-f) are the f-k spectra of a-c, respectively.

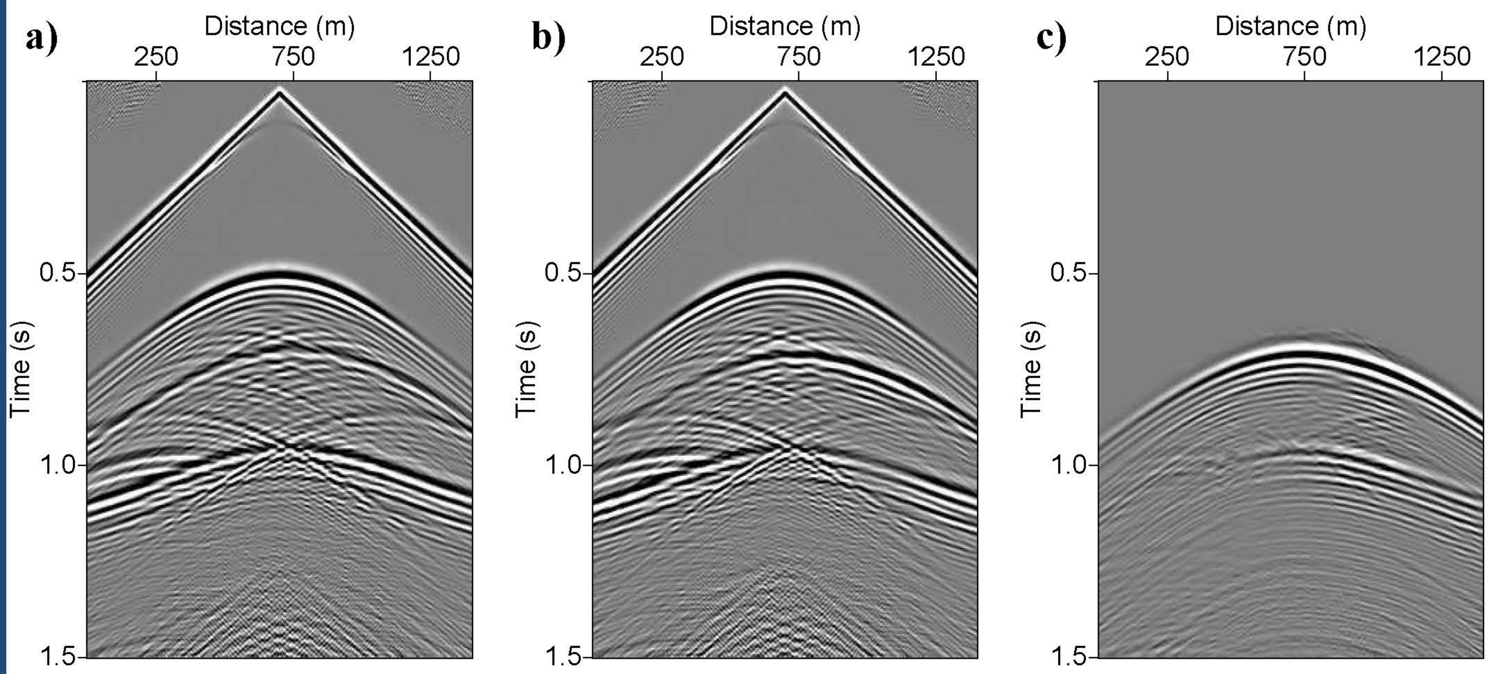


Modeled time-lapse survey

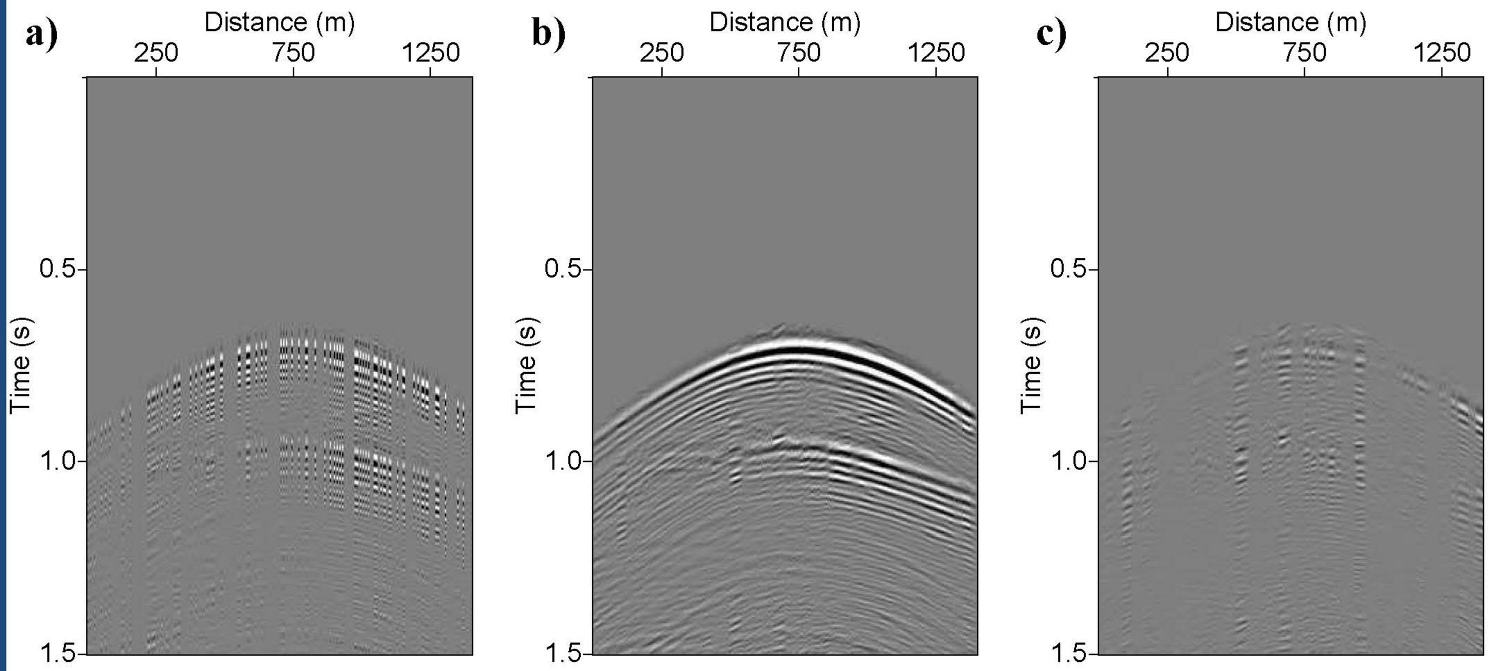
a) Subsurface model used for baseline survey. a) Subsurface model used for monitor survey.



a) A shot gather from baseline survey. a) The correspondent shot gather from monitor survey. a) The difference data.



a) Difference data with 70% randomly missing traces. b) Reconstructed difference data. c) The reconstruction error section.



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