3-D porosity prediction using P-P and P-S seismic inversions

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Introduction
Porosity mapping may be of key importance in the development of a hydrocarbon reservoir. With 3C-3D seismic data and well logs, we may be able to derive a very compelling image of the reservoir porosity. We show an example of porosity prediction in the Blackfoot field, Alberta, using simultaneously results from P-P and P-S inversions. Additional seismic attributes derived from the P-P and P-S data are used to enhance the prediction. Linear multi-regression and neural networks are used to derive the relationship between the seismic attributes and porosity logs at the well locations. The relationship is applied to the attributes and a 3-D porosity volume is generated.

We analyze a 3C-3D seismic survey recorded in October, 1995, with a primary target the Glauconitic member of the Mannville group (Margrave et al., 1998). The reservoir occurs at a depth of around 1550 m. (1060 ms), where Glauconotic sand and shale fill valleys incised into the regional Mannville stratigraphy. The objectives of the survey are to delineate the channel and distinguish between sand-fill and shale-fill.

P-S inversion for shear velocity
The Zoeppritz equations allow us to derive the exact plane wave amplitude of a converted S-wave from an incident P-wave as a function of angle, but do not give us an intuitive understanding of how this amplitudes relate to the various physical parameters. Aki and Richards (1980) approximate the equation assuming small changes in elastic properties across an interface:

\[
R^\text{PS} = c \frac{\Delta \rho}{\rho} + d \frac{\Delta \beta}{\beta}, \quad \text{where}
\]

\[
c = -\frac{\alpha \tan \phi}{2\beta} \left(1 - \frac{2\beta^2}{\alpha^2} \sin^2 \theta + \frac{2\beta}{\alpha} \cos \theta \cos \phi \right)
\]

\[
d = \frac{\alpha \tan \phi}{2\beta} \left(\frac{4\beta^2}{\alpha^2} \sin^2 \theta - \frac{4\beta}{\alpha} \cos \theta \cos \phi \right)
\]

\[
\theta = (\theta_i + \theta_{i+1})/2, \quad \phi = (\phi_i + \phi_{i+1})/2 \quad \text{- average angles across the interface}
\]

\[
\alpha, \beta, \rho \quad \text{- average P-wave velocity, S-wave velocity, and density across the interface}
\]

\[
\Delta \beta/\beta, \Delta \rho/\rho \quad \text{- relative changes in S-wave velocity and density}
\]

Equation (1) can be cast as a least-squares problem and solved for \(\Delta \beta/\beta\) (Stewart, 1990):

\[
\frac{\Delta \beta}{\beta} = \frac{\sum_{i=1}^{n} R^\text{PS}_i d_i - \frac{\Delta \rho}{\rho} \sum_{i=1}^{n} c_i d_i}{\sum_{i=1}^{n} d_i^2}
\]

To obtain the \(\Delta \beta/\beta\) weighted stack, we need a geological model containing P-wave velocity, S-wave velocity, and density. The model, in P-S time, can be built in the following way:

- at the well locations compute the P-S pseudo-velocity logs, defined by:

\[
P - S \text{ pseudo - velocity log} = 2 \left(\frac{V_P V_S}{V_P + V_S}\right)
\]
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where $V_P$ and $V_S$ are the measured P-wave and S-wave velocity logs

- using the computed P-S pseudo-velocity log convert the $V_P$, $V_S$, and density logs into P-S time
- build a 3-D $V_P$, $V_S$, and density volumes in P-S time by 3-D interpolation

The $\Delta\beta/\beta$ weighted stack calculation requires knowledge of the incident angle at any particular interface (the reflection and transmission angles can be found using Snell’s law). The incident angle can be found using ray tracing, but in a complex model, as the one discussed above, the required time may be large and thus unattractive. The problem can be solved by deriving an approximation for the incident angle as a function of the P-S offset $X_{PS}$, P-wave velocity model $\alpha$, and S-wave velocity model $\beta$ (Todorov and Stewart, 1998):

$$\sin \theta_i = \frac{2gX_{PS}\alpha_i}{\alpha_{RMS}^2 \sqrt{\left(\frac{2\beta}{\alpha + \beta} t_{PS0}\right)^2 + \frac{4g^2X_{PS}^2}{\alpha_{RMS}^2}}} \quad \text{where} \quad g = \frac{1}{1 + \frac{\alpha}{\beta} \frac{\beta_{RMS}^2}{\alpha_{RMS}^2}} \quad (6)$$

The angle goes into equations (2) and (3) to calculate the weights for the $\Delta\beta/\beta$ stack.

The P-S inversion flow begins with building the geological model in P-S time, containing P-wave, S-wave and density information for each seismic sample. Then using the model, we calculate the stacking weights for each NMO-corrected CCP gather and perform weighted stacking. The resulted $\Delta\beta/\beta$ volume can be inverted using any available P-P inversion routine to derive the shear velocity.

The derived algorithm has been applied to the Blackfoot data set (Figures 1 and 2).

**Figure 1:** Weighted stack traces from P-S gathers.  **Figure 2:** Shear velocity from P-S inversion.

**Model-based conversion of P-S data to P-P time**

To simultaneously use the seismic attributes extracted from P-P and P-S data, we have to convert the P-S data to P-P time. To do so we need P-S velocity information. The two-way zero-offset P-S time $t_{PS}$ to a particular depth $z$ is:

$$t_{PS} = \frac{z}{V_P} + \frac{z}{V_S} = z \left(\frac{V_P + V_S}{V_P V_S}\right) = \frac{2z}{V_{PS}}, \quad \text{where:} \quad V_{PS} = 2 \left(\frac{V_P V_S}{V_P + V_S}\right) \quad \text{is a P-S pseudo-velocity logs.}$$

The model-based conversion scheme is done in the following way:

- at the well locations compute the P-S pseudo-velocity logs and $V_{PS}/V_P$ logs
- using the computed P-S pseudo-velocity logs convert the $V_{PS}/V_P$ logs into P-S time
- build a 3-D $V_{PS}/V_P$ model in P-S time by 3-D interpolation
- compute the P-P time for each P-S sample using the following equation:

$$PP_{time \ (n-th \ sample)} = \sum_{i=1}^{n} \left(\frac{V_{PS}}{V_P}\right)_i \Delta t$$
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where \( \Delta t \) is the sampling rate.
Note that the sampling rate in the resulting seismic trace is not a constant due to varying \( V_{ps}/V_p \) ratio. Linear interpolation is used to resample the converted to P-P time trace in a regularly sampled sequence. Using the described procedure the Blackfoot converted wave (P-S) data volume is converted to P-P time.

Prediction of porosity logs
The use of linear multi-regression for well log prediction has been described by Todorov et al. (1998). We show an example of prediction of porosity logs for the Blackfoot field using simultaneously attributes from P-P and P-S data. The calculation of the attributes for both data sets is the same, however we have to convert the P-S data to P-P time. Since the porosity is very often correlated with the impedance, the results from the model-based P-P and P-S inversions are used as additional attributes.

Table 1 shows the result of the performed multi-regression using 3-point convolution operator. The Mannville - Mississippian interval has been used to derive the result and it is reliable for the later interval. In the current example, we see that by adding the seventh attribute, Seismic amplitude of the P-P trace, the validation error increases. So we choose to use the first six attributes in the prediction process.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>RMS error %</th>
<th>Validation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance from P-P inversion</td>
<td>4.377</td>
<td>4.440</td>
</tr>
<tr>
<td>S-velocity from P-S inversion</td>
<td>4.317</td>
<td>4.398</td>
</tr>
<tr>
<td>Integrated trace (P-P)</td>
<td>4.248</td>
<td>4.389</td>
</tr>
<tr>
<td>Amplitude envelope (P-S)</td>
<td>4.222</td>
<td>4.386</td>
</tr>
<tr>
<td>Integrated trace (P-S)</td>
<td>4.191</td>
<td>4.381</td>
</tr>
<tr>
<td>Cosine instantaneous phase (P-S)</td>
<td>4.126</td>
<td>4.341</td>
</tr>
<tr>
<td>Seismic amplitude (P-P)</td>
<td>4.093</td>
<td>4.359</td>
</tr>
</tbody>
</table>

Table 1: Result from the multi-regression.

Two types of neural networks, feed-forward and probabilistic (Masters, 1995), are trained using the same six attributes with 3-point convolutional operator. The probabilistic neural network shows superior results, i.e. lower prediction and validation error. Figures 3 and 4 show the measured (in black) and the predicted (in red) porosity logs at 08-08 and 09-08 locations. We can see that the neural network predicts the logs with higher accuracy. The multi-regression predicted the logs with correlation 0.76 while the neural network predicted them with correlation 0.91.

Figure 3: Measured porosity log (in black) and the predicted one (in red) using multi-regression. The correlation is 0.76.

Figure 4: Measured porosity log (in black) and the predicted one (in red) using neural network. The correlation is 0.91.
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Once the relationship between the seismic attributes and the porosity logs has been determined it is applied to the data volumes. Figure 5 shows a cross-line and a data slice at the channel level from the predicted porosity cubes. The sand channel can be distinguished very well as a high porosity anomaly (in blue) around 1100 ms. Note the high resolution achieved using the neural network.

![Figure 5: Predicted porosity using probabilistic neural network, a data slice at the channel level and a line. The high porosity anomaly correlates with the producing oil wells.](image)

References
Aki, K., and Richards, P., 1980, Quantitative seismology, theory and methods: W. H. Freeman and company
Margrave, G., Lawton, D., and Stewart, R., 1998, Interpreting channel sands with 3C-3D seismic data: The Leading Edge, 17(4)
Todorov, T., and Stewart, R., 1998, 3-D converted wave inversion for shear velocity: The CREWES Project Research Report, 10
Todorov T., Stewart, R., Hampson, D., Russell, B., 1998, Well log prediction using attributes from 3C-3D seismic data: SEG Expended Abstracts

Biographical Note:
Todor Todorov graduated from university of Mining and Geology, Sofia, Bulgaria in 1996 with a B.Sc. in exploration geophysics. Currently he is working as Special Projects Geophysicist at Hampson-Russell Software and is taking his M.Sc. at University of Calgary (The CREWES Project).