Summary

Seismic data is arguably always nonstationary in its Fourier spectral character since anelastic attenuation processes are present everywhere. We present a nonstationary deconvolution technique, designed to estimate and remove both the source signature and the subsequent earth attenuation effects. Our method is a direct extension of the familiar Wiener deconvolution when the latter is posed in the Fourier domain. We use the Gabor transform, a nonstationary generalization of the Fourier transform, to perform a time-frequency decomposition of a seismic trace. We then smooth the magnitude of this Gabor spectrum in such a way that we estimate the product of the magnitudes of the attenuation function and the source signature. A phase function is then calculated as in the stationary case by using the minimum phase assumption. At this stage, we have an estimate of a complex-valued, time-frequency function that we call the spectrum of the propagating wavelet. When the original Gabor spectrum is divided by this time-frequency spectrum of the propagating wavelet, the result is an estimate of the Gabor spectrum of the reflectivity. Testing on nonstationary synthetic signals shows that Gabor deconvolution is dramatically effective in producing a very broadband reflectivity estimate, far better than Wiener’s algorithm. Testing on real data also shows very highly resolved images from Gabor deconvolution. When compared with a “standard” image created using surface-consistent Wiener deconvolution together with time-variant spectral whitening, the Gabor images show subtle enhancement of detail and greater robustness in the presence of coherent noise and coverage gaps.

Introduction

Norbert Wiener’s deconvolution algorithm, brought to geophysics by Enders Robinson (e.g. Robinson, 1967) is both elegant and important. In a simple prescription, it tells us how to whiten and phase-correct seismic data. Underlying this algorithm is the convolutional model of a seismic trace, which postulates that the seismic trace is the convolution of two other signals, one which we call the “wavelet” and another that we call the “reflectivity”. Wiener deconvolution adopts this convolutional model and proceeds to make assumptions about both wavelet and reflectivity in order to estimate and remove (i.e. de-convolve) the former, thus revealing the latter. Plausible though this model is, it does not accommodate many physical effects such as attenuation and the general multiple series. For this reason, we have developed a nonstationary extension of the convolutional model and use it as the basis for a new deconvolution technique that directly estimates and removes both Q attenuation and source signature. The initial work on this new algorithm was presented at the 2002 CSEG National Convention (Margrave et al, 2002).

The Gabor Transform

We have developed a particular implementation of the Gabor transform, appropriate for discretely sampled, bandlimited signals based on the concept of a partition of unity (POU). A POU is a set of windows defined on the real line and so chosen that they always sum to unity. If \( \Omega(t) > 0 \) denotes an always positive window function, then it forms a POU if
where by $\Omega_n$ we mean the window $\Omega$ translated (shifted) to the time $n\Delta\tau$. The POU defined by equation (1) uses a single window function translated at regular intervals along the real line to form a POU. Next we define an analysis window, $g$, and a synthesis window, $\gamma$ by $g_n = \Omega_n^p$, $p \in [0,1]$ and $\gamma_n = \Omega_n^{1-p}$. So $g$ and $\gamma$ are just fractional powers of $\Omega$ such that $\sum g_n = \sum \Omega_n = 1$. We define the Gabor slice of a seismic trace $s$ using the analysis window $g$ as $s_n(t) = s(t)g_n(t) = s(t)g(t-n\Delta\tau)$ and the Gabor transform is the Fourier transform over the set of Gabor slices

$$G[s](n,f) = \hat{s}_n(f)$$

where the “hat” over a variable indicates the (forward) Fourier transform. This equation emphasizes that the Gabor transform of $s$ is a two-dimensional time-frequency decomposition. Here time is denoted by the integer $n$ and frequency by the real number $f$. The use of a continuous variable for frequency is just a notational convenience as, in any numerical implementation we use the discrete Fourier transform and calculate equation (2) over a discrete set of frequencies as well as times. The inverse the Gabor transform is easily accomplished using the synthesis window $\gamma$, the inverse Fourier transform, $F^{-1}$, and summation over $n$, by the equation

$$G^{-1}[G(s)] = \sum \gamma_n F^{-1}[\hat{s}_n] = s.$$

**Deconvolution in the Gabor Domain**

We adopt a nonstationary seismic trace model that includes the effects of attenuation (i.e. $Q$) and source signature but excludes multiples. When the Gabor transform is applied to such a nonstationary seismic trace the result is approximately a product of factors given by

$$G[s](n,f) = \hat{w}(f)\alpha(t = n\Delta\tau, f)G[r](n,f) + \cdots$$

where $w$ is the source waveform and $\alpha$ is the time and frequency dependent attenuation function. We will not present the detailed mathematical justification of this equation here; but, it is easy to see why this result makes sense. Isolate a small portion of a seismic trace with a window; then, from the perspective of the stationary convolutional model, we expect its Fourier spectrum to factor into the product of an effective wavelet (Fourier) spectrum times a reflectivity (Fourier) spectrum. Now, if we repeat the process for the same window shifted to a later time, we expect the Fourier spectrum to be the product of the spectra of a different effective wavelet and a different reflectivity. If we consider only attenuation and neglect multiples, then these two different effective wavelets are both the product of the Fourier spectrum of the actual waveform emitted by the source times a different $Q$ filter for each window position. That is precisely what the first two factors on the right-hand-side of equation (4) are. Then, having a different reflectivity spectrum involved in each window is precisely what is meant by having the third factor since the Gabor transform is simply a windowed Fourier transform. In this light, for fixed $n$, equation (4) is simply a temporally localized version of the stationary convolutional model.

As with Wiener deconvolution, the first step in our algorithm is to smooth the magnitude of $G[s](n,f)$ in such a way to eliminate all features attributable to $G[r](n,f)$. The simplest possible smoothing procedure is to convolve with a 2D boxcar, whose time and frequency dimensions must be chosen empirically. Unfortunately, the resulting deconvolution has an amplitude equalization effect, much like an AGC. Usually better is a technique we call
**hyperbolic smoothing.** Since the level-lines of the attenuation function, for $Q$ a constant, are the hyperbolic family defined by $tf = \text{constant}$, we calculate the mean value of the Gabor magnitude spectrum along such contours as an estimate of $\alpha(n, f)$. We divide $|G[s](n, f)|$ by the estimate for $\alpha(n, f)$, average over $t$, and smooth in $f$ with a convolution operator to estimate $|\hat{\hat{w}}(f)|$. This becomes an estimate of the Gabor magnitude spectrum of the propagating wavelet that we will call $|\hat{w}_\alpha|$ and we assert that

$$|\hat{w}_\alpha|(n, f) \approx |\hat{\hat{w}}(f)|$$

(5)

where the approximation holds with a hopefully small, but admittedly unknown, error term. Using the minimum-phase assumption to calculate the phase of $\hat{w}_\alpha$ and including a small stability constant $\varepsilon$, as in Wiener deconvolution, we estimate the entire Gabor spectrum of the propagating wavelet as

$$\hat{w}_\alpha = (|\hat{w}_\alpha| + \varepsilon)^{-1} e^{-it|\ln|\hat{w}_\alpha|+\varepsilon|}$$

(6)

where we understand that both sides of the equation depend on discrete time $n$ and frequency $f$, and the Hilbert transform, $H$, is to be taken over frequency. The actual deconvolution is accomplished by spectral division in the Gabor domain

$$G[d](n, f) = \frac{G[s](n, f)}{\hat{w}_\alpha(n, f)}$$

(7)

where we have used $d$ to symbolize the deconvolved seismic trace and the division is understood to be a point-by-point operation using complex arithmetic. $d$ itself is recovered by an inverse Gabor transform of the result from equation (7). Usually, it is convenient to bandlimit $G[d](n, f)$ in the Gabor domain prior to the inverse transform. Since the attenuation function may be nearly constant along the hyperbolae $tf = \text{constant}$, we find it suitable to define a (time-variant) high frequency cutoff whose value follows a particular hyperbolic trajectory.

**Examples**

Figure 1 shows an example with a synthetic nonstationary signal. The time-domain traces are (from the bottom up) an input attenuated seismogram, the Wiener deconvolution result, the Gabor deconvolution result, and the bandlimited true reflectivity. The Gabor deconvolution result has estimated a very high resolution reflectivity that has a strong correlation with the true reflectivity. We emphasize that this was done without knowing the value of $Q$. The Wiener deconvolution result in the design gate (.8-1.2 seconds) is a reasonably good result, with some resolution of the major events. However, even over this design gate, where the result should be ‘optimal’ there is clear evidence of spectral decay. At earlier times, there is evidence of overwhitening, and also there is obvious underwhitening (poor resolution) at later times.

Figure 2 shows a real data comparison. The data processing was not done by us, but rather by Sensor Geophysical of Calgary, Alberta. The standard processing flow begins with true-amplitude gain recovery and continues with surface consistent Wiener deconvolution. This is followed by TVSW (time variant spectral whitening), CMP Stack, TVSW again, and FX noise reduction. For the Gabor flow, we substituted a pre-stack Gabor deconvolution for the sequence “true-amplitude recovery, surface consistent deconvolution, and TVSW” and replaced the post-stack TVSW with a post-stack Gabor deconvolution. Our test line was provided courtesy of Husky Energy and was acquired over a sedimentary basin using a dynamite source. The outlined feature in Figure 2 is likely a geological anomaly. It is more precisely delineated by the increased bandwidth of the Gabor result. In addition to greater bandwidth, the Gabor result appears to have better lateral continuity.
Conclusions

We have presented a nonstationary deconvolution algorithm based on the Gabor transform that effectively corrects seismic data for source signature and attenuation. Gabor deconvolution easily beats Wiener deconvolution on a nonstationary synthetic. On real data, Gabor deconvolution is very competitive with the combination of Wiener deconvolution and TVSW.

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References


Figure 1: A comparison of Gabor and Wiener deconvolution on a nonstationary synthetic.

Figure 2. A comparison of Gabor and Wiener deconvolution on real data.