Examining the phase property of the nonstationary vibroseis wavelet
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Summary
We have observed that Vibroseis wavelets behave very much as if they are minimum phase. This was discovered by applying minimum-phase Wiener deconvolution to the separated vibroseis VSP downgoing waves and observing that the result is effectively a band-limited spike. Motivated by this finding, we simulated a synthetic nonstationary vibroseis wavelet in a constant-Q medium by nonstationary convolution and again we found it to be close to minimum phase. Furthermore, we applied minimum-phase Gabor deconvolution to the sweep-removed vibroseis record and correlated vibroseis data. We obtained very similar results from these two approaches. For the synthetic data, the deconvolved traces from both approaches are consistent with the input reflectivity. For a real shot gather, there are few differences between the deconvolved gathers from these two methods. These comparisons further confirm that the nonstationary wavelet embedded in the correlated surface vibroseis seismic data is approximately minimum phase.

Introduction
In the 1980s and earlier, the wavelet embedded in correlated vibroseis data was assumed to be the zero-phase autocorrelation of the sweep (see Brötz et al., 1987, Bickel 1982). This assumption has been challenged by Sallas (1984) and Baeten and Ziólkowski (1990) who argued that the far field wavelet is not an autocorrelation function but the cross-correlation between the pilot sweep and the time derivative of the ground force, which can be estimated from a weighted sum of the vertical acceleration of the base plate and the reaction mass. Thus, the embedded vibroseis wavelet for propagation in an elastic medium is not zero phase. This argument ignores attenuation. Others (e.g. Brittle, 2001) have stated that the presence of earth-attenuation results in a mixed-phase wavelet that is the convolution of the Klauder wavelet and the earth-attenuation minimum-phase filter, but this assumption is not verified. Therefore, the phase property of the wavelet embedded in the vibroseis trace is still an unsolved problem. The theoretical investigation of this phase property is very difficult, but it can be examined empirically by applying a minimum-phase deconvolution. In time-domain Wiener spiking deconvolution, if the wavelet is band-limited minimum-phase and the reflectivity is white, the output of the deconvolution would resemble a band-limited zero-phase impulse, we can infer that the wavelet is approximately minimum phase.

In this investigation, first the phase property of the vibroseis wavelet is examined on both the simulated wavelet and the directly observed wavelet. Then, the uncorrelated data are convolved with two different deconvolution operators including minimum-phase Gabor deconvolution and the frequency domain sweep deconvolution called FDSD or sweep deconvolution (Brittle and Lines, 2001). The correlated data are deconvolved directly by Gabor deconvolution. Finally the deconvolved data from these two approaches are compared. Since the remaining wavelets in the FDSD result are assumed to be minimum phase, we can infer that the wavelets embedded in the correlated gather are effectively minimum phase if the two deconvolved gathers are similar.

Gabor and sweep deconvolution
Suppose the uncorrelated vibroseis record can be modeled as the nonstationary convolution of the pilot sweep, the constant Q attenuation function and reflectivity, and written as (adapted from Margrave et al., 2003)
\[
x(t) = \frac{1}{2\pi} \int [\hat{s}(w)\hat{r}(\tau, w)\alpha(\tau, w)e^{-i\tau t} d\tau] e^{iwt} dw ,
\]
where \(x(t)\) represents the displacement of the seismic wave, \(\hat{s}\) denotes the spectrum of the sweep, \(r\) is reflectivity and \(\alpha\) is the Q attenuation function (Margrave et al, 2003). The Gabor spectrum of the seismic trace is expressed as (Margrave and Lamoureux, 2002)
\[
G[x](\tau, w) = \int x(t)g(t - \tau)e^{-iwt} dt ,
\]
where \(g\) is the Gabor analysis window and \(\tau\) is the window centre. Equation (2) can be approximately factorized as (Margrave et al, 2003)
\[
G[x](t, w) \approx \hat{s}(w)\alpha(t, w)G[r](t, w),
\]
where \(G[r]\) denotes Gabor spectrum of the reflectivity. Now we define a new function as (adapted from Brittle and Lines, 2001)
\[
Y(\tau, w) = \frac{G[x](\tau, w)}{\hat{s}(w) + \mu \max|\hat{s}(w)|}.
\]
which is the sweep-removed Gabor spectrum. Here $\mu$ is a small stability factor. After smoothing $Y(\tau, \omega)$ and solving for phase spectrum by Hilbert transform, the Gabor deconvolution operator for the uncorrelated data is given by

$$D(\tau, \omega) = e^{-i\mu \max \left\{ \ln |Y(\tau, \omega)| + \varepsilon \max \left\{ |Y(\tau, \omega)| \right\} \right\} \cdot (5)$$

where $H$ denotes Hilbert transform, $\varepsilon$ is the stability factor and the over bar represents a smoother. The result of the Gabor deconvolution is given by

$$r(\tau) = IG[Y \cdot D](\tau), \quad (6)$$

where $IG$ denotes inverse Gabor transform.

Since the embedded wavelet estimate, $Y(\tau, \omega)$, is now close to minimum phase, the result of the minimum-phase Gabor deconvolution is approximately zero phase.

**Synthetic data example**

When the resulting wave travels in a horizontally-layered constant-Q medium, with Q equal to 60 for all the layers, it will be attenuated, reflected and finally recorded on the surface. This process can be simulated by the nonstationary convolution of the sweep, the constant-Q attenuation function and the reflectivity as shown in equation (1). Here the sweep is stationary and the attenuation function is nonstationary since its Fourier spectrum changes as the wave propagates. Figure 1 shows the Q-attenuated sweep for a range of possible first arrival times. The amplitude decreases and the high frequency component decays with increasing traveltime. Figure 2 shows the reflectivity and the first two seconds of the uncorrelated record.

In the uncorrelated record, the embedded wavelet should be nonstationary and equal to the sweep convolved with a Q filter appropriate for the first arrival time. Figure 3 illustrates this by comparing the impulse responses of the attenuation filter and the result of FDSD on the attenuated sweeps, deconvolved attenuated sweeps. Except for numerical noise, the results are very similar. We emphasize that the forward Q filter is a nonstationary, but minimum phase, process.

We can also generate correlated synthetic vibroseis data by crosscorrelation of the sweep and the uncorrelated data. We made no attempt to generate a synthetic ground-force recording because we are mainly interested here in the consequences of attenuation. Figure 4 shows the correlated and nonstationary vibroseis trace. The embedded nonstationary wavelet in this trace is the crosscorrelation of the sweep and the attenuated sweep and is shown in Figure 5a. If this wavelet is effectively minimum phase, then minimum-phase spiking deconvolution, run on each trace of Figure 5a, should result in a nearly zero-phase impulse. It is observed from Figure 5b that the deconvolved wavelets are close to band-limited zero-phase impulse.

![Figure 1 Attenuated sweeps. Each sweep has been convolved with a forward Q filter appropriate for the first arrival time.](image)

![Figure 2 Reflectivity and the uncorrelated synthetic data.](image)

![Figure 3 Impulse response of the forward Q filter (a) and the output (b) of FDSD on the uncorrelated data shown in Figure 2.](image)

![Figure 4 Correlated and nonstationary vibroseis trace.](image)

![Figure 5 (a) Normalized nonstationary vibroseis wavelet and (b) the minimum-phase deconvolution of each trace in (a).](image)

We next applied FDSD followed by Gabor deconvolution to the uncorrelated synthetic data and Gabor deconvolution without FDSD to the correlated synthetic vibroseis trace. Figure 6 shows the attenuated trace, deconvolved traces and
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reflectivity. The results from both approaches are quite close to the input reflectivity and do not show large spurious phase rotations. Thus the wavelet embedded in the correlated synthetic data seems to behave as a minimum-phase wavelet.

Figure 6 Comparison of deconvolved trace with the input reflectivity.

Real data example 1, Rosedale, Drumheller
Provided by Encana, this dataset consists of 5 vibroseis source points recorded simultaneously into VSP and surface spreads. In the borehole, receivers were positioned between 322 and 1820 m depth at an interval of 20 m. An additional 78 receivers were placed on the surface between 30 and 2310 m from the borehole at a 30 m interval. The five source points were located 27, 430, 960, 1350 and 1700 m from the borehole and used a 12 s, 10-96 Hz linear sweep. Sixteen second long, uncorrelated, surface records and VSP records were recorded at a 2 ms sample rate.

The downgoing wavelets were directly estimated from the correlated VSP data using standard wavefield separation methods and are shown, time-aligned and normalized, in Figure 7(a). Wiener spiking deconvolution was then applied to these down-going wavelet estimates. The Wiener inverse operator was designed with data from 0.2 to 0.3 s of Figure 7(a). Operator length was 0.12 s and the stabilization factor was 0.0001. Figure 7(b) shows that the normalized, deconvolved wavelets are nearly identical approximations to band-limited zero-phase wavelets. This suggests that the observed wavelets in correlated VSP downgoing waves are effectively minimum phase.

Figure 7 (a) Directly observed wavelets (normalized to peak amplitude) and (b) the result of minimum phase spiking deconvolution on the wavelets of (a).

Figures 8(a) and 8(b) show the deconvolved, surface-recorded gathers from minimum-phase Gabor deconvolution on the correlated data and FDSD followed by minimum phase Gabor deconvolution on the uncorrelated data respectively. Figure 8(c) shows the difference between these data. If the wavelet in the sweep-removed data is minimum-phase, it can be inferred from the small and random differences shown in Figure 8(c) that the nonstationary wavelet embedded in the correlated surface vibroseis data is close to being minimum phase.

Real data example 2, Pikes Peak, Saskatchewan
This experiment was conducted by Husky Energy over the Pikes Peak heavy oil field in west-central Saskatchewan. Receivers were placed in a borehole at 7.5 m intervals from 27 m to 514.5 m. The vibration point was 23 m from the well and the sweep was linear from 8 to 200 Hz. Figure 9(a) shows the downgoing wavelets isolated from the correlated data, comparable to Figure 7(a). Though the results of Wiener deconvolution on these wavelets also give symmetrical, band limited impulses, we choose to show an alternate test of minimum phase. In Figure 9(b) we show the minimum-phase equivalent wavelet for each observed wavelet in Figure 9(a). These were calculated by inverting the operator obtained by running the Levinson algorithm on the downgoing wavelets, using all autocorrelation lags, with a white noise stabilization factor of 0.0001. The minimum-phase equivalent wavelets are nearly identical to the observed; and, to demonstrate this, we show the difference plot in Figure 9(c). The differences were calculated after careful alignment of the wavelets using a cross correlation.

Real data example 3, Ross Lake

Figure 8 (a) The gather after minimum-phase Gabor deconvolution of the correlated data, (b) the gather after minimum-phase Gabor deconvolution of the uncorrelated data and (c) the differences of the data shown in (a) and (c).

Figure 9 (a) Directly observed wavelets in VSP downgoing waves from Pikes Peak (normalized to peak amplitude), (b) the minimum phase equivalents of the wavelets shown in (a) and (c) the difference between (a) and (b).

Real data example 3, Ross Lake
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This experiment was conducted by Husky Energy and CREWES over the Ross Lake oil field in south-west Saskatchewan. Receivers were placed in a borehole at 7.5 m intervals from 198 m to 1165 m. The vibration point was 54 m from the well and the sweep was linear from 8 to 180 Hz. Figure 10(a) shows the downgoing wavelets isolated from the correlated data, comparable to Figure 7(a). Though the results of Wiener deconvolution on these wavelets also give symmetrical, band limited impulses, we choose again to show an alternate test of minimum phase. In Figure 10(b) we show the minimum-phase equivalent wavelet for each observed wavelet in Figure 10(a). The minimum-phase equivalent wavelets are nearly identical to the observed; and, to demonstrate this, we show the difference plot in Figure 10(c). The differences were calculated after careful alignment of the wavelets using a cross correlation.

Discussion

These results may seem confusing since they seem to contradict theoretical expectations. It is a fundamental point of signal theory that a composite signal, formed as a convolution of two primitive signals, can only be minimum phase if both of the primitive signals are minimum phase. The uncorrelated data model expressed by equation (1) is conceptually \( x = s \ast \alpha \odot r \) where \( s \) is the sweep, \( \alpha \) is the earth filter, \( r \) is the reflectivity, \( \ast \) is a stationary convolution, and \( \odot \) is a nonstationary convolution. It follows immediately that the correlated data is \( x_c = w \ast \alpha \odot r \), where \( w \) is the zero-phase Klauder wavelet. So, it seems inescapable that correlated vibroseis data cannot be minimum phase. Yet, our observations speak for themselves. The reconciliation may lie in the fact that a perfect impulse is mathematically both zero and minimum phase. That is, if the amplitude spectrum is unity for all frequencies, then the natural logarithm is zero and the Hilbert transform gives zero. As a sweep becomes more and more broad band, its corresponding Klauder wavelet (autocorrelation) must approach a spike. On the other hand, the earth filter is a strong, nonstationary, minimum-phase process. It seems plausible that the minimum-phase nature of the earth filter plays a much more important role in shaping the vibroseis wavelet than the zero-phase Klauder wavelet. If this is the case, we do not yet understand why our final example seems to be getting less minimum phase with increasing depth. This may have something to do with a progressively decreasing highest signal frequency, something we did not try to account for in our computation of the minimum-phase equivalents. While the vibroseis wavelet cannot, in theory, be minimum phase, our experimental evidence suggests that it is so in a practical sense.

Conclusions

We have presented a combination of synthetic and real field experiments that support the conjecture that the embedded wavelet found in correlated vibroseis data is, for practical purposes, minimum phase. This implies that vibroseis data does not require a phase correction to agree with the minimum phase assumption in a typical deconvolution algorithm.

References


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