Spatial prediction filtering in fractional Fourier domains

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Summary:

We generalize the time and frequency spatial predictive filtering techniques by means of the fractional Fourier transform. This time-frequency transform is a generalization of the Fourier transform by introducing a fractional parameter that allows transformation to any of a continuous family of spaces intermediate to the time and frequency domains. The family of fractional Fourier transforms of a signal can be considered as interpolated representations between the signal and its Fourier transform. Prediction techniques, such as spatial prediction filtering, are based on the assumption that the signal to be filtered is composed of two parts: one predictable, the coherent signal and other unpredictable, the random noise. A lateral prediction algorithm estimates the predictable component of a trace from its neighboring traces. In the conventional spatial prediction process, lateral coordinates are always spatial and the vertical coordinate can be either time or frequency. By applying the fractional Fourier transform in the vertical direction we extend the prediction techniques to a continuum of mixed time-frequency domains in which time and frequency are just particular cases. We test the method in the new domains using stationary a non-stationary synthetic seismic data.

Introduction:

The seismic trace, especially in the case of converted-wave datasets, has a non-stationary spectrum. Several time-frequency representations have been introduced to handle non-stationary signals. The Gabor transform (e.g. Feichtinger 1998), the wavelet transform (e.g. Chakraborty and Okaya, 1994) and the Wigner distribution (e.g. Claasen, 1980) are among the most widely used. Traditional processing methods performed in the time or the frequency domain may improve their performance on non-stationary signals when extended to time-frequency domains.

Spatial prediction filtering techniques have been developed to attenuate random noise, uncorrelated from trace to trace, that remains after stacking. The main advantage of spatial prediction methods is the preservation of the relative amplitudes and the signal character without amplitude distortion.

In the spatial prediction process, lateral coordinates are always spatial and the vertical coordinate can be either time or frequency. If the vertical coordinate is time, the filtering method is known as the t-x prediction method (Hornbostel, 1991). Alternately if frequency is used as the vertical axis the prediction technique is known as the f-x prediction method (Canales, 1984). In this paper we show how both methods can be incorporated in a more general method by means of the fractional Fourier transform.

The fractional Fourier transform

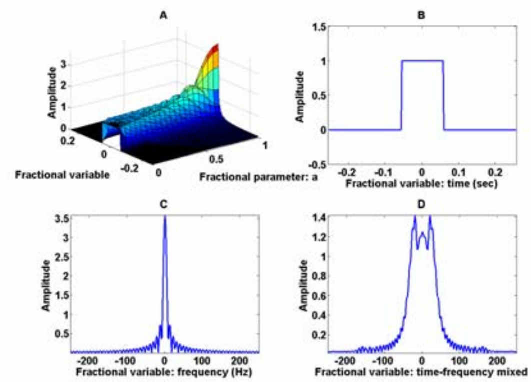

Figure 1: Amplitude component of the Fractional Fourier transform for a boxcar function. A: the continuum of fractional transforms, B: a slice at a=0 is the original boxcar function. C: a slice at a=1 is its Fourier transform. D: a slice at a=0.5 is the fractional Fourier transform of order 0.5

The ordinary Fourier domain is just a special case of a continuum of fractional Fourier domains, which has a rich mathematical theory and potential applications in every area where the Fourier transform has been used.

$$f_a(u) = A_a \int_{-\infty}^{\infty} f(t)\, e^{i\pi\left[\cot(a\pi/2)u^2 - 2\csc(a\pi/2)ut + \cot(a\pi/2)t^2\right]} dt \quad (1)$$

The definition of the fractional transform, equation (1), introduces a parameter *a*. When *a* is equal to 0, 2 or -2 the transform is the function itself. When *a* is equal to one the ordinary Fourier transform is obtained and for -1 the inverse Fourier transform is obtained. For any other *a* value between -2 and 2, a new representation of the signal is generated. Equation (1) is periodic in *a* with period 4. $A_a = \sqrt{1 - i\cot(a\pi/2)}$ is a complex scalar defined in such a way that the argument of the result lies in the interval [- /2, /2] for *a* in [-2,2]. These new representations may be interpreted as interpolated representations between the signal and its Fourier transform. An algorithm by Ozaktas (2001) was used to generate the fractional Fourier transform showed in figure 1.

Prediction filtering

Spatial prediction in fractional domains

A multichannel approach can be used to obtain a predicted version $\boldsymbol{z_{pr}}$=[$z_{p1}\ \ z_{p2} \dots\ z_{pn}$] of a series $\boldsymbol{z}$=[$z_1\ \ z_2 \dots\ z_n$] from the series $\boldsymbol{x}$=[$x_1\ \ x_2\ \dots\ x_n$] and $\boldsymbol{y}$=[$y_1\ \ y_2 \dots\ y_n$]. The filtering process is defined by the following matrix multiplication:

$$\begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \\ \vdots \\ z_{p(n-2)} \\ z_{p(n-1)} \\ z_{pn} \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 0 & 0 \\ x_2 & y_2 & x_1 & y_1 \\ x_3 & y_2 & x_1 & y_2 \\ \vdots & \vdots & \vdots & \vdots \\ x_{n-2} & y_{n-2} & x_{n-3} & y_{n-3} \\ x_{n-1} & y_{n-1} & x_{n-2} & y_{n-2} \\ x_n & y_n & x_{n-1} & y_{n-1} \\ 0 & 0 & x_n & y_n \end{bmatrix} \begin{bmatrix} f_{1,1} \\ f_{1,2} \\ f_{2,1} \\ f_{2,2} \end{bmatrix}. \qquad (2)$$

$\boldsymbol{f}$=[$f_{1,1}\ \ f_{1,2}\ \ f_{2,1}\ \ f_{2,2}$] is called the filter series. $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ are consecutive traces. Equation (2) can be generalized to predict a trace from any number of neighboring traces. An operator notation for equation (2) is:

$$\boldsymbol{z} = \boldsymbol{Bf} \qquad .(3)$$

The problem is to determine the optimum filter $\boldsymbol{f}$ for which the difference between the actual and the desired output $\boldsymbol{e} = \boldsymbol{z} - \boldsymbol{z_{pr}}$, which is called the error, is minimized. A least-squares solution (Claerbout, 1985) ends up with the solution of the normal equations

$$\boldsymbol{Rf} = \boldsymbol{g}, \qquad (4)$$

where

$$\boldsymbol{R} = \boldsymbol{B}^{\boldsymbol{T*}}\boldsymbol{B} \qquad (5)$$

and

$$\boldsymbol{g} = \boldsymbol{B}^{\boldsymbol{T*}}\boldsymbol{z} \qquad (6)$$

The symbol ^{T*} indicates complex congujate transposed matrix. The examples shown in this paper were obtained from the solution of equation (4) using a biconjugate gradient solver (Claerbout, 1992).

Spatial prediction in practice is performed by windowing data into small tiles where the events can be considered approximately linear. The filter is designed by choosing a number of neighboring traces and a number of samples in each trace. These design parameters and the length of the window in the vertical direction are the main prediction parameters. Equation (2) is what is normally called a "forward" prediction filter. A "backward" prediction filter is performed if $\boldsymbol{x}$ is predicted from $\boldsymbol{y}$ and $\boldsymbol{z}$. For a particular location, signal is calculated as the average of a forward prediction and a backward prediction. If the prediction process is performed on frequency representations of the traces, the method is called frequency-space (f-x) prediction. Linear events in the f-x domain are periodic in x direction, which allows the prediction process to be performed on individual frequencies (Gulunay, 1986). Traces from frequency to frequency are highly uncorrelated and no significant improvement can be achieved by including neighboring frequencies in the prediction process (Hornbostel, 1991). In contrast, in the time-space (t-x) prediction method, traces are correlated both in vertical and horizontal directions in t-x domain which force the inclusion of several traces and time samples in the prediction process.

The trace in the fractional domain is an intermediate representation between the signal and its Fourier transform. It is possible to extend the spatial prediction method to the fractional domains. The u-x section is obtained by taking the a^{th} fractional Fourier transform of the x-t section in the t direction. The prediction process is applied to the u-x section, then the predicted x-t section obtained by taking the inverse a^{th} fractional Fourier transform.

Examples

Prediction filtering on fractional domains of orders between 0 and 1 in steps of 0.1 was performed on stationary and non-stationary synthetic seismic datasets. The stationary dataset is shown in figure 2, together with a noisy version generated by adding random noise with signal-to-noise ratio of 1. After the prediction process, the maximum crosscorrelation between each filtered trace and its corresponding clean original trace was computed. The mean value of the maximum correlation vector (mvmc) for a dataset was used as an attribute to estimate the quality of the filtering process. For the original clean dataset mvmc is 1 and for the noisy datasets is 0.66. It was expected that the prediction filtering process will raise this value; a perfect prediction filtering would have mvmc=1. The mvmc values obtained after filtering the data in different fractional domains are shown in figure 3 (left).

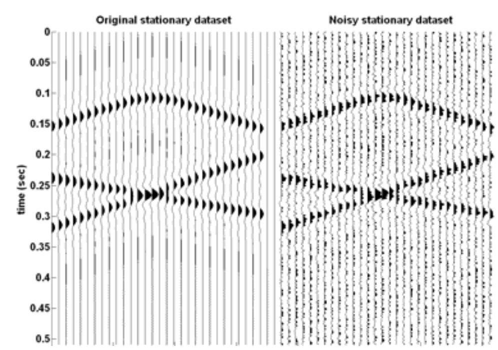

Figure 2: Stationary synthetic dataset (left). Sample rate: 2 msec. Right: same synthetic plus random noise (s/n=1).

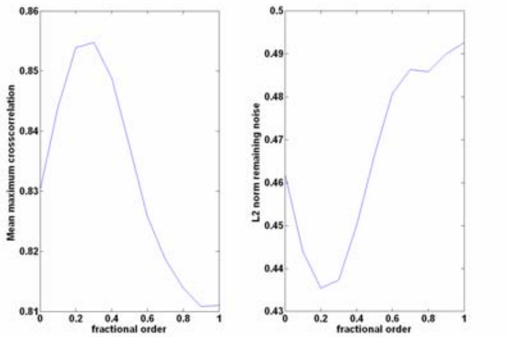

Figure 3: Mean maximum crosscorrelation (left) of the predicted data after filtering noisy data of figure 2 on different fractional domains. L2 norm (right) of the difference between actual and predicted signal.

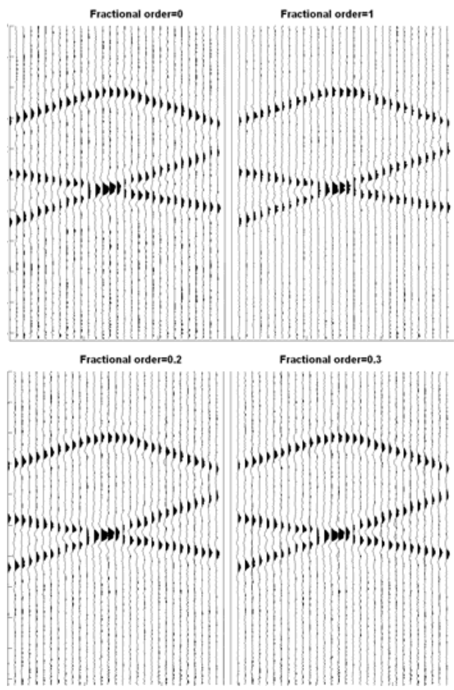

Figure 4: Stationary dataset after prediction filtering in different fractional domains.

The L2 norm of the difference between the original signal and the predicted signal is a second attribute used to measure the success of the filtering; a perfect prediction filter would have zero L2 norm. Figure 3 (right) shows this attribute vs. fractional order for the noisy data of figure 2. There is an excellent correlation between mvmc and L2 norm indicators as can be seen in figures 3 and 6.

For the filtered data shown in figure 4, the noisiest result is reached in order 0 domain (time domain). The cleanest data is obtained in order 1 domain (frequency domain), but the character of the signal is the most distorted. In order 0.2 and 0.3 domains the predicted data show a better balance between noise removal and signal character conservation. The filters were designed with 3 traces, 5 samples and 128 samples window length.

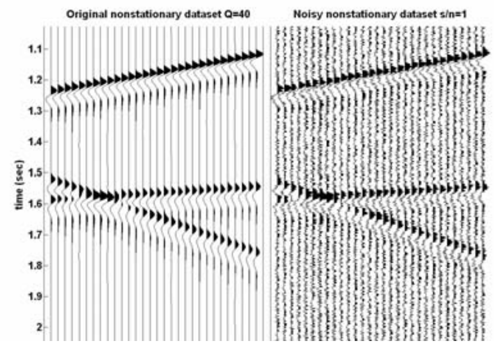

Figure 5: Nonstationary original dataset (left), Q=40 and its noisy version (right), signal to noise ratio=1.

The nonstationary seismic dataset shown in figure 5 was subjected to prediction filtering in the same different fractional domains as the previous dataset. A value of 40 for the attenuation parameter Q was used to induce the temporal nonstationarity. To generate the noisy synthetic a signal to noise ratio of 1 was used. Figure 6 shows the mean value of the maximum correlation for the predicted data and the L2 norm for the remaining noise. The mvcm for the original noisy data is 0.66.

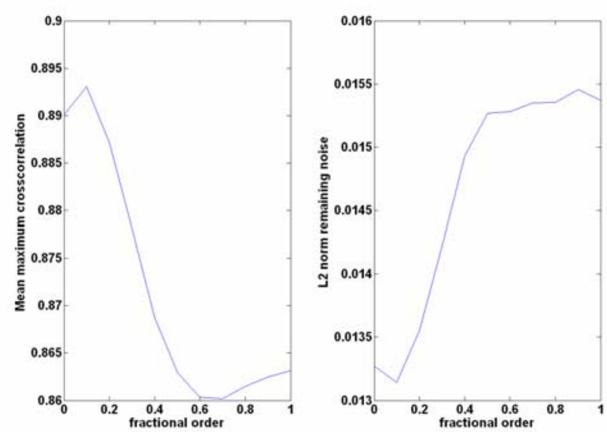

Figure 6: Mean maximum crosscorrelation (left) of the predicted data after filtering noisy data of figure 5 on different fractional domains. L2 norm (right) of the difference between actual and predicted signal.

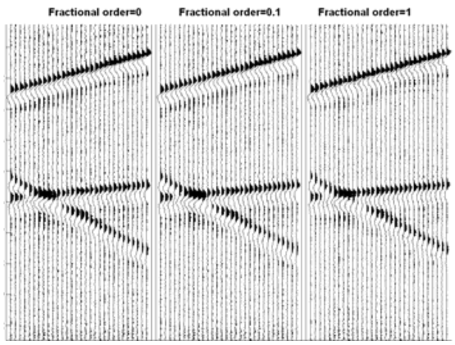

Figure 7: Nonstationary dataset after prediction filtering in different fractional domains.

Figure 7 shows the obtained results after prediction filtering in different fractional domains. The best result is obtained for fractional order 0.1. It has better signal character preservation than the result in frequency domain (fractional order 1) and a slightly less noisy aspect than the result in time domain (fractional order 0).

Conclusions

We introduced the fractional Fourier transform to extend the spatial prediction filter method to fractional Fourier domains. Thus frequency-space prediction and time-space prediction become particular cases of a more general method. We tested the generalized method on stationary and nonstationary synthetic seismic data for different values of the fractional parameter. We used mean maximum correlation of the predicted data and L2 norm of the remaining noise as attributes to evaluate the quality of the results. Both attributes showed excellent correlation and according to them the best results were obtained for fractional values of 0.3 and 0.1. The results obtained in the frequency domain tend to be less noisy but to have more distorted signal character. In the time domain tends to occur the opposite behavior. In a certain intermediate fractional domain a balance between signal character preservation and noise removal occurs producing the best prediction performance. Further research should be accomplished in order to find a methodology to identify the optimal fractional domain to filter real data.

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