

# Comparison of estimation of Thomsen's anisotropy parameters by anisotropic NMO analysis

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## Abstract

Anisotropic NMO analysis is used to estimate anisotropy parameters in a VTI medium in combination with check-shot or well-log data. Analysis of four reflection-traveltime inversions in weakly anisotropic media shows that inversion accuracy is related to the spread length and subsurface anisotropic parameters. Under their own offset range, the accuracy of estimated  $\delta$  decreases with their offsets and the accuracy of estimated  $\varepsilon$  increases with their offsets, and the accuracy of the estimated Thomsen anisotropy parameter  $\delta$  depends not only on the accuracy of the picked NMO velocity but also on the subsurface anisotropy parameters. The smaller the value of  $(\varepsilon - \delta)$ , the higher the accuracy of the estimated  $\delta$  value. The results of the four reflection-traveltime inversions by semblance analysis for synthetic seismic examples demonstrate that in estimating  $\delta$ , the nonhyperbolic and the shifted-hyperbolic estimations are better than the three-term Taylor-series method. Only the nonhyperbolic approximation can be used to estimate the anisotropy parameter  $\varepsilon$  accurately. Hyperbolic estimation is only suitable for estimation of elliptical anisotropy which is rarely happened in practice.

## Introduction

Alkhalifah and Larner (1994) showed that accurate 2-D imaging in transversely isotropic media requires good knowledge of the Thomsen anisotropy parameters  $\delta$  and  $\varepsilon$ . There are various traveltimes inversion approaches for estimating anisotropy parameters (Alkhalifah and Tsvankin, 1995; Brown et al., 2000; Elapavuluri and Bancroft, 2002; Gaiser, 1990; Isaac and Lawton, 2004; White et al., 1983). Thomsen (1986) has derived relations between normal-moveout (NMO) velocities and anisotropy parameters in a homogeneous anisotropic layer. In combination with check-shot or well-log data, we are able to use various analytic reflection-traveltime approximations over limited spread lengths to obtain anisotropy parameters in VTI media by NMO-velocity analysis and through a Dix-type differentiation procedure. Besides hyperbolic approximation, a popular approach for estimating anisotropy is a modified three-term Taylor series approximation to the reflection moveout curve (Tsvankin and Thomsen, 1994; Alkhalifah and Larner, 1994; Tsvankin, 1995).

If one ignores the contribution of the vertical shear-wave velocity, a modified three-term Taylor-series approximation to the reflection moveout curve can be fully determined by two parameters,  $V_{\text{NMO}}$  (NMO velocity) and  $\eta = (\varepsilon - \delta)/(1 + 2\delta)$ ; [Alkhalifah and Tsvankin, 1995], or by  $V_{\text{NMO}}$  and  $V_h$  (horizontal velocity). Based on the nonhyperbolic moveout equation developed by Tsvankin and Thomsen (1994), a 2-D semblance scan can be used to estimate anisotropy parameters. For convenience, we refer to this method as nonhyperbolic reflection-traveltime inversion. Elapavuluri and Bancroft (2002) showed the shifted hyperbolic approximation can also be used to estimate anisotropy parameters from P-wave reflection data.

In this paper, we carry out these four inversions on synthetic seismic data examples and try to determine the relation between the estimated anisotropy parameters and the true anisotropy parameters. Finally, we formulate some conclusions for guiding the application of these approximations.

## Anisotropic NMO equations

The P-wave traveltimes approximations for four reflection-traveltime inversion methods are given as follows.

1) The hyperbolic reflection-traveltime approximation:

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{NMO}}^2}, \quad (1)$$

2) The modified three-term Taylor-series approximations (Tsvankin and Thomsen, 1994) in the limit of weak anisotropy:

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{NMO}}^2} + \frac{A_4 x^4}{1 + Ax^2}$$

(2)

3) The shifted-hyperbolic approximation (Castle, 1994):

$$t = \tau_s + \sqrt{\tau_x^2 + \frac{x^2}{SV_{\text{NMO}}^2}}$$

(3)

4) The nonhyperbolic approximation:

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\text{NMO}}^2} - \frac{[V_h^2 - V_{\text{NMO}}^2]x^4}{V_{\text{NMO}}^2[t_0^2 V_{\text{NMO}}^4 + V_h^2 x^2]} \quad (4)$$

From equation (1) to equation(4),

$$V_{\text{NMO}}^2(P) = \alpha_0^2(1 + 2\delta), \quad V_h^2 = \alpha_0^2(1 + 2\epsilon), \quad (5)$$

$$A = 1/(\alpha_0 t_0)^2, \quad A_4(P) = -\frac{2(\epsilon - \delta)}{t_0^2 \alpha_0^4},$$

(6)

$$\tau_s = t_0(1 - \frac{1}{S}), \quad \tau_x = \frac{t_0}{S}, \quad S = 1 + \frac{8(\epsilon - \delta)}{(1 + 2\delta)}. \quad (7)$$

where  $\alpha_0$  is vertical velocity for P waves,  $\beta_0$  is vertical SV-wave velocity,  $\delta$  and  $\epsilon$  are Thomsen's anisotropy parameter;  $V_{\text{NMO}}$  is NMO velocity,  $V_h$  is horizontal velocity for P-waves,  $t_0$  and  $t$  are the two-way traveltimes for zero-offset and offset  $x$ , respectively, and  $S$  is the shift parameter. Xiao et al (2004) have demonstrated that these traveltimes approximations have their own ranges of the offset and anisotropy parameters.

### Dix-type inversions

For simplicity, we consider a series of single-layer case in order to determine how both actual anisotropy parameters and spread length affect the estimation of anisotropy parameters. The input CMP gather for anisotropy-parameter estimation contains a single reflection from a flat interface. The depth of this interface is 500 m. Vertical P- and S-wave velocities above the reflector are 3000 m/s and 1500 m/s, respectively. The values of  $\epsilon$  are fixed at 0.2, 0.1 and 0.0, respectively, and those of  $\delta$  range from  $-0.2$  to  $0.2$  at increments of 0.02.

Using equations (1) to (4), we can pick up effective coefficients  $A_4$ ,  $V_{\text{NMO}}$ ,  $V_h$  and  $S$ , and then obtain anisotropic parameters  $\epsilon$  and  $\delta$  by using equations (5) to (7) through a Dix-type differentiation procedure (here vertical P-wave velocity  $\alpha_0$  is known from well log, check shot or VSP). Semblance scanning is employed to estimate effective coefficients.

Our research shows that under their own offset range, the accuracy of estimated delta decreases with their offsets and the accuracy of estimated epsilon increases with their offsets. Figures 1(a) shows the errors in estimated  $\delta$ , plotted versus  $\delta$  when offset/depth = 1.0, for  $\epsilon$  values of 0.2, 0.1 and 0.0. and  $\delta$  values ranging from  $-0.2$  to  $0.2$  at increments of 0.02. From Figure 1(a) it appears that i) the smaller the value of  $(\epsilon - \delta)$ , the higher the accuracy of the estimated  $\delta$  value; ii) the estimated values deviate greatly from the true values when  $|\epsilon - \delta| > 0.2$ , and iii) the nonhyperbolic and the shifted-hyperbolic estimations are better than the three-term Taylor-series method while hyperbolic estimation is accurate only for elliptical anisotropy ( $\epsilon = \delta$ ). Figures 1(b) shows the errors in estimated  $\epsilon$ , plotted versus  $\delta$  when offset/depth = 2.0, for  $\epsilon$  values of 0.2, 0.1 and 0.0. and  $\delta$  values ranging from  $-0.2$  to  $0.2$  at increments of 0.02. From Figure 1(b) we can see that only nonhyperbolic inversion is able to estimate parameters  $\epsilon$  with any accuracy.

### Examples

Table 1 demonstrates the model parameters and estimated anisotropy parameters for a four-layer model. Note that all  $(\epsilon - \delta)$  values in model 1 are less than 0.2. The only difference between model 2 and model 1 is that the value of  $(\epsilon - \delta)$  in the second layer is larger than 0.2. Figure 2 and 3 show estimated anisotropy-parameter values, actual values and CDP gathers with the

traveltime calculated by their estimated effective coefficients from model1 and model 2, respectively . These estimation results from multilayer VTI media also demonstrate that the estimated interval anisotropy parameters are very close to the true parameter values. Only when  $(\epsilon - \delta)$  is larger than 0.2 do the estimated interval parameter values depart significantly from the true value.

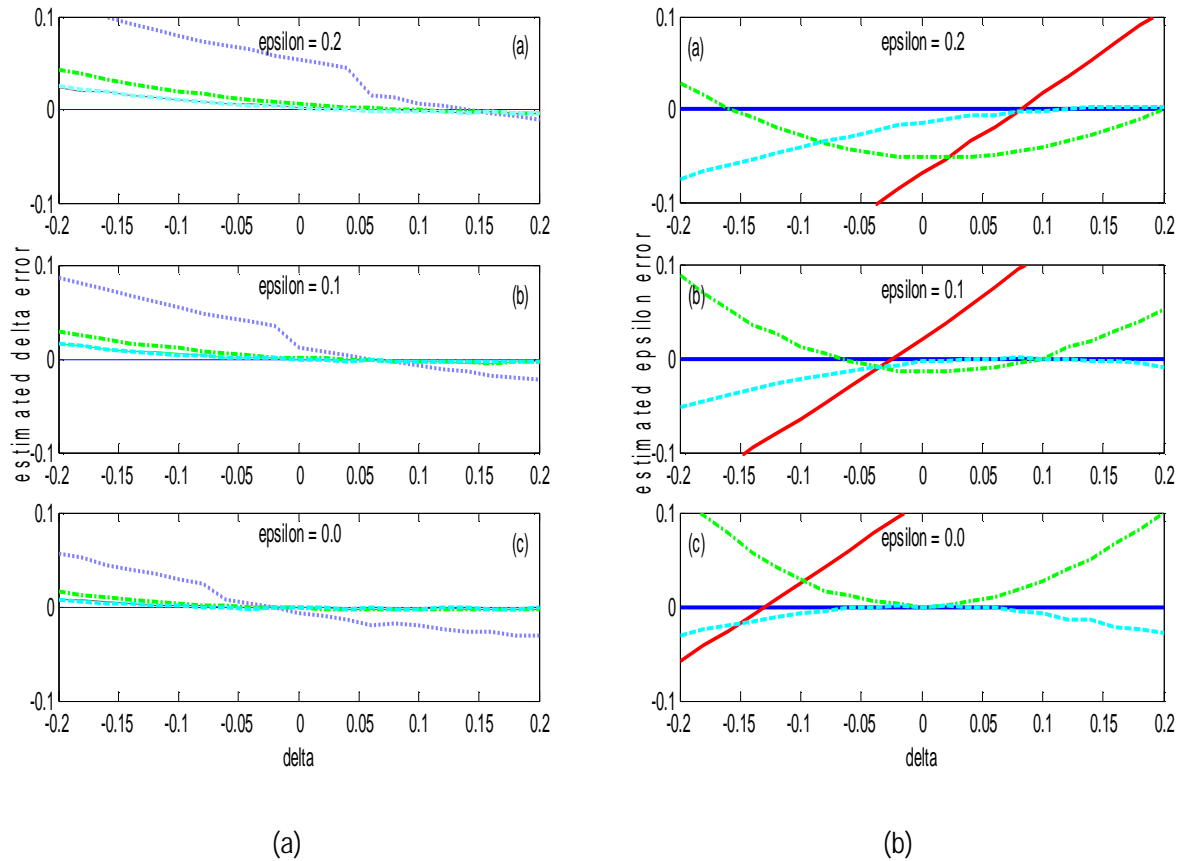


Fig. 1.(a) The error in estimated  $\delta$  plotted vs. true  $\delta$  when offset/depth = 1.0. (b) The error in estimated  $\epsilon$  plotted vs. true  $\delta$  when offset/depth = 2.0. Blue solid line: the exact anisotropic parameters; purple dotted line: hyperbolic traveltime inversion; green dash-dot line: the modified three-term Taylor-series inversion; red solid line: the shifted hyperbolic inversion; cyan dashed line: nonhyperbolic inversion.

Table 1. Model parameters and estimated anisotropic parameters for layered VTI media

Thickness (m)	$\alpha_0$ (m/s)	$\beta_0$ (m/s)	Model 1 $\epsilon, \delta$	Model 2 $\epsilon, \delta$
500	2800	1400	0.20, 0.10	0.20, 0.10
500	3000	1500	0.15, 0.08	0.20, -0.20
500	3200	1600	0.10, 0.04	0.10, 0.04
500	3500	1750	0.08, 0.02	0.08, 0.02

## Conclusions

The accuracy of the estimated anisotropic parameter  $\delta$  depends not only on the accuracy of the picked NMO velocity but also on the value of  $(\epsilon - \delta)$ . The smaller the value of  $(\epsilon - \delta)$  and the value of  $\epsilon$ , the higher the accuracy of estimated  $\delta$ . The results of the four traveltime inversions by semblance analysis for the seismic examples demonstrate that the nonhyperbolic and shifted-hyperbolic estimations are better than the three-term Taylor-series method. Only nonhyperbolic inversion can be used to estimate accurately the anisotropy parameter  $\epsilon$ . Hyperbolic estimation is only suitable for estimation of elliptical anisotropy which is rarely happened in practice.

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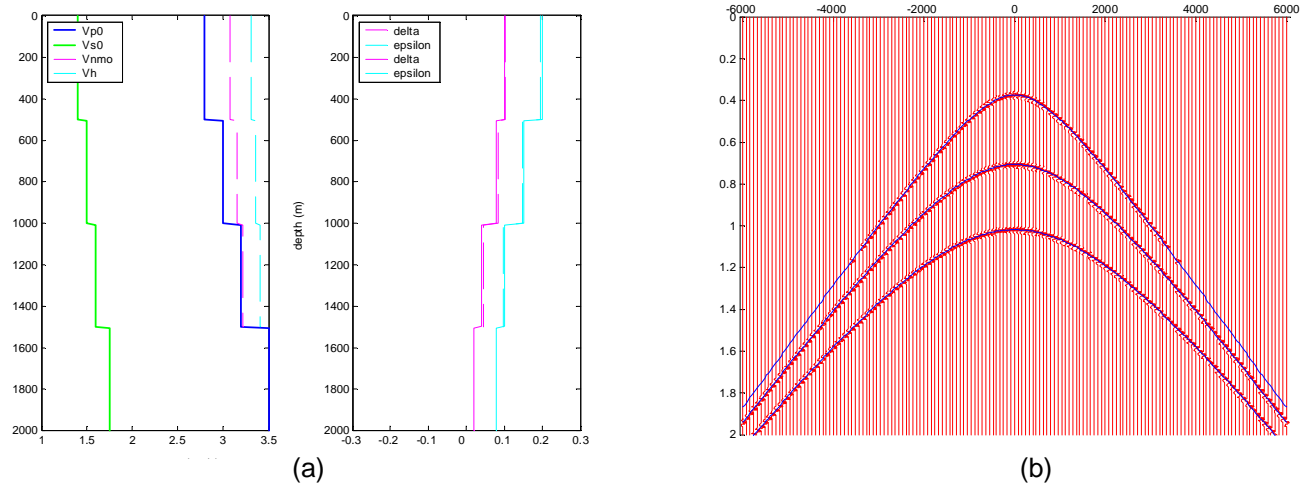


Fig. 2.(a) Model 1 parameters (solid lines) with estimated coefficients (dashed lines); (b) the CDP gather with the traveltine calculated by their estimated effective coefficients.

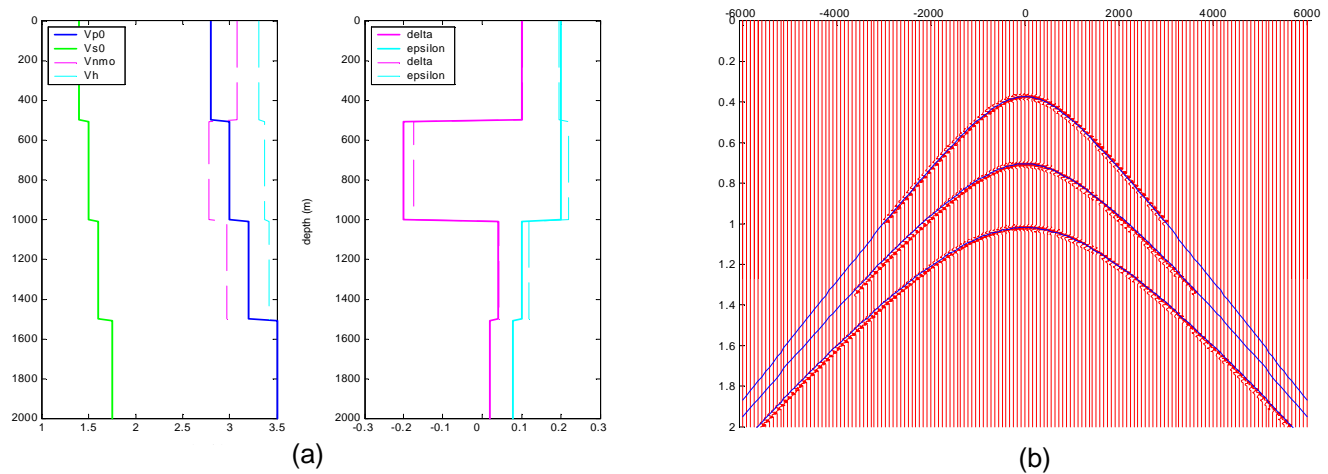


Fig. 3.(a) Model 2 parameters (solid lines) with estimated coefficients (dashed lines); (b) the CDP gather with the traveltine calculated by their estimated effective coefficients.