# Z-99 PP AND PS POLARITIES IN CORRELATION OF EVENTS AND APPROXIMATING $R_{PS}$ FOR AVO

R.J. BROWN[1,2] and A. VANT[1,3]
[1] University of Calgary, Department of Geology & Geophysics, Calgary, Alberta, Canada T2N 1N4
[2] University of the Faroe Islands, Faculty of Science & Technology, PO Box 2109, FO-165 Argir, Faroe Islands
[3] Enervant Consulting Ltd

**Summary**

For most interfaces, $R_{PP}$ and $R_{PS}$ have opposite sign, which means an event usually has the same display polarity on PP and PS seismic sections. In processing and interpreting PS sections, we usually rely on correlation of corresponding PS and PP events. Assuming the wrong polarity relationship can lead to a mistie of half a cycle between corresponding PP and PS events and, consequently, over- or underestimated $V_P/V_S$ ratios for the affected intervals. We examine the conditions that determine the display polarities of PP and PS and show that the sign of $R_{PS}$ depends on the sign of $\Delta U$, where $U = \rho\beta^h$ and $h$ is a function of velocities and densities. We also derive a small-angle $R_{PS}$ approximation that is more accurate than the Aki-Richards expression in many situations, not just for small angles.

**Introduction**

Correlation of PS and PP events enables determination of preliminary $V_P/V_S$ ratios, needed in the PS processing flow. Typically, this correlation relies on synthetic seismograms generated using log data for $V_P$, $V_S$ and $\rho$ from a nearby well, if available. These synthetics are then used to match events on PP and PS field sections and, from traveltime ratios, to calculate $V_P/V_S$ ratios. But these efforts can be frustrated if the logs for $V_S$ or $\rho$ are unavailable. If so, $\rho$ may be estimated from $V_P$ using empirical relationships like Gardner's equation (Gardner et al., 1974) or modified versions thereof for specific lithologies (Castagna et al., 1993). If $V_S$ information is unavailable, estimates of interval $V_P/V_S$ ratios have to be used to create PS synthetic stacks (Lawton & Howell, 1992), which may then be fine-tuned to give optimal correlation with the stacked field data (Miller, 1996). In the absence of any of the logs, there may be polarity errors on particular events on the PS synthetics relative to the PP (Figure 1).

The SEG polarity standard (Thigpen et al., 1975) implies, for a minimum-phase wavelet from a compressive source, that a PP reflection from an interface with a *positive* PP reflection coefficient ($R_{PP} > 0$) will begin with a downward (negative) deflection on the recorded seismogram (Sheriff, 2002). Recommended SEG standards for horizontal-component geophones (Landrum et al., 1994) and subsequent proposed standards (Brown et al., 2002) imply that a PS reflection from the same source, when the interface has a *negative* PS reflection coefficient ($R_{PS} < 0$), will also lead to a downward (negative) deflection on the inline geophone on positive-offset traces. Negative-offset traces, which are flipped in preprocessing, originally have the opposite polarity (e.g. Tessmer & Behle, 1988; Brown et al., 2002). Therefore, because for most interfaces, $R_{PP}$ and $R_{PS}$ have opposite sign, an event usually has the same display polarity on PP and PS. In this paper, we study the 'unusual situation', where events on the two sections display opposite apparent polarities, i.e., where $R_{PS}/R_{PP} > 0$.

**Approximations to the Zoeppritz equations**

In adopting approximations for polarity relations, we do not want to restrict ourselves to low-contrast interfaces, so we assume small angles of incidence, $i$. We also assume the polarity relationship at small angles to be representative of that of the stacked events in all but the rarest of cases.

Many approximations to the Knott-Zoeppritz equations governing P-SV waves at a welded interface have been derived (mainly for $R_{PP}$ and $R_{PS}$), some of the earliest being those of Bortfeld (1961), Richards & Frasier (1976) and Aki & Richards (1980), whose notation we essentially follow. We use the symbol $r$ to represent rock parameters generally (e.g. $\alpha$, $\beta$, $\rho$, $\sigma$ or $\mu$) and we express an arbitrary rock parameter and its change over an interface in terms of $r$ (the average of $r_1$ and $r_2$) and $\Delta r$ (the difference $r_2 - r_1$), where 1 and 2 denote media 1 and 2, respectively.

For $R_{PP}$, many approximations have been published (e.g. Bortfeld, 1961; Richards & Frasier, 1976; Aki & Richards, 1980; Shuey, 1985; Wang, 1999; Ursenbach, 2002) However, for $R_{PP}$, we use the zero-offset expression as sufficient for characterizing polarity. Some of the published $R_{PS}$ approximations assume small parameter changes (Aki & Richards, 1980; Xu & Bancroft, 1997; Ursenbach, 2002), some, like ours (below) assume small angles (Bortfeld, 1961; Richards & Frasier, 1976; Zaengle & Frasier, 1993; Wang, 1999; Ramos & Castagna, 2001; Carcuz, 2001; Geldart & Sheriff, 2004). It turns out that the differences between the two are not as great as one might think because in the Taylor-series expansions the terms of higher order in $i$, or sin$i$, or $p$ (horizontal slowness) tend also to be the terms of higher order in $\Delta r/r$.

### Approximating $R_{PS}$

In deriving an $R_{PS}$ approximation, we start with the exact formula as given by Aki & Richards (1980):

$$R_{PS} = -2\left[\frac{\cos i_1}{\alpha_1}\left(ab + cd\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)p\alpha_1\right]\Big/\left(\beta_1 D\right) \tag{1}$$

where $a$, $b$, $c$, $d$ and $D$ are defined by Aki & Richards (1980). These are reprinted by Xu & Bancroft (1997), Ramos & Castagna (2001) and Vant (2003), though Ramos & Castagna have an error in their expression for $d$. It should be:

$$d = 2\left(\rho_2\beta_2^2 - \rho_1\beta_2^2\right) \quad \text{and not} \quad d = 2\left(\rho_2\beta_2^2 p^2 - \rho_1\beta_2^2\right). \tag{2}$$

To get our $R_{PS}$ approximation we first rewrite (1) as:

$$R_{PS} = -\left[\sin 2i_1\left(ab + cd\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)\right]\Big/\left(\alpha_1\beta_1 D\right) \tag{3}$$

then apply the small-angle approximation by setting $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ for sines and cosines of incidence angles, $i_1$, $i_2$, $j_1$ and $j_2$, in the expressions for $a$, $b$, $c$, $d$ and $D$, giving:

$$R_{PS} = \frac{-\sin 2i_1\left(\alpha_2\beta_2\rho_2\Delta\rho + 2\rho_1\Delta\mu\right)}{\left(\rho_1\alpha_1 + \rho_2\alpha_2\right)\left(\rho_1\beta_1 + \rho_2\beta_2\right)}. \tag{4}$$

If we also approximate the explicit sine factor, it simply reduces from sin$2i_1$ to $2i_1$ and we have an expression equivalent to one given by Geldart & Sheriff (2004, p. 70). However this is only a slight simplification and it costs significantly in accuracy at moderate-to-large angles (Figure 2); so we usually choose to retain the explicit sine factor.

Wang (1999) started with the exact formulae for $R_{PP}$ and $R_{PS}$ (Aki & Richards, 1980) and developed Taylor-series expansions in powers of $p$, and therefore in powers of sin $\theta$, or $\theta$, i.e., small-angle approximations. His first $R_{PS}$ approximation [his equation (C-3)] is correct up to terms in $p^5$. However, to obtain his second, simplified, approximation [his equation (C-5)], he made two assumptions, one of which is quite unjustified [coming from his equation (A-10)]. For $R_{PS}$ this amounts to assuming that $\Delta\rho = 0$, which eliminates one of the two first-order terms in his expressions, even though he retains other terms up to fifth order. Vant (2003) found Wang's first fifth-order approximation [his (C-3)] to be extremely accurate for a selection of interfaces, but the second [his (C-5)] to be quite inaccurate. Truncation of Wang's first $R_{PS}$ approximation after first order gives an expression that is about as accurate as our equation (4) but much more complicated, while the second truncates to an expression that is much less accurate than equation (4).

### Conditions for opposite PP and PS polarities

We use the two small-angle approximations to $R_{PP}$ and $R_{PS}$ to determine under what conditions we get opposite display polarities on the same event on PP versus PS data, that is, when $R_{PP}/R_{PS} > 0$. Thus, we start with our own approximation for $R_{PS}$ [equation (4)] and the normal-incidence expression for $R_{PP}$:

$$R_{PP} = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}. \tag{5}$$

With respect to equation (5):

$$\operatorname{sgn} R_{\mathrm{PP}} = \operatorname{sgn}(\rho_2\alpha_2 - \rho_1\alpha_1) = \operatorname{sgn}(\rho\Delta\alpha + \alpha\Delta\rho) = \operatorname{sgn}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right). \tag{6}$$

With respect to equation (4):

$$\operatorname{sgn} R_{\mathrm{PS}} = -\operatorname{sgn}\left\{\left[\alpha_2\beta_2\rho_2 + \rho_1\left(\beta_2^2 + \beta_1^2\right)\right]\Delta\rho + 2\rho_1\left(\rho_2\beta_2 + \rho_1\beta_1\right)\Delta\beta\right\} = -\operatorname{sgn}\left[f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta}\right] \tag{7}$$

where

$$f = \rho\left[\alpha_2\beta_2\rho_2 + \rho_1\left(\beta_2^2 + \beta_1^2\right)\right] \quad \text{and} \quad g = 2\rho_1\beta\left(\rho_2\beta_2 + \rho_1\beta_1\right). \tag{8}$$

So, for opposite display polarities, or $R_{\mathrm{PP}}/R_{\mathrm{PS}} > 0$, we need either:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho} > 0 \ \text{and} \ f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta} < 0; \ \text{i.e.} \ -\frac{\Delta\alpha}{\alpha} < \frac{\Delta\rho}{\rho} < -\frac{g\Delta\beta}{f\beta} \quad \text{for } R_{\mathrm{PP}} > 0 \text{ and } R_{\mathrm{PS}} > 0 \tag{9}$$

or:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho} < 0 \ \text{and} \ f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta} > 0; \ \text{i.e.} \ -\frac{g\Delta\beta}{f\beta} < \frac{\Delta\rho}{\rho} < -\frac{\Delta\alpha}{\alpha} \quad \text{for } R_{\mathrm{PP}} < 0 \text{ and } R_{\mathrm{PS}} < 0. \tag{10}$$

Equations (8) to (10) give conditions for occurrence of the unusual situation, that is, reversed display polarity of a reflection event on PS versus PP. We can formulate these conditions from (9) and (10) in another way by first noticing that:

$$\frac{\Delta\rho}{\rho} + \frac{g\Delta\beta}{f\beta} = \frac{\Delta\rho}{\rho} + \Delta\ln\beta^h = \frac{\Delta U}{U}, \quad \text{where} \quad h = \frac{g}{f} \quad \text{and} \quad U = \rho\beta^h. \tag{11}$$

Then we will get the unusual polarity situation, $R_{\mathrm{PP}}/R_{\mathrm{PS}} > 0$, if either:

$$\frac{\Delta U}{U} < 0 \quad (\text{i.e. } R_{\mathrm{PS}} > 0) \quad \text{when} \quad \frac{\Delta Z}{Z} > 0 \quad (\text{i.e. } R_{\mathrm{PP}} > 0) \tag{12}$$

or

$$\frac{\Delta U}{U} > 0 \quad (\text{i.e. } R_{\mathrm{PS}} < 0) \quad \text{when} \quad \frac{\Delta Z}{Z} < 0 \quad (\text{i.e. } R_{\mathrm{PP}} < 0). \tag{13}$$

## Conclusions

We have derived a small-angle $R_{\mathrm{PS}}$ approximation that is more accurate than the Aki-Richards approximation for three interfaces tested: beyond 25° incidence in two cases and beyond 40° in the third case. We have also derived mathematical expressions for the conditions on the interface rock parameters for an event to be recorded with opposite display polarities on PP and PS data.

## References


Aki, K. & Richards, P.G., 1980, Quantitative seismology: Theory and Methods: W.H. Freeman and Co.

Bortfeld, R., 1961, Approximation to the reflection and transmission coefficients of plane longitudinal and transverse waves: Geophysical Prospecting **9**, 485-503.

Brown, R.J., Stewart, R.R. & Lawton, D.C., 2002, A proposed polarity standard for multicomponent seismic data: Geophysics **67**, 1028-1037.

Carcuz, J.R., 2001, A combined AVO analysis of P-P and P-S reflection data: SEG Expanded Abstracts **20**, 323-325.

Castagna, J.P., Batzle, M.L. & Kan, T.K., 1993, Rock physics—The link between rock properties and AVO response, in J.P. Castagna & M.M. Backus, eds., Offset-dependent reflectivity—Theory and practice of AVO analysis: SEG, Investigations in Geophysics, **8**, 135-171.

Gardner, G.H.F., Gardner, L.W. & Gregory, A.R., 1974, Formation velocity and density: The diagnostic basis of stratigraphic traps: Geophysics **39**, 770-780.

Geldart, L.P. & Sheriff, R.E., 2004, Problems in exploration seismology and their solutions: SEG, Geophysical Reference Series, **14**.

Landrum, R.A., Brook, R.A. & Sallas, J.J., 1994, Polarity convention for vibratory source/recording systems: Geophysics **59**, 315-322.

Lawton, D.C. & Howell, C.E., 1992, P-SV and P-P synthetic stacks: SEG Expanded Abstracts **11**, 1344-1347.

Miller, S.L.M., 1996 Multicomponent seismic data interpretation: M.Sc. thesis, University of Calgary.

Ramos, A.C.B. & Castagna, J.P., 2001, Useful approximations for converted-wave AVO: Geophysics **66**, 1721-1734

Richards, P.G. & Frasier, C.W., 1976, Scattering of elastic waves from depth-dependent inhomogeneities: Geophysics **41**, 441-449.

Sheriff, R.E., 2002, Encyclopedic dictionary of exploration geophysics, 4th ed.: SEG, Geophysical Reference Series, **13**.

Shuey, R.T., 1985, A simplification of the Zoeppritz equations: Geophysics **50**, 609-614.

Tessmer, G. & Behle, A., 1988, Common reflection point data-stacking technique for converted waves: Geophysical Prospecting **36**, 671-688.

Thigpen, B.B., Dalby, A.E. & Landrum, R., 1975, Special report of the subcommittee on polarity standards: Geophysics **40**, 694-699.

Ursenbach, C.P., 2002, Optimal Zoeppritz approximations: CREWES Research Report **14**, 14, 1-10.

Vant, A., 2003, Studies of multicomponent seismic polarity and amplitude: M.Sc. thesis, University of Calgary.

Wang, Y., 1999, Approximations to the Zoeppritz equations and their use in AVO analysis: Geophysics **64**, 1920-1927.

Xu, Y. & Bancroft, J.C., 1997, Joint AVO analysis of PP and PS seismic data: CREWES Research Report **9**, 34, 1-44.

Zaengle, J.F. & Frasier, C.W., 1993, Correlation and interpretation of P-P and P-S data, Zamora gas field, Yoho County, California, in J.P. Castagna & M.M. Backus, eds., Offset-dependent reflectivity—Theory and practice of AVO analysis: SEG, Investigations in Geophysics **8**, 333-345.

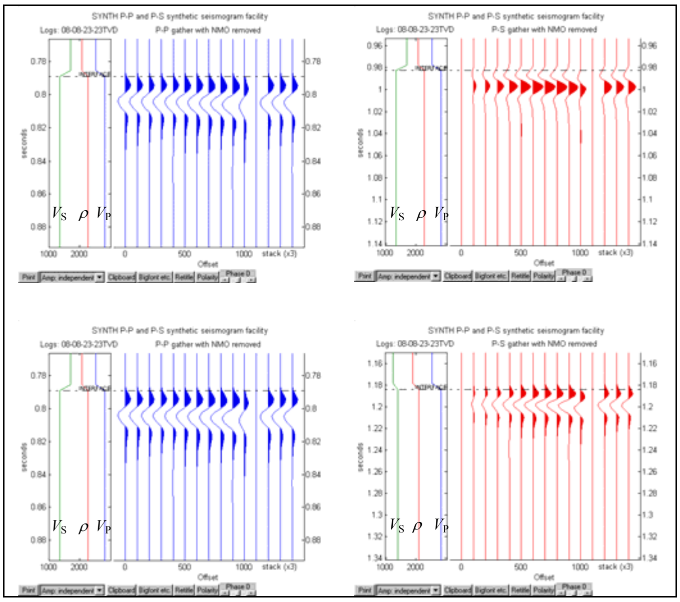


**Figure 1.** AVO responses and synthetic stacks [PP (blue) and PS (red)] for a model of a sand over a coal, an example of opposite PP-PS display polarities. In modelling this interface (upper figure) all logs were used. A $V_P/V_S$ ratio of 2.0 was used to construct the lower PS synthetics for the scenario of a missing shear-wave log.

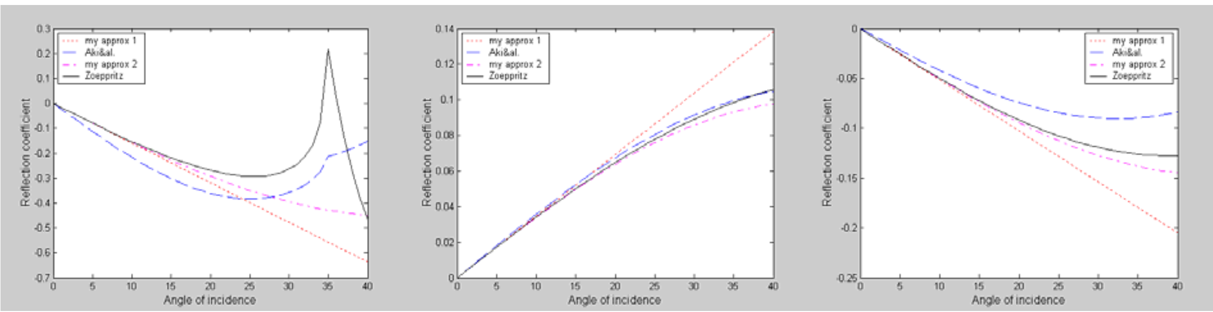


**Figure 2.** Comparison of the exact $R_{PS}$ curve (solid black) with three approximations: our equation (4) (dashed magenta); Geldart & Sheriff (2004) (dotted red) [equivalent to (4) with $\sin 2i_1 \rightarrow 2i_1$], and Aki & Richards (1980) (coarsely dashed blue). The interface models, left to right, are: young shale over old shale, sandstone over salt, and shale over gas-sand (Brown et al., 2002).