Q-14 Compensating for attenuation by Gabor deconvolution

CARLOS A. MONTAÑA AND GARY F. MARGRAVE
CREWES Project, The University of Calgary, Calgary, Alberta, Canada

Summary

Both Gabor deconvolution and inverse-*Q* filtering methods are based on the constant *Q*-model theory for attenuation of seismic waves. Gabor deconvolution methodologically differs from inverse-*Q* filter in that the compensation for attenuation and the inversion of the minimum phase wavelet component of the seismic trace are carried out simultaneously. An additional remarkable difference is that Gabor deconvolution does not use any estimation of *Q* whereas in inverse-*Q* filtering this estimation is a critical issue. Related to the difference between digital and analog Hilbert transforms, a phase-shift difference between a seismic trace at a well and a synthetic trace created from the well log remains after application of Gabor deconvolution. This phase shift can be removed either by adding a function linear in time and quadratic in frequency, to the digital Hilbert transform estimation of the minimum phase function of the Gabor deconvolution operator or by resampling the trace to a smaller sample rate.

Introduction

The constant *Q* model (Kjartansson, 1979) is generally accepted as a good theoretical model to represent inelastic attenuation in seismic waves. Its favorable reception among geophysicists is explained by its simplicity, and the satisfactory agreement found between its predictions and laboratory measurements. It is a linear theory that completely characterizes attenuation in a medium in terms of only 2 parameters: the attenuation factor *Q* and a reference frequency ω_0 for which the wave-propagation velocity (i.e. phase velocity) in the medium is known. Different seismic data processing methods to compensate for the attenuation effects, such as inverse-*Q* filtering and Gabor deconvolution, use the constant-*Q* theory as a background theoretical model. In inverse-*Q* filtering, the seismic data are compensated for attenuation effects before dealing with the inversion of the minimum-phase earth filter by spiking deconvolution. Theoretically, this ordering is incorrect. Gabor deconvolution, which is in essence an extension of the stationary Wiener deconvolution to the nonstationary case, approaches the compensation for attenuation in a radically different way, compensating for attenuation effects and inverting the minimum-phase earth wavelet simultaneously. Although this difference in methodology for tackling the attenuation problem defines already an important distinction between the two methods, there is one additional even more important difference: an estimation of *Q* is necessary for applying an inverse-*Q* filter, whereas to apply Gabor deconvolution knowledge of *Q* is not required.

Modeling attenuation, the constant Q model

A nonstationary convolutional model for an attenuated seismic trace, *s,* can be derived from the constant-*Q* theory by applying the forward Q filter, as a pseudodifferential operator, to a reflectivity function and then by convolving the result with a minimum phase wavelet. Such a model has been used, for example by Margrave and Lamourex (2002), and can be expressed in the frequency domain,

$$\hat{s}(\omega) = \frac{1}{2\pi}\hat{w}(\omega)\int_{-\infty}^{\infty}\alpha_Q(\omega,\tau)r(\tau)e^{i\omega(t-\tau)}d\tau . \tag{1}$$

where the 'hat' symbol indicates the Fourier transform, ω is the frequency, r is the reflectivity function, w is the wavelet and $\alpha_Q(\omega,\tau)$ is the time-frequency exponential attenuation function, defined as

$$\alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right). \tag{2}$$

in which the real and imaginary components in the exponent and connected through the Hilbert transform H, result that is consistent with the minimum phase characteristic associated with the attenuated pulse. As written, equation (1) assumes a spatially constant Q and models only primaries though both of these simplifications can be removed with a slight complication in the formula.

Gabor deconvolution

The Gabor transform, G, maps functions of time to complex-valued functions of time and frequency using a windowed Fourier transform. Gabor deconvolution is a nonstationary extension of Wiener's deconvolution method, based on an approximate, asymptotic factorization of the nonstationary trace model of equation (1)

$$Gs(t,\omega) \approx \hat{w}(\omega)\alpha_Q(t,\omega)Gr(t,\omega), \tag{3}$$

which states that the Gabor transform of the seismic trace, $Gs(t,\omega)$, is approximately equal to the product of the Fourier transform of the source wavelet, $\widehat{w}(\omega)$, the time-frequency attenuation function, $\alpha_Q(t,\omega)$, and the Gabor transform of the reflectivity $Gr(t,\omega)$. The method assumes that $|Gr(t,\omega)|$ is a rapidly varying function in both variables τ and ω, $|\widehat{w}(\omega)|$ is smoothly varying in ω, and $\alpha_Q(\tau,\omega)$ is an exponentially decaying function in both variables τ and ω, and constant over hyperbolic families of $\tau\omega$=*constant.* Analogously to the Wiener's method, Gabor deconvolution smooths the Gabor magnitude spectrum of the seismic signal $|Gs(t,\omega)|$ to estimate the product of the magnitudes of the attenuation function and the source signature $|\sigma(\tau,\omega)| = |\widehat{w}(\omega)|\,|\alpha_Q(\tau,\omega)|$. A phase function is then estimated, with the help of the Hilbert transform, H, using the minimum phase assumption,

$$\varphi(t,\omega) = H\left(\ln|s(t,\omega|\right) \tag{4}$$

where the Hilbert transform is taken over frequency.

Finally the Gabor spectrum of the reflectivity is estimated in the Gabor domain as:

$$Gr(t,\omega)_{est} = \frac{Gs(t,\omega)}{\sigma(t,\omega)} \tag{5}$$

and the reflectivity estimate is then recovered as the inverse Gabor transform of the result of equation (5).

Amplitude correction in Gabor deconvolution

Gabor deconvolution amplitude compensation for attenuation is carried out over the whole frequency-band of the signal with a high degree of stability, allowing an excellent amplitude correction of the attenuation effects even for very small values of *Q*; a couple of examples are shown in figure 2. The reported stability problems that affect amplitude restoration in inverse-*Q* filtering (e.g. Wang, 2001) beyond a time-variant critical frequency are not present in Gabor deconvolution. This is due to the fact that the operator for amplitude correction in Gabor deconvolution $1/|\sigma(t,\omega)|$, which is build by smoothing the Gabor transform of the input signal, turns out to be stable inside a frequency-band which is broader than the band inside which the amplitude recovery operator used in inverse-Q filtering by downward continuation is stable.

Phase correction in Gabor deconvolution

An important component of the Q-constant theory is velocity dispersion. Each monochromatic component of the seismic wave travels with a different velocity. Aki and Richards (2002) use the following relation, which is used in the examples below,

$$\upsilon(\omega) = \upsilon(\omega_0)\left[1 + \frac{1}{\pi Q}\log\left(\frac{\omega}{\omega_0}\right)\right], \tag{6}$$

where ω_0 is a reference frequency for which the velocity $\upsilon(\omega_0)$ is known. Velocity dispersion is the cause of the positive drift observed at tying seismic data at a well with a synthetic trace built from its well logs. Digitization of the seismic data imposes a constraint in the maximum frequency recoverable from seismic data; for a typical sample rate of 2 ms, the Nyquist frequency is 250 Hz. Hence the maximum phase velocity for any monochromatic component of a recorded seismic wave will be much lower than the reference velocity used to generate a synthetic seismic trace from well logs. When an algorithm for Gabor deconvolution as described above is applied to seismic data there is a partial restoration of the phase lag due to attenuation, as in the example shown in figures 2d, 2e, 3d and 3e. The restoration is partial because the minimum phase function of the deconvolution operator is built by using a digital Hilbert transform. The phase correction, obtained with the digital Hilbert transform, does not match the phase of the synthetic seismic trace generated from a well log because the digital Hilbert transform is an imperfect estimation, which depends on the sample rate, of the analog Hilbert transform (figure 1). Two different methods can be used to correct the remaining phase-shift. A first method consists of adding a time-frequency correction function to the phase operator estimated from the digital Hilbert transform to remove the remaining phase lag. The corrected phase is

$$\varphi_c(t,\omega) = H\left(\ln\left|\sigma(\tau,\omega)\right|\right) + \frac{t}{Q}\left(a + b\omega + c\omega^2\right), \tag{7}$$

where *a, b* and *c* depend on the Nyquist frequency and ω_0. Numerical values for a, b, and c are estimated by regression of a second order polynomial in ω upon the difference between the digitally computed phase and the exact phase expected from constant Q theory. This method is illustrated in the example shown in figure 3. A good compensation for the phase-shift can be achieved if a good estimation of Q is available, and just a partial correction is obtained using a poor estimation of Q. A second method is suggested by the variation of the digital Hilbert transform of the attenuation impulse response with the Nyquist frequency, (figure 1). A better matching between the analog and the digital Hilbert transforms can be obtained by resampling the signal. This fact can be used to apply a correction by interpolating the signal to a lower sample rate. In the example shown in figure 4 an excellent correction is obtained by resampling to 1 ms. a signal with a weak attenuation level corresponding to Q=200.

Examples

A reference synthetic nonattenuated trace was created using a well log from Alberta. An attenuated seismic trace was generated applying a forward *Q* filter to the reference trace, using Kjartansson theory, and then convolving the result with a minimum phase wavelet (40 Hz dominant frequency). The reference frequency ω_0 value used is 40,000π rad/sec. Then Gabor deconvolution was applied to the attenuated traces. The examples in figure 3 and 4 illustrate phase recovery in Gabor deconvolution for attenuation levels corresponding to Q=50 and Q=200 respectively. The instantaneous crosscorrelation, the maximum coefficient of the crosscorrelation between corresponding windows (40 ms. long, in the shown examples) in the real and the reference output, is used in figure 2 as attribute of the similarity between the expected and the real output (amplitude recovery). The instantaneous phase lag, the lag of the maximum coefficient of the windowed crosscorrelation is used in figures 3 and 4 as an attribute of the phase lag.

Conclusions

Knowledge of Q is not required in Gabor deconvolution to compensate for the amplitude losses due to attenuation. The minimum phase spectrum of the Gabor deconvolved trace is computed by using the Hilbert transform. The remaining phase-shift can be corrected by reconstituting the signal to a smaller sample rate if attenuation is weak. If an estimation of Q is available an alternative method for correcting the phase can be implemented by adding a function, linear in time and quadratic in frequency, to the digital Hilbert transform.

References

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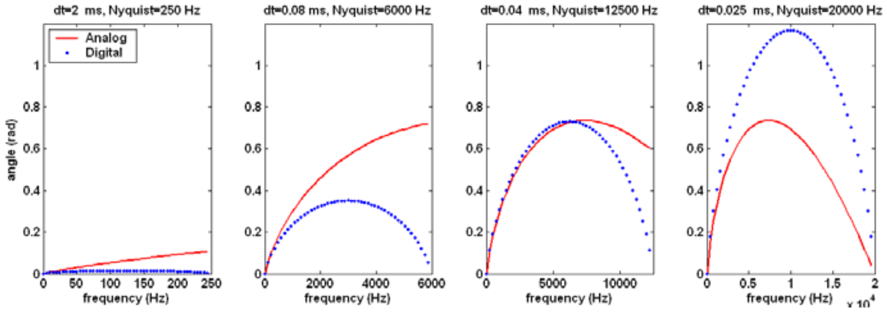

Figure 1. Minimum phase spectrum estimation for the attenuation impulse response. The minimum phase function can be computed either by analog or by digital Hilbert transform. A better matching between the two transforms is achieved by broadening the spectrum using a higher Nyquist frequency or equivalently a lower sample rate.

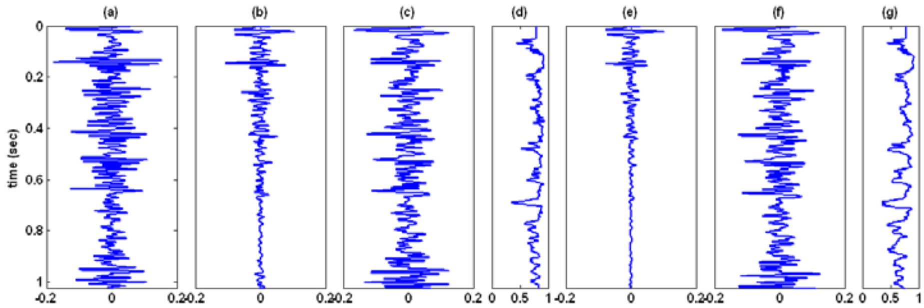

Figure 2. (a): synthetic band-limited reflectivity generated from a well log. (b) and (e): attenuated traces generated by applying a forward-Q filter to (a), Q=100 and Q=50 respectively. (c) and (f): band-limited reflectivity obtained by applying Gabor deconvolution to traces (b) and (e) respectively. (d) and (g): the instantaneous crosscorrelation gives an estimation of the amplitude recovery in traces (c) and (f) with respect to the reference trace (a).

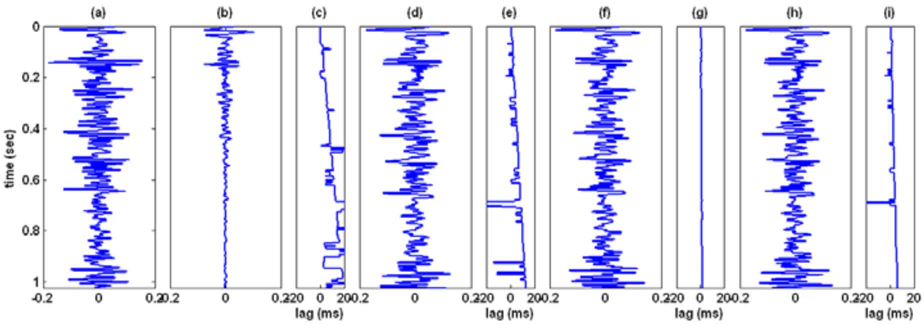

Figure 3. Phase correction using equation (7). (a): Reference reflectivity from a well log. b): attenuated trace using forward-Q=50 filter on (a). (c): phase-shift for trace (b) with respect to trace (a), this is the phase-shift due to attenuation. (d): After Gabor deconvolution on (b), using equation (4) for the phase. (e): remaining phase-shift for trace (d) with respect to trace (a). (f): after Gabor deconvolution on (b), using equation (7) for the phase, with Q=60 (a good estimation of Q). (g): phase lag for trace (f) with respect to trace (a). (h): after Gabor deconvolution on (b), using equation (7) for the phase, with Q=100, (a poor estimation of Q). (i): phase-shift for trace (h) with respect to trace (a).

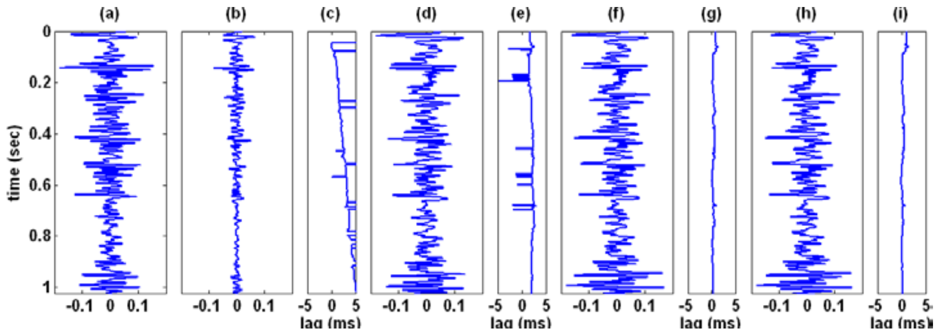

Figure 4. Phase correction using equation (4) and resampling to a smaller sample rate. (a), (b), and (c): same as in figure 3, but using Q=200 in the forward-Q filter. (d): After Gabor deconvolution on (b), using equation (4) for the phase. (e): remaining phase-shift for trace (d) with respect to trace (a). (f): after Gabor deconvolution on (b), using equation (4) for the phase, and resampling to 1 ms. (g): phase lag for trace (f) with respect to trace (a). (h): after Gabor deconvolution on (b), using equation (4) for the phase, and resampling to 0.5 ms. (i): phase-shift for trace (h) with respect to trace (a).