The FOCI method versus the WLSQ and Hale's wavefield extrapolation methods

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Summary

Recursive wavefield extrapolation methods are more powerful than ray theory based methods because of their greater ability to handle strong lateral velocity variations. Wavefield extrapolation methods have two major problems: (1) the extrapolator instability and (2) the computational expense. The forward operator and conjugate inverse (FOCI) method is an appropriate method for designing accurate and efficient extrapolation operators that remain stable in a recursive algorithm. FOCI's results are comparable with results obtained with other known methods such as Hale's and the weighted least square (WLSQ) extrapolation methods. Further, the spatial resampling in the FOCI method offers computational advantages.

Introduction

There are different ways to design spatial convolution operators for recursive wavefield extrapolation. The most common approach is to design an operator that approximates the exact phase-shift operator in the frequency-wavenumber domain then transform it to the spatial domain. Hale (1991) introduced a method to calculate a stable explicit extrapolator. This method is based on the Taylor expansion of the exact constant-velocity, phase-shift operator in the frequency-wavenumber domain and the use of novel basis functions. While Hale's method does lead to stable operator designs, it is numerically and analytically cumbersome and requires long operators to propagate high-angle wavefields. As well, a simple change of the operator length using Hale's method usually requires the use of both symbolic and numerical mathematical software packages.

Thorbecke et al. (2004) have introduced a weighted least-squares method (WLSQ), which is not perfectly stable but has a controlled instability. The WLSQ extrapolator can handle high angles of propagation and is faster and simpler to compute. Margrave et al. (2004, 2005) introduced a new method for designing spatial operators called the FOCI method. "FOCI" is an acronym for forward operator and conjugate inverse, which suggests the key concept in operator stabilization by Wiener filtering. However, there are three key innovations in the method with the other two being: (2) the use of dual operator tables to reduce evanescent filtering, and (3) spatial resampling of the lower frequencies to increase operator accuracy and decrease run times. In this paper, comparisons of the FOCI method versus Hales and the WLSQ extrapolation methods are shown.

Theory

To design a wavefield extrapolator, we start with the 2-D scalar wave equation for a homogeneous medium

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}. \tag{1}$$

After 2-D Fourier transformation equation (1) reduces to

$$\frac{\partial^2\bar{\bar{\psi}}}{\partial z^2}=-k_z{}^2\bar{\bar{\psi}}, \tag{2}$$

where

$$k_z{}^2=\frac{\omega^2}{v^2}-k_x{}^2. \tag{3}$$

Equation 2 is just a 1D Helmholtz equation whose solution, for upgoing or downgoing waves, is

$$\bar{\bar{\psi}}\left(k_x,z,\omega\right)=\bar{\bar{\psi}}\left(k_x,z=0,\omega\right)e^{ik_z z}, \tag{4}$$

where the sign of k_z is chosen based on the direction of wavefield propagation, ψ is the pressure wavefield, $\bar{\bar{\psi}}$ represents its 2-D Fourier transform, *t* is the two-way travel time, and *x* and *z* are the spatial and depth coordinates. Thus the wavefield at some depth *z*, $\bar{\bar{\psi}}\left(k_x,z,\omega\right)$, can be obtained by multiplying the recorded wavefield at the surface, $\bar{\bar{\psi}}\left(k_x,z=0,\omega\right)$, by a phase shift operator, $e^{ik_z z}$, in a homogeneous medium (Gazdag, 1978).

It is not immediately apparent how lateral velocity variations can be handled using the Gazdag phase shift method because the space coordinate has been Fourier transformed. As a result, extrapolation techniques for a laterally variable velocity field are usually formulated in the space-frequency domain (Berkhout 1981, Holberg 1988, Hale 1991, and others) as a dip-limited approximation to the inverse Fourier transform of the phase shift operator. To handle lateral velocity variations, the extrapolator is varied with the local velocity of the computation grid. The extrapolator has to have a finite length, which means the inverse Fourier transform of the extrapolator from the frequency-wavenumber to the space-frequency must be truncated. Truncation with a boxcar in the space-frequency domain is equivalent to convolving the Fourier transform of the extrapolator with a *sinc* function, which is the Fourier transform of the boxcar. As a result of this convolution, overshoots can occur at the evanescent boundaries. Usually, a truncated operator with a boxcar is not stable, as the amplitude exceeds unity, and repeated applications of this extrapolator will cause amplitudes greater than one to accumulate. The extrapolation methods such as Hale's, WLSQ, and FOCI try to design an operator in the spatial domain whose Fourier transform approximates the phase shift operator and remains stable in a recursive scheme.

Discussion

While Hale's extrapolator is stable, it requires long operators to handle high-angle wavefields. On the other hand, both the WLSQ and FOCI extrapolators have controlled instabilities, but they can handle higher angles of propagation efficiently. Furthermore, it is computationally more efficient to calculate WLSQ and FOCI extrapolators. Figure 1a shows a comparison of the three extrapolators. The FOCI extrapolator exhibits a better stability than the WLSQ but it is less stable than Hale's with a broader amplitude spectrum, which means that the FOCI extrapolator is more effective in handling the high angles of propagation than Hale's. When increasing the depth step size from 2 m to 6 m, the stability of the FOCI extrapolator does not change as much as the WLSQ extrapolator (Figure 1b).

The impulse responses of the three extrapolators are used to analyze their accuracies. The zero-offset experiment is done with an operator length of 31 points in a homogenous medium, a receiver spread of 1280 m, a maximum extrapolation depth of 1280 m, a velocity of 2000 m/s, a spatial spacing of Δx=10 m, and a vertical spacing of Δz=10 m. The trace in the center of the zero-offset section contains five Ricker wavelets at 0.0600, 0.1240, 0.1880, 0.2520, and 0.3160 seconds. The sample rate is 4 ms and the dominant frequency of the Ricker wavelet is 30 Hz. Figure 2 shows the impulse responses of the Hale's, WLSQ, and FOCI extrapolators compared with the result from phase-shift migration. While Hale's extrapolator could not handle the high angles of propagation, the WLSQ and FOCI extrapolators show that they can better handle the high angles of propagation.

The Marmousi dataset is usually used to test the accuracy of migration algorithms because of the strong lateral velocity variations and steep dips. These data will be used as a further test of the WLSQ and FOCI extrapolators in the presence of strong lateral velocity variations and steep dips. Figure 3a shows the velocity model of the Marmousi dataset. Figures 3b and 3c show the migration results using WLSQ and FOCI extrapolators, respectively. The operator length used for both is 51 points and the images have $\Delta x = 12.5$ m and $\Delta z = 12.5$ m spacing. The run times on a standard PC were 23.7 and 15.8 hours for the WLSQ and FOCI results, respectively. Both of these methods handled the strong lateral velocity variations and the steeply dipping events. However, due to spatial resampling, the FOCI method is more efficient than WLSQ. Spatial resampling can conceivably be incorporated into any method though it requires great flexibility in operator design, which FOCI excels at.

Conclusions

Unlike Hale's extrapolator, both the WLSQ and FOCI extrapolators are not perfectly stable but have controllable instabilities. However, they can handle higher angles of propagation than Hale's. The stability of the WLSQ extrapolator is more sensitive to the size of the depth step than the FOCI extrapolator. Calculating tables of extrapolators using the WLSQ and FOCI methods is computationally more efficient than using Hale's method. Further, the FOCI method with spatial resampling is computationally less expensive than the other two methods and this is promising for 3-D prestack depth migration.

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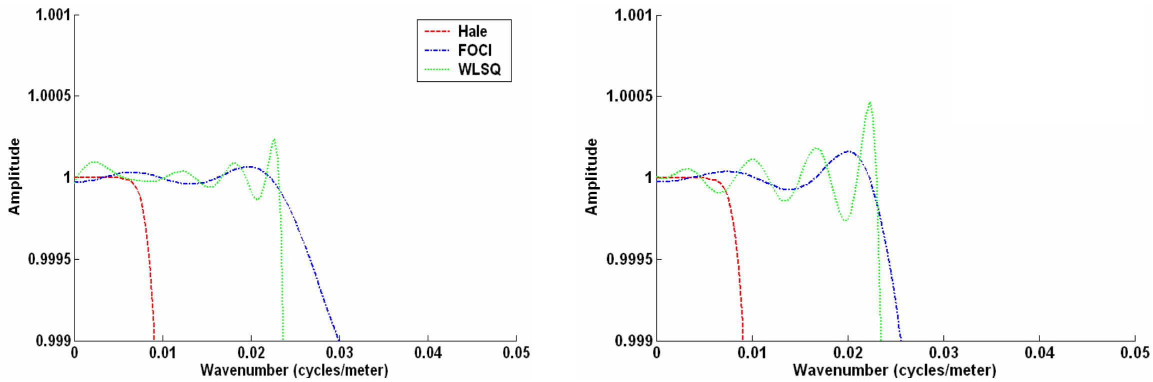
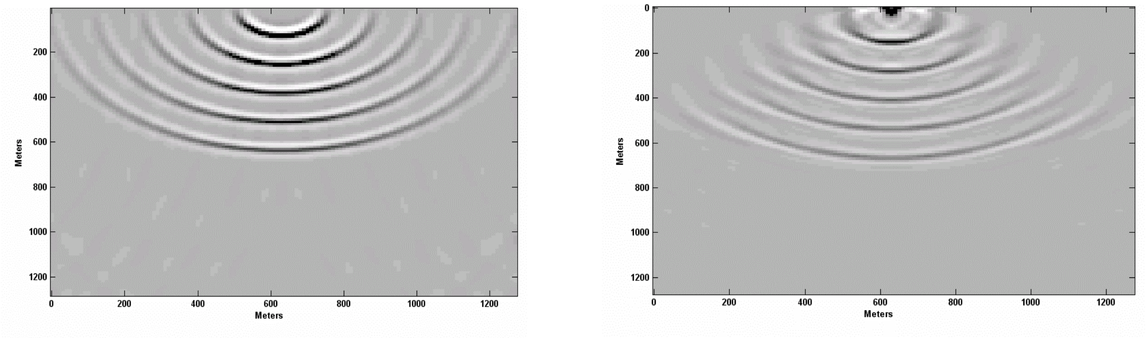


Figure 1: A comparison of the amplitudes of Hale's, FOCI, and WLSQ extrapolators. (a) Δx =10 m and Δz =2 m, operator length=25 points, velocity=2000m/s, and frequency=50 Hz and (b) the same parameters as in (a) but with Δz = 6 m.



WLSQ

FOCI

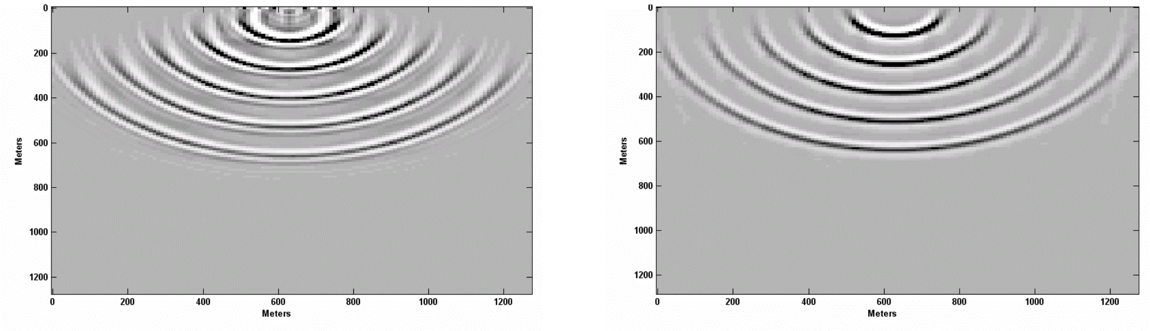


Figure 2: Impulse responses of Hale's, WLSQ, and FOCI extrapolators compared with the result from phase-shift migration. The velocity is 2000 m/s, Δx =10 m, Δz =10 m, and the operator length is 31 points.

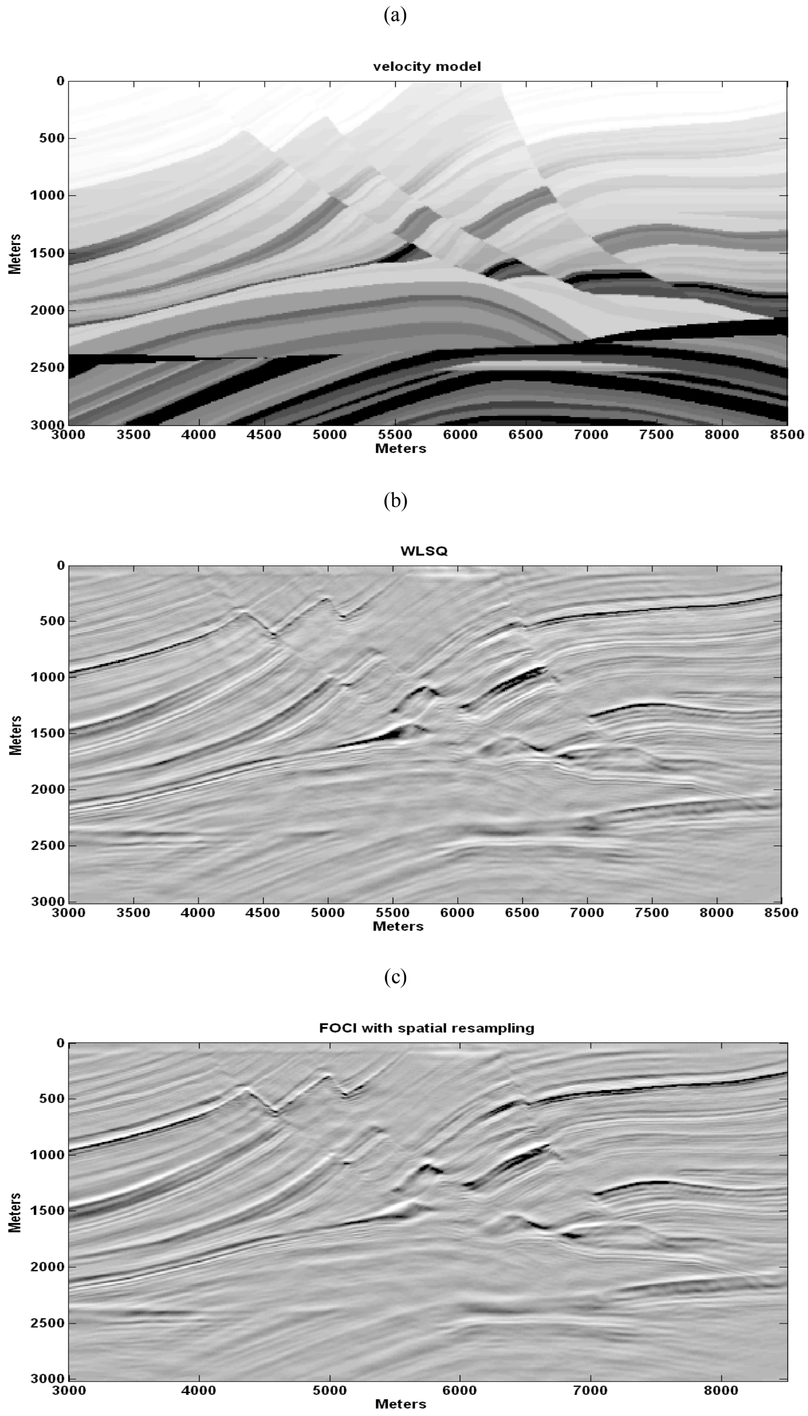


Figure 3: Prestack depth migration results of the Marmousi data where (a) shows the velocity model of Marmousi data, (b) shows the WLSQ result with run time=23.7 hours, and (c) shows the FOCI result with run time=15.8 hours. A 51-point operator was used in both (b) and (c).