# Relative polarity in correlating P-P and P-S events and approximations to $R_{PS}$ for polarity and AVO

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## Summary

In processing and interpreting P-S seismic sections, one usually relies on correlation of corresponding P-S and P-P events. Getting the polarity wrong on an event on the P-S section can lead to a mistie of half a cycle between correlated events on the two sections and, consequently, over- or underestimated $V_P/V_S$ ratios for the affected intervals. We examine under just what conditions an event will be recorded with opposite display polarities on P-P and P-S and show that the sign of $R_{PS}$ depends on the sign of $\Delta U$, where $U = \rho\beta^h$ and $h$ is a function of velocities and densities. In this analysis we derived an accurate small-angle $R_{PS}$ approximation, one that we show is better than the Aki-Richards expression in many situations, not just for small angles.

## Introduction

Correlation of P-S and P-P events enables determination of preliminary $V_P/V_S$ ratios, needed in the P-S processing flow. Typically, the correlation of P-P and P-S seismic sections relies on synthetic seismograms generated using log data for $V_P$, $V_S$ and $\rho$ from a nearby well, if available. These synthetics are then used to match events on P-P and P-S field sections and, from traveltime ratios, to calculate $V_P/V_S$ ratios. But these efforts can be frustrated if the logs for $V_S$ or $\rho$ are unavailable. If so, $\rho$ can be estimated from $V_P$ using empirical relationships like Gardner's equation (Gardner et al., 1974) or modified versions thereof for specific lithologies (Castagna et al., 1993). If $V_S$ information is unavailable, user-defined (or guesstimated) $V_P/V_S$ ratios have to be used to create a P-S synthetic stack (Lawton & Howell, 1992). Interval $V_P/V_S$ is then adjusted on the P-S synthetic stack to give optimal correlation with the stacked field data (Miller, 1996). In the absence of any of the logs, there may be polarity errors on particular events on the P-S synthetics relative to the P-P (Figure 1).

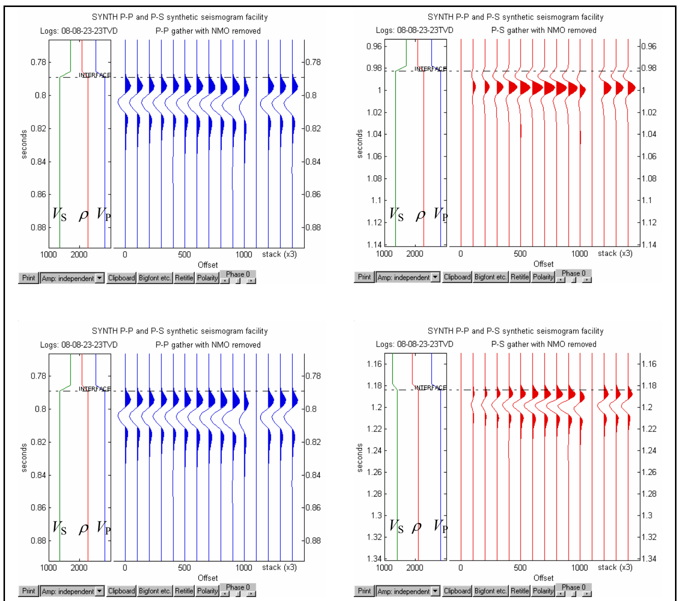


Figure 1. AVO responses and synthetic stacks [P-P (blue) and P-S (red)] for a model of a sand over a coal, an example of opposite PP-PS display polarities. In modeling this interface (upper figure) all logs were used. A $V_P/V_S$ ratio of 2.0 was used to construct the lower P-S synthetics for the scenario of a missing shear-wave log.

The SEG polarity standard (Thigpen et al., 1975) implies, when using for display a minimum-phase wavelet from a compressive source, that a P-P reflection from an interface with a *positive* P-P reflection coefficient ($R_{PP} > 0$) will begin with a downward (negative) deflection on the recorded seismogram (Sheriff, 2002). Recommended SEG standards for horizontal-component geophones (Landrum et al., 1994) and subsequent proposed standards (Brown et al., 2002) imply that a P-S reflection from the same source, when the interface has a *negative* P-S reflection coefficient ($R_{PS} < 0$), will also lead to a downward (negative) deflection on the inline geophone on positive-offset traces. Negative-offset traces, which are flipped in preprocessing, originally have the opposite polarity (e.g. Tessmer & Behle, 1988; Brown et al., 2002). Therefore, because for most interfaces, $R_{PP}$ and $R_{PS}$ have opposite sign, an event usually has the same display polarity on P-P and P-S. In this paper, we study the 'unusual situation', where events on the two sections display opposite apparent polarities, i.e., where $R_{PS}/R_{PP} > 0$.

## Approximations to the Zoeppritz equations

In adopting approximations for polarity relations, we did not wish to restrict ourselves to low contrasts, or small changes in rock parameters, so we assume small angles of incidence. Such approximations should also be valid for high-contrast interfaces. We also assume the polarity relationship at small angles to be representative of the polarity relationship of the stacked events in all but the rarest of cases. Figure 2 shows a rather extreme case in which there is a polarity change in $R_{PP}$. However, it occurs at sufficiently large offset that the stacked-trace polarity is the same as the small-offset polarity, even though this stack includes some rather long-offset opposite-polarity traces.

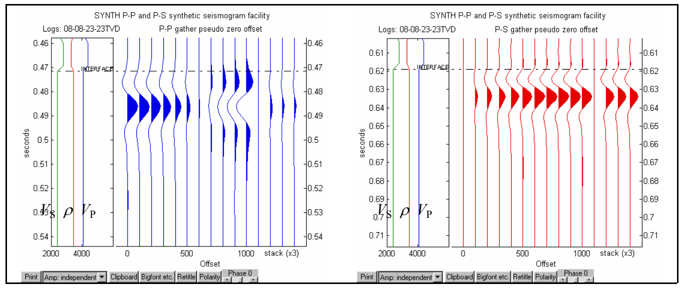


Figure 2. AVO responses and synthetic stacks [P-P (blue) and P-S (red)] for an interface model of water-saturated sandstone over chalk at a depth of 1000 m. The polarity of P-P changes at a moderate offset but the stack retains the small-offset polarity.

Many approximations to the Knott-Zoeppritz equations governing P-SV waves at a welded interface have been derived (mainly for $R_{PP}$ and $R_{PS}$), some of the earliest being those of Bortfeld (1961), Richards & Frasier (1976) and Aki & Richards (1980), whose notation we essentially follow. We use the symbol $r$ to represent rock parameters generally (e.g. $\alpha$, $\beta$, $\rho$, $\sigma$ or $\mu$) and we express any rock parameter and its changes over an interface in terms of $r$ (the average of $r_1$ and $r_2$) and $\Delta r$ (the difference $r_2 - r_1$), where 1 and 2 denote media 1 and 2, respectively. Contrary to what has been implied by some authors, the definitions of $r$ and $\Delta r$ are exact and do not require any assumption of small parameter changes. However, caution is required in expressions like:

$$\Delta Z = \Delta(\rho\alpha) = \rho\Delta\alpha + \alpha\Delta\rho \qquad \text{and} \qquad \Delta\mu = \Delta(\rho\beta^2) = \rho\Delta\beta^2 + \beta^2\Delta\rho \tag{1}$$

(where $\mu$ is rigidity). Both are exact if one defines notation like $\beta^2$ or $\rho\alpha$ to be the average of the squares $\beta_2^2$ and $\beta_1^2$ or the average of the products $\rho_2\alpha_2$ and $\rho_1\alpha_1$, not the square of the average of $\beta_1$ and $\beta_2$ or the product of the averages $\rho$ and $\alpha$. The difference between these two is second-order in $\Delta\beta$ or in $\Delta\rho\Delta\alpha$, so no such caution is needed in first-order low-contrast theory.

For $R_{PP}$, many approximations have been published (e.g. Bortfeld, 1961; Richards & Frasier, 1976; Aki & Richards, 1980; Shuey, 1985; Zheng, 1991; Wang, 1999; Ursenbach, 2002) However, for $R_{PP}$, we use the zero-offset expression as sufficient for characterizing polarity. Some of the published $R_{PS}$ approximations assume small parameter changes (Aki & Richards, 1980; Zheng, 1991; Xu & Bancroft, 1997; Gulati & Stewart, 1997; Donati & Martin, 1998; Ursenbach, 2002), some, like ours (below) assume small angles (Bortfeld, 1961; Richards & Frasier, 1976; Zaengle & Frasier, 1993; Wang, 1999; Ramos & Castagna, 2001; Carcuz, 2001; Geldart & Sheriff, 2004). It turns out that the differences between the two are not as great as one might think because in the Taylor expansions the terms of higher order in sin$i$ tend also to be the terms of higher order in $\Delta r/r$.

## Our approximation for $R_{PS}$

In deriving our own approximation to $R_{PS}$, we started with the exact formula given by Aki & Richards (1980, p. 150):

$$R_{PS} = -2\left[\frac{\cos i_1}{\alpha_1}\left(ab + cd\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)p\alpha_1\right]\Big/(\beta_1 D) \tag{2}$$

where $p$ is horizontal slowness. The definitions of $a$, $b$, $c$, $d$ and $D$ (Aki & Richards, 1980) were reprinted by Xu & Bancroft (1997), Ramos & Castagna (2001) and Vant (2003), though Ramos & Castagna err in their expression for $d$. It should be:

$$d = 2\left(\rho_2\beta_2^2 - \rho_1\beta_2^2\right) \qquad \text{and not} \qquad d = 2\left(\rho_2\beta_2^2 p^2 - \rho_1\beta_2^2\right). \tag{3}$$

To get our $R_{PS}$ approximation we first rewrite (2) as:

$$R_{PS} = -\left[\sin 2i_1\left(ab + cd\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)\right]\Big/(\alpha_1\beta_1 D) \tag{4}$$

then apply the small-angle approximation by setting $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ for sines and cosines of all incidence angles, $i_1$, $i_2$, $j_1$ and $j_2$, in the expressions for $a$, $b$, $c$, $d$ and $D$, giving:

$$R_{PS} = \frac{-\sin 2i_1\left(\alpha_2\beta_2\rho_2\Delta\rho + 2\rho_1\Delta\mu\right)}{(\rho_1\alpha_1 + \rho_2\alpha_2)(\rho_1\beta_1 + \rho_2\beta_2)}. \tag{5}$$

If we also approximate the explicit sine factor, it simply reduces from $\sin 2i_1$ to $2i_1$ and we have an expression equivalent to one given by Geldart & Sheriff (2004, p. 70). However this is only a slight simplification and it costs significantly in accuracy at moderate-to-large angles (Figure 3); so we usually choose to retain the explicit sine factor.

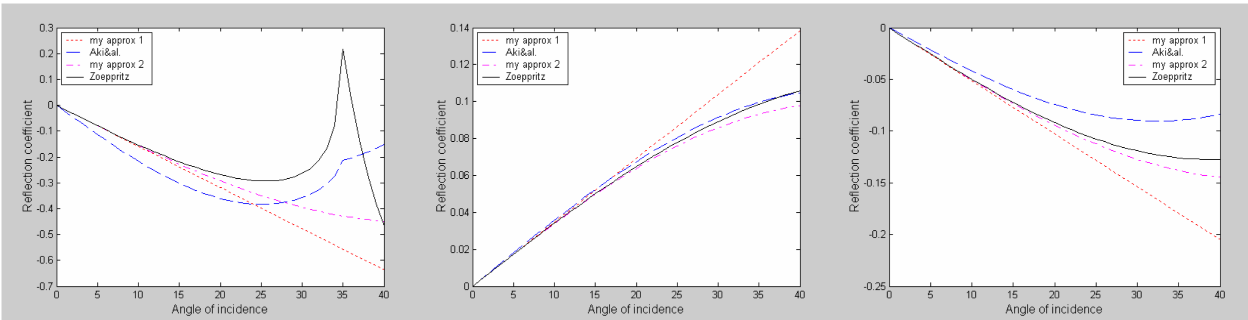


Figure 3. Comparison of the exact $R_{PS}$ curve (solid black) with three approximations: our equation (5) (dashed magenta); Geldart & Sheriff (2004) (dotted red) [equivalent to (5) with $\sin 2i_1 \to 2i_1$], and Aki & Richards (1980) (coarsely dashed blue). The interface models, left to right, are: young shale over old shale, sandstone over salt, and shale over gas-sand (Brown et al., 2002).

Wang (1999) started with the exact formulae for $R_{PP}$ and $R_{PS}$ (Aki & Richards, 1980) and developed Taylor-series expansions in powers of $p$, and therefore in powers of sin $\theta$, or $\theta$, i.e., small-angle approximations. He presents one $R_{PS}$ approximation [his equation (C-3)] that is correct up to terms in $p^5$. However, to obtain a second simplified approximation [his equation (C-5)], he introduces two assumptions, one of which is quite unjustified [coming from his equation (A-10)]. For $R_{PS}$ this amounts to assuming that $\Delta\rho = 0$, which eliminates one of the two first-order terms in Wang's expressions, even though he retains other terms up to fifth order. In comparing the accuracy of Wang's and other $R_{PS}$ approximations, for reasonable interfaces, Vant (2003) found Wang's first fifth-order approximation [his (C-3)] to be extremely accurate but the second [his (C-5)] to be quite inaccurate. Truncation of Wang's $R_{PS}$ approximations after first order gives, first, an expression whose accuracy is about the same as that of our equation (5) but which is much more complicated, second, an expression that is much less accurate than (5).

**Conditions for opposite P-P and P-S polarities**

We shall use the two small-angle approximations to $R_{PP}$ and $R_{PS}$ to determine under what conditions we get opposite display polarities on the same event on P-P versus P-S data, that is, when $R_{PP}/R_{PS} > 0$. Thus, we start with our own approximation for $R_{PS}$ [equation (5)] and the normal-incidence expression for $R_{PP}$:

$$R_{PP} = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}. \tag{6}$$

With respect to equation (6):

$$\operatorname{sgn} R_{PP} = \operatorname{sgn}(\rho_2\alpha_2 - \rho_1\alpha_1) = \operatorname{sgn}(\rho\Delta\alpha + \alpha\Delta\rho) = \operatorname{sgn}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right). \tag{7}$$

And with respect to equation (5):

$$\operatorname{sgn} R_{PS} = -\operatorname{sgn}\left\{\left[\alpha_2\beta_2\rho_2 + \rho_1\left(\beta_2^2 + \beta_1^2\right)\right]\Delta\rho + 2\rho_1(\rho_2\beta_2 + \rho_1\beta_1)\Delta\beta\right\} = -\operatorname{sgn}\left[f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta}\right] \tag{8}$$

where

$$f = \rho\left[\alpha_2\beta_2\rho_2 + \rho_1\left(\beta_2^2 + \beta_1^2\right)\right] \quad \text{and} \quad g = 2\rho_1\beta(\rho_2\beta_2 + \rho_1\beta_1). \tag{9}$$

So, for opposite display polarities, or $R_{PP}/R_{PS} > 0$, we need either:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho} > 0 \quad \text{and} \quad f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta} < 0\,; \quad \text{i.e.} \quad -\frac{\Delta\alpha}{\alpha} < \frac{\Delta\rho}{\rho} < -\frac{g\Delta\beta}{f\beta} \quad \text{for } R_{PP} > 0 \text{ and } R_{PS} > 0 \tag{10}$$

or:

$$\frac{\Delta\alpha}{\alpha}+\frac{\Delta\rho}{\rho}<0 \quad \text{and} \quad f\frac{\Delta\rho}{\rho}+g\frac{\Delta\beta}{\beta}>0\,; \quad \text{i.e.} \quad -\frac{g\Delta\beta}{f\beta}<\frac{\Delta\rho}{\rho}<-\frac{\Delta\alpha}{\alpha} \quad \text{for } R_{PP}<0 \text{ and } R_{PS}<0. \tag{11}$$

Equations (9) to (11) give conditions for occurrence of the unusual situation, that is, reversed display polarity of a reflection event on P-S versus P-P. We can formulate these conditions from (10) and (11) in another way by first noticing that:

$$\frac{\Delta\rho}{\rho}+\frac{g\Delta\beta}{f\beta}=\frac{\Delta\rho}{\rho}+\Delta\ln\beta^h=\frac{\Delta U}{U}, \quad \text{where} \quad h=\frac{g}{f} \quad \text{and} \quad U=\rho\beta^h\,. \tag{12}$$

Then we will get the unusual polarity situation, $R_{PP}/R_{PS} > 0$, if either:

$$\frac{\Delta U}{U}<0 \quad (\text{i.e. } R_{PS}>0) \quad \text{when} \quad \frac{\Delta Z}{Z}>0 \quad (\text{i.e. } R_{PP}>0) \tag{13}$$

or:

$$\frac{\Delta U}{U}>0 \quad (\text{i.e. } R_{PS}<0) \quad \text{when} \quad \frac{\Delta Z}{Z}<0 \quad (\text{i.e. } R_{PP}<0). \tag{14}$$

## Conclusions

We have derived a small-angle $R_{PS}$ approximation that is more accurate than the Aki-Richards approximation for three interfaces tested: beyond 25° incidence in two cases and beyond 40° in the third case. We have also derived mathematical expressions for the conditions on the interface rock parameters for an event to be recorded with opposite display polarities on P-P and P-S data.

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