Finite difference modeling in structurally complex anisotropic medium

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Summary

Modeling seismic data is a very important aspect in exploration seismology. In this paper, we derive a wave equation for a tilted TI medium and use it to generate seismic data over a numerical thrust sheet model. We also compare the data generated to the physical modeling data. It is also shown that the traveltimes of both physical modeling data and the numerical data match.

Introduction

The velocity structure of the earth is fundamentally anisotropic, i.e. the velocity varies with the direction of propagating of energy. Modeling algorithms which are used to model seismic data need to take into account the velocity anisotropy. Seismic modeling plays a very important role in exploration seismology. It is used in planning and designing seismic surveys, processing of data acquired, and in the interpretation of the data.

Anisotropy is an area of active research as shown by the number of publications over the last few decades. Helbig (Helbig, 1980), Thomsen (Thomsen, 1986), Alkhalifah (Alkhalifah et al., 1996), and Tsvankin (Tsvankin and Thomsen, 1994) have published many papers on the topic of anisotropy. Alkhalifah (Alkhalifah, 2000) derived a wave equation for acoustic medium in the paper titled "Acoustic wave equation for VTI medium." Later, he proposed a scheme for numerically modeling seismic data in orthorhombic medium (Alkhalifah, 2003). Zhang *et.al* (2002) extended Alkhalifah's VTI formulation to TTI medium to solve for traveltimes.

The most common symmetry observed in the context of exploration seismology, is Vertical Transverse Isotropy (VTI). VTI symmetry, as the name implies, is only valid when the symmetry axis is vertical. The approximation of VTI symmetry is not valid when the axis of symmetry is tilted, such as in structurally complex areas. The case in which the axis of symmetry is not vertical is termed 'tilted transverse isotropy (TTI).' Zhang (Zhang et al., 2002) proposed a seismic modeling scheme in a 2-D TTI medium. Most of the commercial seismic modeling programs that are available, like *NORSAR* and *GX II*, simulate VTI medium, but can't handle TTI medium.

In this paper we develop a modeling algorithm which can be used to model 2-D seismic data in TTI medium. This modeling technique will be implemented using a Finite Difference technique.

Wave Equation in TTI medium

In order to derive the wave equation in TTI medium, we start with the phase velocity formulation of Daley (Daley et al., 1999). The equation for phase velocity in an anisotropic medium can be written as Equation 1. Phase velocity (V_{ph})in 2-D VTI can be written as (Daley et al., 1999):

$$V_{ph}^2(\theta) = V_e^2(\theta) + \frac{A_D^2 \sin^2(\theta)\cos^2(\theta)}{V_e^2(\theta)}, \tag{1}$$

where for acoustic case A_D can be written as

$${A_D}^2 = A_{13}^2 - (A_{11} * A_{33}), \tag{2}$$

where for acoustic case V_e can be written as

$${V_e}^2 = A_{11}\sin^2(\theta) + A_{33}\cos^2(\theta), \tag{3}$$

A_{11} can be written as

$$A_{11} = A_{33} * (1 + 2\epsilon). \tag{4}$$

VTI Wave Equation in 2-D Media

Equation 1 is now used to derive the wave equation in 2-D VTI medium. Dividing both sides by ${V_{ph}}^2$ and multiplying by $i\omega^2$ we get the following equation

$$(i\omega)^2 = (i\omega)^2(A_{11}p^2 + A_{33}q^2 + \frac{A_{D_{13}}^2 p^2 q^2}{A_{11}p^2 + A_{33}q^2}), \tag{5}$$

where p, q are the slownesses in the x and z directions respectively. They can be written as

$$-i\omega p = \frac{\partial}{\partial x}, \tag{6}$$

and

$$-i\omega q = \frac{\partial}{\partial z}. \tag{7}$$

Substituting Equations 6 and 7 into 5 we get

$$(i\omega)^2 = A_{11}\frac{\partial^2}{\partial x^2} + A_{33}\frac{\partial^2}{\partial z^2} + A_{D_{13}}^2\frac{\partial^4}{\partial x^2 \partial z^2}. \tag{8}$$

Using

$$-i\omega = \frac{\partial}{\partial \tau}, \tag{9}$$

Equation 8 can be written as

$$\frac{\partial}{\partial t^2} = A_{11}\frac{\partial^2}{\partial x^2} + A_{33}\frac{\partial^2}{\partial z^2} + A_\delta\frac{\partial^4}{\partial x^2 \partial z^2}, \tag{10}$$

where,

$$A_\delta = 2 * A_{33} * (\delta - \epsilon). \tag{11}$$

Equation 11 becomes equals to zero when both anisotropy parameters are equal and the symmetry of the medium reduces to the degenerate case of elliptical anisotropy. Equation 10 can be solved using a finite-difference scheme.

This equation is valid in VTI medium. If the media has TTI symmetry, we need to transform this wave equation to be valid in TTI medium. The following method is used to achieve this transformation.

Axes Rotation in 2-D TTI Media

Up to this point we have defined the velocities, A_{11}, A_{33}, $A_{D_{13}}$, with respect to the principal crystallographic axes. Therefore, for structurally deformed medium, we need to define an angle of rotation θ, from unprimed model coordinates to primed model coordinates (Daley et al., 1999). The following recipe is used to rotate unprimed model coordinates to primed coordinates.

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -sin(\theta) \\ \sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x\prime \\ z\prime \end{bmatrix},$$

where the primed coordinates are the unrotated coordinates and unprimed are the rotated coordinates. Using the above orthogonal matrix, the unrotated directional space derivatives can be written in rotated coordinates as follows:

$$\frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial x\prime} - \sin\theta \frac{\partial}{\partial z\prime} \qquad (12)$$

$$\frac{\partial}{\partial z} = \sin\theta \frac{\partial}{\partial x\prime} + \cos\theta \frac{\partial}{\partial z\prime}. \qquad (13)$$

Equations 12 and 13 are substituted in the wave equation derived for VTI medium (Equation 10), the resulting equation then can be used propagate seismic waves in TTI medium. This equation is then numerically solved using the finite-difference technique.

Boundary Conditions

We have implemented sponge boundary conditions as specified by Bording (2004). The boundaries on all the four sides contain 20 discrete points and they mitigate the boundary reflections quite well.

Testing

The equation is tested on different models. The method was first tested on constant velocity models and then it was tested on complicated velocity model. The data is then compared to the physical modeling data acquired over the same model.

Model 1

The model 1 parameters are

- Velocity=3755 m/s

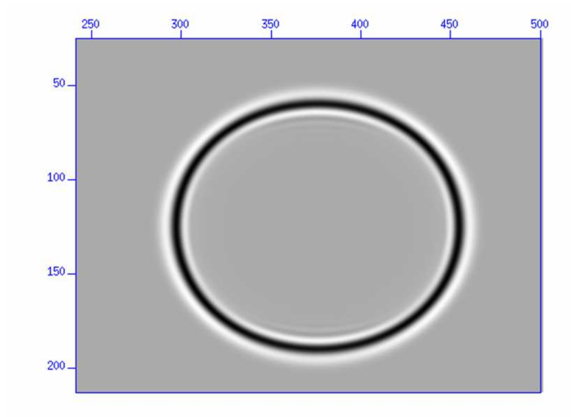

Fig. 1: The snap shot of a wave in a media with ϵ=0.2 and δ=0.2

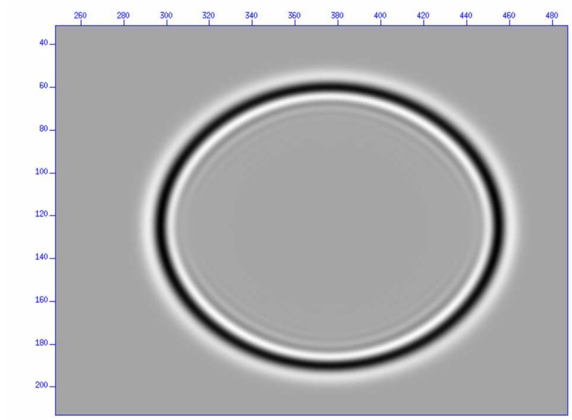

Fig. 2: The snap shot of a wave in a media with ϵ=0.2 and δ=-0.2

- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 1. The shot is located at the center of the model.

Model 2

The model 2 parameters are

- Velocity=3755 m/s
- ϵ=0.2
- δ=-0.2

A snapshot of the wave propagating is shown in Figure 2. The shot is located at the center of the model.

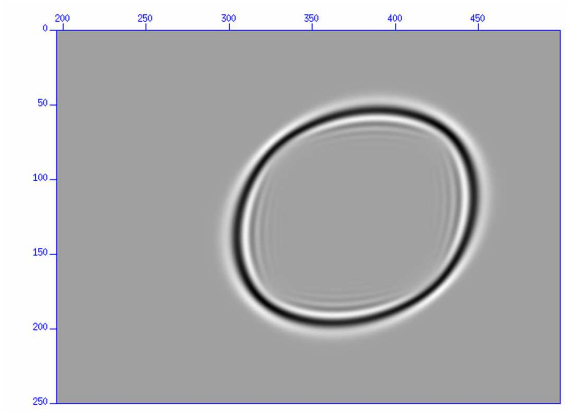

Fig. 3: The snap shot of a wave in a media with ϵ=0.2 and δ=-0.2 and dip=45°

Model 3

The model 3 parameters are

- Velocity=3755 m/s
- dip=45°
- $\epsilon = 0.2$
- $\delta = -0.2$

A snapshot of the wave propagating is shown in Figure 3. The shot is located at the center of the model.

Thrust Sheet Model

Lesie and Lawton (2001) acquired seismic data over a physical model of an anisotropic thrust sheet embedded in isotropic media. The Thrust sheet has the velocity of 2840 m/s, ϵ=0.22 and δ=0.10, while the isotropic material's velocity is 2745 m/s. The anisotropic material is phenol embedded in isotropic Plexiglas. The base of the model rests on a aluminum plate which has a velocity of 5402 m/s. The model is laterally heterogeneous, and the tilt ($0°, 30°, 50°$ and $60°$) of the axis of symmetry for the TI thrust sheet is variable. The dimensions of the model are 5100X2200 m.

The data was shot with a piezo-electric receivers and sources. A total of 86 shots worth of data was acquired. The physical model is illustrated in Figure 4. The algorithm described above is now tested on this model.

Comparison

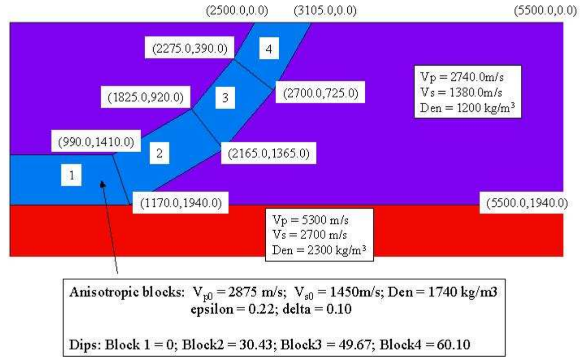

Fig. 4: The velocity model of the thrust sheet. Courtesy Prof.Don Lawton

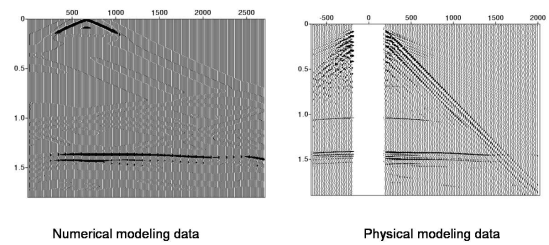

Fig. 5: The comparison between physical model and numerical model shots with shot at 660m

Case 1

Figure 5 shows the comparison between the shot records acquired over the physical modeling data and the numerical modeling data with the shot location at 660m. It can be seen that the traveltimes in both sections match with each other (Figure 5).

Case 2

In this case show in Figure (6) the shots are located at the middle of the model where the anisotropic thrust sheet outcrops. The interesting section in the data is where the dipping anisotropic section meets the surface. As the anisotropic layer's fast direction is oriented up wards to wards the surface, a pull up in traveltime is expected. We see a pull up in traveltime of the same magnitude in both the physical model data and the numerical model data.

Please note that somehow the offsets in the physical modeling data are flipped.

Case 3

In the case the shot shown in Figure (7) at the same position as Case 2 but the data is generated by making

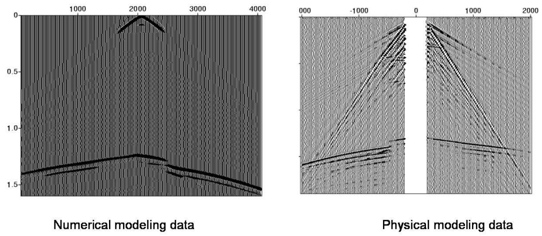

Fig. 6: The comparison between physical model and numerical model shots with shot at 2580m

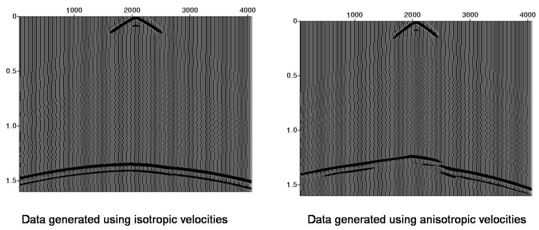

Fig. 7: The comparison between numerical model generated by using isotropic velocity model and Anisotropic model

the anisotropic parameters zero. The pull-up is less pronounced in the isotropic case.

Conclusions

In this paper we derived a wave equation for a TTI medium. We tested this equation on various simple models and displayed snapshots. The algorithm is then applied to a numerical version of a thrust sheet model. The data is then compared to the data acquired on a physical version of the same model. In future, we plan to extend this method to 3-D medium, and to improve the boundary condition implementation in the modeling code.

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References

Alkhalifah, T., Tsvankin, I., Larner, K., and Toldi, J., 1996, Velocity analysis and imaging in transversely isotropic media: Methodology and a case study: The Leading Edge, **15**, no. 05, 371–378.

Alkhalifah, T., 2000, An acoustic wave equation for anisotropic media: Geophysics, **65**, no. 4, 1239–1250.

Alkhalifah, T., 2003, An acoustic wave equation for orthorhombic anisotropy:, **68**, no. 4, 1169–1172.

Bording, R. P., 2004, Finite difference modeling - nearly optimal sponge boundary conditions: 74th Ann. Internat. Mtg., 1921–1924.

Daley, P. F., Marfurt, K. J., and McCarron, E. B., 1999, Finite-element ray tracing through structurally deformed transversely isotropic formations: Geophysics, **64**, no. 3, 954–962.

Helbig, K., 1980, Modeling of transversely isotropic media by thin isotropic layers: 50th Ann. Internat. Mtg, Session:R.30.

Leslie, J. M., and Lawton, D. C., 2001, P-wave traveltime anomalies below a dipping anisotropic thrust sheet: Advances, 77–84.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, no. 10, 1954–1966.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, **59**, no. 08, 1290–1304.

Zhang, L., Rector, J., and Hoversten, G., 2002, An Eikonal solver in tilted TI media: 71st Ann. Internat. Mtg, 1955–1958.