Estimation of Thomsen's anisotropy parameters from compressional and converted wave surface seismic traveltime data using NMO equations, neural networks and regridding inversion

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Summary:

This study evaluates algorithms that estimate Thomsen's anisotropic parameters (ε and δ) that describe P- and PS-waves. It is investigated if better results for the estimation of ε and δ are found from inversion of compressional wave data, converted wave data or these in combination and to determine the method that best estimates these parameters. Synthetic data is used to develop and evaluate a number of inversion algorithms including: NMO equations, neural networks and regridding inversions. Optimal inversion techniques are applied to the Blackfoot data set.

Introduction:

A synthetic model consisting of 9 horizontal layers, each with unique material properties including δ and ε is constructed. Anisotropic ray tracing is applied to solve for a data set that is convolved with a wavelet to create synthetic seismograms. Semblance analysis is preformed on CSP gathers to acquire velocity information and the subsequent anisotropy parameters are estimated.

Several methods for the estimation of Thomsen's anisotropy parameters are investigated:

1. P-wave NMO equations
2. PS-wave NMO equations
3. P-wave regridding inversion
4. PS-wave regridding inversion
5. Neural networks applied to P-wave data estimating δ
6. Neural networks applied to P-wave data estimating ε
7. Neural networks applied to P-wave data estimating δ and ε
8. Neural networks applied to PS-wave data estimating δ
9. Neural networks applied to PS-wave data estimating ε
10. Neural networks applied to PS-wave data estimating δ and ε
11. Neural networks applied to PP- and PS-wave data estimating δ
12. Neural networks applied to PP- and PS-wave data estimating ε
13. Neural networks applied to PP- and PS-wave data estimating δ and ε

Methods are assessed by comparing the RMS errors of each method.

For regridding inversion, two traveltime NMO approximations are selected that estimate Taner and Keohler's (1969) Taylor series expansion. First is Thomsen and Tsvankin's (1994) description of NMO for a VTI media and the second Castle's (1994) formulation of the shifted hyperbola NMO. Equations were implemented equating like coefficients and solving for unknown parameters. The solution for δ is determined in directly. However, solving for ε requires the use of a non-linear inversion technique: regridding inversion. Regridding inversion consists of discretizing the unknown parameters on a rough grid and calculating the error surface. The area that gives the best solution is rediscretized on a finer grid until the estimated solution is within a specified tolerance of the known solution. Neural network are also used to solve for the anisotropic parameters. Multilayer feedforward networks that use the standard backpropagation algorithm are brought into play.

Theory:

<u>Regridding Inversion</u>

Regridding inversion is the first of two non-linear inversion algorithms invoked for this study. In a regridding inversion algorithm multiple unknown parameters are estimated to evaluate a function with a known solution, velocity in this case. Initially a guess at the range of possible values for the unknown parameters is made. In this sense, the unknowns are discretized. The ensuing function is evaluated at each point. **Error! Reference source not found.** illustrates the error surface and picks. The true solution is marked by a red asterisk. After this first iteration, the optimal solution is marked by a black asterisk. The area around this optimal solution is further discretized into a finer grid and again the function is evaluated. This regridding is continued until the estimated solution is within a specified tolerance of the real solution; thereby obtaining estimates of the unknown parameters. The final solution is marked by a green square.

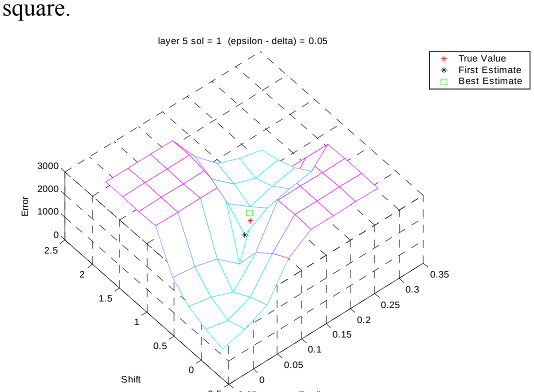

Figure 1 Regridding inversion error surface. Initial guess, best guess and true parameter estimations are marked

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An anticipated hurdle is the phenomenon of non-uniqueness. The system is underdetermined and therefore a unique solution may not exist. This is circumvented by evaluating the gradient of the error. When ε is greater than or equal to δ the best solution is obtained by discretizing the area with the steepest gradient conversely when δ is greater than ε the flattest area is discretized.

Multilayer feedforward network

The second inversion algorithm is neural networks. An artificial neural network referred to simply as a neural network is an information processing algorithm that is inspired by the way biological nervous systems, such as the brain, process information. In the simplest sense, a neural network is a mathematical algorithm that can be trained to solve a problem. The key element is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. Neural networks, like people, learn by example. A neural network is configured for a specific application, such as pattern recognition or data classification, through a learning process. The learning process in biological systems involves adjustments to the synaptic connections that exist between the neurons. This is true of neural networks as well.

The architecture of a multilayer feedforward neural network is illustrated in Figure 2. A layer consists of a weight matrix, a bias vector and an output vector. There can be any number of 'hidden' layers or layers that reside between the inputs and output layer. However networks with biases, a single sigmoid layer (hidden layer) and a single linear layer (output layer) are capable of approximating any function with a finite number of discontinuities (Higham and Higham, 2000). A single hidden layer is used in all of our applications.

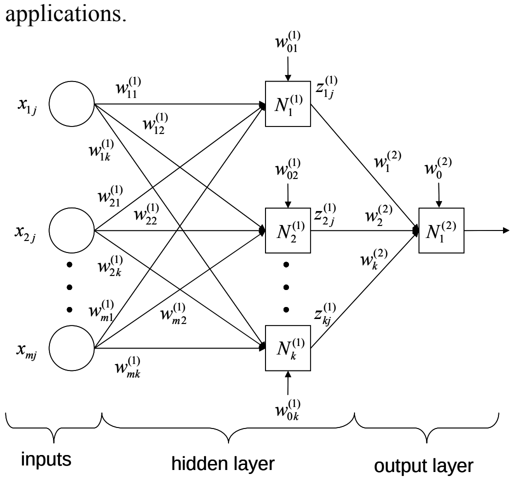

Figure 2 A multilayer feedforward network with *m* inputs, *k* neurons and 1 output

From Figure 2 and following Russell (2004) the input to the multilayer feedforward network is a vector, x, of m attributes where j =1:N is indicative of the number of seismic samples. Weights for the first layer connect inputs and neurons in the hidden layer and are written as w^1_{ij}, where i represents the input attribute number and j represents the neuron number. Each neuron, N, consists of weights, a summation step, and a transfer function. The weighting and summation is written as

$$y^1_{Kj} = \sum_{i=0}^{m} w^{(1)}_{Ki} x_{ij}, K = 1:k \qquad (1)$$

In equation 1 a bias term, w_{0K}, has been included by letting $x_{oj} = 1$. The sum of the weighted inputs and the bias forms the input to the transfer function, f. The transfer function is written as

$$z^{(1)}_{Kj} = f(y^{(1)}_{Kj}) \qquad (2)$$

The output from layer 1 is then fed into layer 2 and the resultant anisotropic parameter(s) are solved for.

Examples:

Results for estimating Thomsen's anisotropy parameters from P-wave data are shown, first for δ in Figure and next for ε in Figure 3. Three techniques are applied to the P-wave data to estimate δ: Thomsen's (1986) NMO equation for a P-wave propagating ain a VTI media (*PP NMO*) and two different neural networks (*PP NN estimating δ* and *PP NN estimating δ and ε*). Three techniques were also applied to the P-wave data to estimate ε: two different neural networks (*PP NN estimating ε* and *PP NN estimating δ and ε*) and P-wave regridding inversion (*P-wave regrid*).

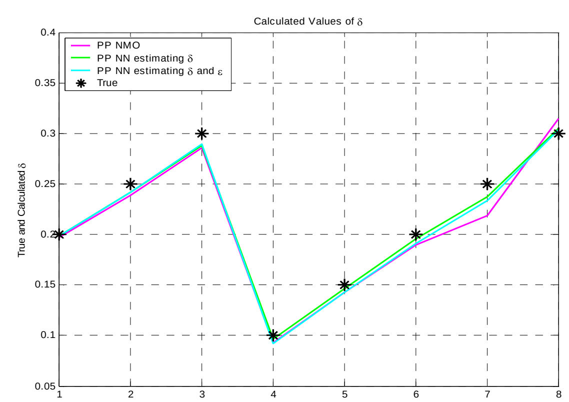

Figure True and calculated δ values from P-wave inversion methods

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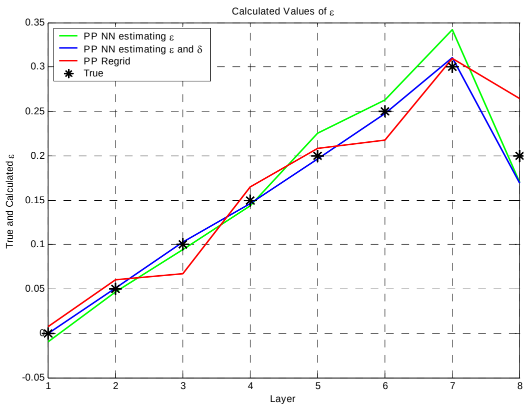

Figure 3 True and calculated ε values from P-wave inversion methods

Conclusions:

Methods were assessed by comparing the RMS errors of each method. Overall, neural networks applied to compressional wave data gave the best results. Other methods were very close in their performance, but it was the P-wave neural networks that proved to be most promising. The method determined to best recover δ was a neural network applied to P-wave data solving for δ. It was able to delimit δ to within 5% of the true value. Conversely, the method that best determined ε was a neural network applied to P-wave data that recovered both δ and ε. It was able to delimit ε to within 16% of the true value. PS-wave regridding inversion gave the best result for both ε and δ, when considering only converted wave data the. Neural networks designed to recover both parameters gave the best results for δ and when designed to recover only ε gave the best results for ε, when considering compressional and converted wave data were used in combination.

Algorithms that best estimated the anisotropy parameters were applied to the Blackfoot data set. Evidence of anisotropy in the area had been shown. Inversion results were consistent with Elapavuluri (2003) who had previously used P-wave NMO equations to determine δ and a Monte Carlo inversion to determine ε. Results are also in agreement with laboratory results of Thomsen (1986) where the coals and shales displayed a higher degree of anisotropy than the sands.

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