Phase correction in Gabor deconvolution

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Summary

Constant Q theory is a simple and robust theoretical model for seismic waves attenuation which needs just two parameters to characterize anelastic attenuation in a medium: the quality factor Q and a reference frequency ω_0. The convolutional model for a seismic trace can be extended to nonstationary attenuated traces using the constant-*Q* theory. Gabor deconvolution is a nonstationary extension of Wiener's deconvolution by factorizing the attenuated trace with the help of the Gabor transform. Gabor deconvolution compensates for the amplitude losses due to attenuation without necessity of any estimation of *Q*. A phase shift difference between a recorded seismic trace and a synthetic trace generated from a well log remains after Gabor deconvolution is applied. This phase shift can be removed either by adding a function, linear in time and quadratic in frequency, to the digital Hilbert transform estimation of the minimum phase function of the Gabor deconvolution operator or, by resampling the trace to a smaller sample rate.

Introduction

Gabor deconvolution is an extension of the stationary Wiener's deconvolution algorithm to apply nonstationary deconvolution. Wiener's algorithm posed in the Fourier domain is generalized by using a nonstationary extension of the Fourier transform such us the Gabor transform (Margrave et al, 2005). Gabor deconvolution is an alternate method to inverse-*Q* filter to compensate for attenuation effects. Although both methods use the constant-*Q* theory (Kjartansson, 1979) as a background theoretical model to represent attenuation there are essential differences between them. In inverse-*Q* filtering, the seismic data are compensated for attenuation effects before dealing with the inversion of the minimum-phase earth filter by spiking deconvolution. Theoretically, this ordering is incorrect. Gabor deconvolution approaches the compensation for attenuation in a radically different way, compensating for attenuation effects and inverting the minimum-phase earth wavelet simultaneously. Although this difference in methodology for tackling the attenuation problem defines already an important distinction between the two methods, there is one additional even more important difference: an estimation of *Q* is necessary for applying an inverse-*Q* filter, whereas to apply Gabor deconvolution knowledge of *Q* is not required. The nonstationary convolution model for the seismic trace considers the attenuated trace as the convolution between a minimum phase wavelet and a pseudodifferential operator which symbol is a minimum phase time-frequency attenuation function, applied on the reflectivity (Margrave et al., 2005). In Gabor deconvolution a minimum phase deconvolution operator is designed to compensate for the effects of the wavelet and the attenuation effects. The phase spectrum of this minimum phase deconvolution operator is the Hilbert transform of the logarithm of its amplitude spectrum. If the difference between the analog and the digital Hilbert transform and the dependence of the latter on the sample rate is not taken into account a phase-shift appears in the process of tying seismic data with synthetic traces generated from well logs.

Modeling attenuation, the constant Q model

A nonstationary convolutional model for an attenuated seismic trace, *s,* can be derived from the constant-*Q* theory by applying the forward *Q* filter, as a pseudodifferential operator, to a reflectivity function and then by convolving the result with a minimum phase wavelet. Such a model has been used, for example by Margrave et al. (2005), and can be expressed in the frequency domain,

$$\hat{s}(\omega) = \frac{1}{2\pi}\hat{w}(\omega)\int_{-\infty}^{\infty}\alpha_Q(\omega,\tau)r(\tau)e^{i\omega(t-\tau)}d\tau \,. \qquad (1)$$

where the 'hat' symbol indicates the Fourier transform, ω is the frequency, r is the reflectivity function, w is the wavelet and $\alpha_Q(\omega,\tau)$ is the time-frequency exponential attenuation function, defined as

$$\alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right). \qquad (2)$$

in which the real and imaginary components in the exponent and connected through the Hilbert transform *H*, result that is consistent with the minimum phase characteristic associated with the attenuated pulse. As written, equation (1) assumes a spatially constant Q and models only primaries though both of these simplifications can be removed with a slight complication in the formula.

Gabor deconvolution

The Gabor transform, G, maps functions of time to complex-valued functions of time and frequency using a windowed Fourier transform. Gabor deconvolution is a nonstationary extension of Wiener's deconvolution method, based on an approximate, asymptotic factorization of the nonstationary trace model of equation (1)

$$Gs(t,\omega) \approx \hat{w}(\omega)\alpha_Q(t,\omega)Gr(t,\omega)\,, \qquad (3)$$

which states that the Gabor transform of the seismic trace, $Gs(t,\omega)$, is approximately equal to the product of the

Fourier transform of the source wavelet, $\hat{w}(\omega)$, the time-frequency attenuation function, $\alpha_Q(t,\omega)$, and the Gabor transform of the reflectivity $Gr(t,\omega)$. The method assumes that $|Gr(t,\ \omega)|$ is a rapidly varying function in both variables τ and ω, $|\hat{w}(\omega)|$ is smoothly varying in ω, and $\alpha_Q(t,\omega)$ is an exponentially decaying function in both variables τ and ω, and constant over hyperbolic families of $\tau\omega = constant$. Analogously to the Wiener's method, Gabor deconvolution smoothes the Gabor magnitude spectrum of the seismic signal $|Gs(t,\ \omega)|$ to estimate the product of the magnitudes of the attenuation function and the source signature

$$|\sigma(t,\omega)| = |\hat{w}(\omega)|\ |\alpha_Q(t,\omega)|. \tag{4}$$

A phase function for $\sigma(t,\omega)$ is then estimated, with the help of the Hilbert transform, H, using the minimum phase assumption,

$$\varphi(t,\omega) = H(\ln|\sigma(t,\omega|) \tag{5}$$

where the Hilbert transform is taken over frequency. Finally the Gabor spectrum of the reflectivity is estimated in the Gabor domain as:

$$Gr(t,\omega)_{est} = \frac{Gs(t,\omega)}{\sigma(t,\omega)} \tag{6}$$

and the reflectivity estimate is then recovered as the inverse Gabor transform of the result of equation (6).

Phase correction in Gabor deconvolution

An important component of the Q-constant theory is velocity dispersion. Each monochromatic component of the seismic wave travels with a different velocity. Aki and Richards (2002) use the following relation, which is used in the examples below,

$$\upsilon(\omega) = \upsilon(\omega_0)\left[1 + \frac{1}{\pi Q}\log\left(\frac{\omega}{\omega_0}\right)\right], \tag{7}$$

where ω_0 is a reference frequency for which the velocity $\upsilon(\omega_0)$ is known. Velocity dispersion is the cause of the positive drift observed at tying seismic data at a well with a synthetic trace built from its well logs. Digitization of the seismic data imposes a constraint in the maximum frequency recoverable from seismic data; for a typical sample rate of 2 ms, the Nyquist frequency is 250 Hz. Hence the maximum phase velocity for any monochromatic component of a recorded seismic wave will be much lower than the reference velocity used to generate a synthetic seismic trace from well logs. When an algorithm for Gabor deconvolution as described above is applied to seismic data there is a partial restoration of the phase lag due to attenuation, as in the example shown in figures 4 (a) and 4 (b), and 6 (c) and (d). The restoration is partial because the minimum phase function of the deconvolution operator is built by using a digital Hilbert transform. The phase correction, obtained with the digital Hilbert transform, does not match the phase of the synthetic seismic trace generated from a well log because the digital Hilbert transform is an imperfect estimation, which depends on the sample rate, of the analog Hilbert transform (figure 1).

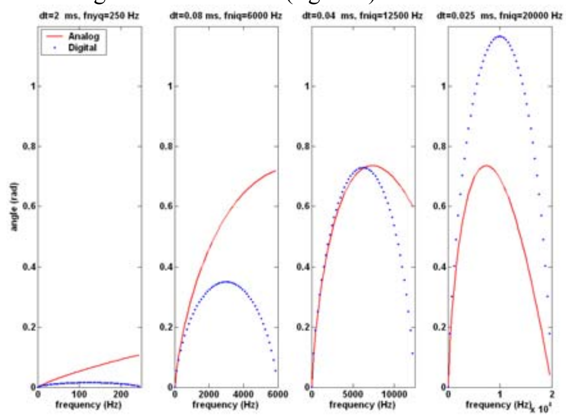

Figure 1. Minimum phase spectrum estimation for the attenuation impulse response. The minimum phase function can be computed either by analog or by digital Hilbert transform. A better matching between the two transforms is achieved by broadening the spectrum using a higher Nyquist frequency.

Two different methods can be used to correct the remaining phase-shift. A first method consists of adding a time-frequency correction function to the phase operator estimated from the digital Hilbert transform to remove the remaining phase lag. The corrected phase is

$$\varphi_c(t,\omega) = H(\ln|\sigma(\tau,\omega)|) + \frac{t}{Q}(a + b\omega + c\omega^2), \tag{8}$$

where a, b and c depend on the Nyquist frequency and ω_0. Numerical values for a, b, and c are estimated by regression of a second order polynomial in ω upon the difference between the digitally computed phase and the exact phase expected from constant Q theory. This method is illustrated in the example shown in figure 3. A good compensation for the phase-shift can be achieved if a good estimation of Q is available, and just a partial correction is obtained using a poor estimation of Q. A second method is suggested by the variation of the digital Hilbert transform of the attenuation impulse response with the Nyquist frequency, (figure 1). A better matching between the analog and the digital Hilbert transforms can be obtained by resampling the signal. This fact can be used to apply a correction by interpolating the signal to a lower sample rate. In the example shown in figures 6 an 7 a correction is obtained by resampling to 1 or 0.5 ms. a signal with an attenuation level corresponding to Q=200. This method is conceptually consistent with the theoretical formulation of the problem, but has practical disadvantages such us the increment in the demand of computational resources and the fact that the correction is not appreciable for strong attenuation.

Examples

A reference synthetic nonattenuated trace was created using a well log from Alberta. An attenuated seismic trace was generated applying a forward *Q* filter to the reference trace, using Kjartansson theory, and then convolving the result with a minimum phase wavelet (40 Hz dominant frequency). The reference frequency ω_0 value used is 40,000π rad/sec. Then Gabor deconvolution was applied to the attenuated traces. The examples in figure 4 and 5 illustrate phase recovery in Gabor deconvolution for attenuation levels corresponding to Q=50. The instantaneous crosscorrelation, the maximum coefficient of the crosscorrelation between corresponding windows (40 ms. long, in the shown examples) in the real and the reference output, (as illustrated in figure 2) as attribute of the similarity between the expected and the real output (amplitude recovery). The instantaneous phase lag, the lag of the maximum coefficient of the windowed crosscorrelation is used in figures 3 to 7 as an attribute of the phase lag.

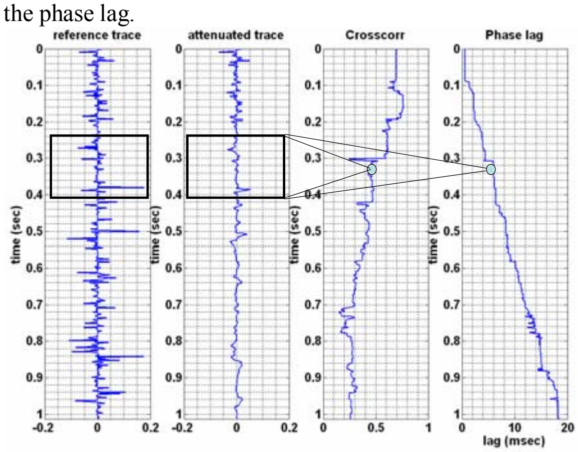

Figure 2. Phase lag estimation. The difference in phase between 2 traces is estimated by the windowed crosscorrelation. The maximum coefficient of the crosscorrelation and its lag indicate similarity and phase lag respectively, for the middle point of the window.

Conclusions

After applying Gabor deconvolution, a phase lag between the deconvolved trace and a synthetic generated from well logs is observed. This phase difference is attributed to the difference between the analog and the digital Hilbert transform. The minimum phase spectrum of the Gabor deconvolved trace is computed by using the Hilbert transform. The remaining phase-shift can be corrected, if the attenuation is weak, by reconstituting the signal to a smaller sample rate. If an estimation of Q is available an alternative method for correcting the phase can be implemented by adding a function, linear in time and quadratic in frequency, to the digital Hilbert transform.

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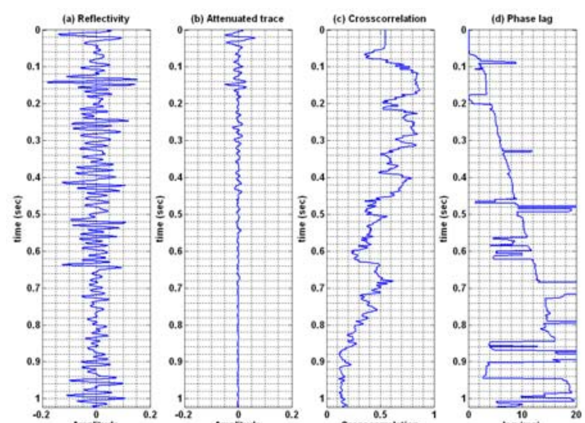

Figure 3. (a): Bandlimited reflectivity used as reference trace for all the examples in figures 4 to 7. b): attenuated trace using forward-Q=50 filter on (a). (c): Continuous crosscorrelation between traces (a) and (b). (d): phase lag for trace (b) with respect to trace (a), this is the phase lag due to attenuation.

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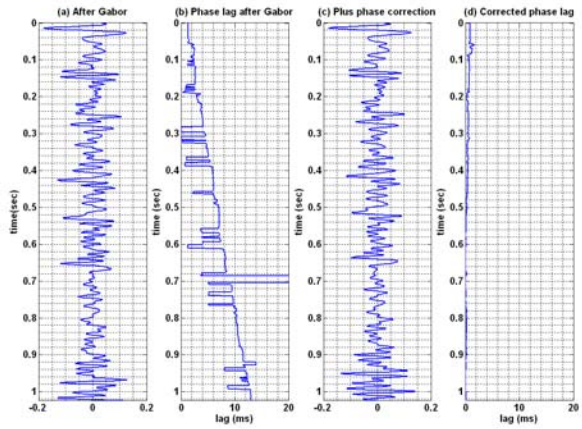

Figure 4. (a): After standard Gabor deconvolution on the attenuated trace in figure 3(b), using equation (5) for the phase. (b): Remaining phase-shift for trace (a) with respect to the reference trace in figure 3(a). (c): after Gabor deconvolution on the attenuated trace in figure 3(b), using equation (8) for the phase, with Q=50 (a perfect estimation of Q). (d): phase lag for trace (c) with respect to the reference trace in figure 3(a).

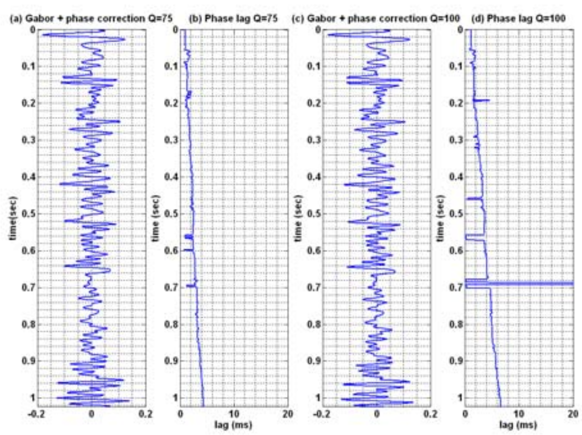

Figure 5. A partial correction for the phase lag can be obtained from an inaccurate estimation of Q. (a): after Gabor deconvolution on the attenuated trace in figure 3(b), using equation (8) for the phase, with Q=75, (an inaccurate estimation of Q). (b): phase-shift for trace (a) with respect to the reference trace in figure 3(a). (c): after Gabor deconvolution on the attenuated trace in figure 3(b), using equation (8) for the phase, with Q=100, (a poor estimation of Q). (d): phase-shift for trace (c) with respect to the reference trace in figure 3(a).

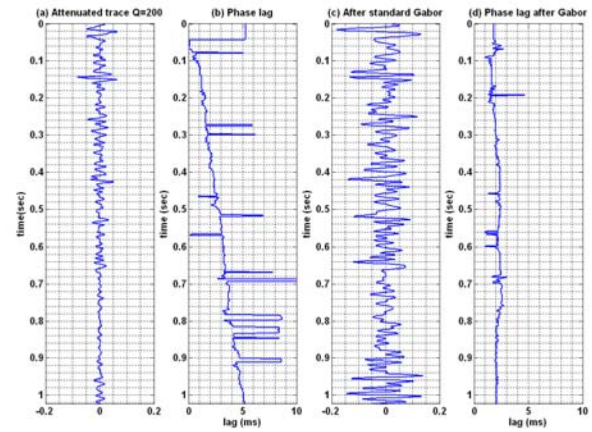

Figure 6. (a): Attenuated trace using forward-Q filter, Q=200, on the trace reference in figure 3(a). (b): Phase lag of trace (a) respect to the reference trace in figure 3(a), this is phase lag is due to attenuation. (c): After standard Gabor deconvolution. (d): Remaining phase lag after standard Gabor deconvolution.

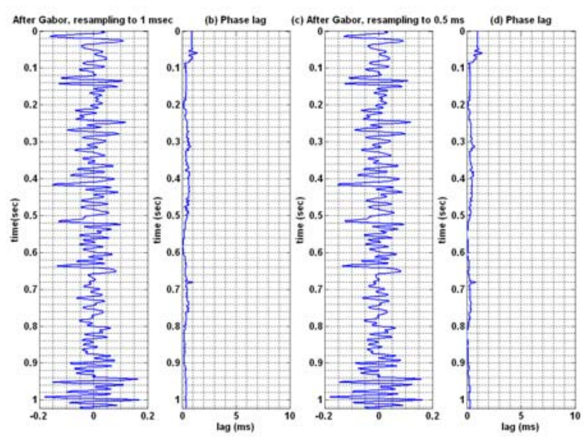

Figure 7. (a): After Gabor deconvolution on the attenuated trace in figure 6(a), using equation (5) and resampling to a 1.5 ms. (b): Residual phase lag for trace (a). (c): Same as (a) but resampling to 1 ms. (d): remaining phase lag for trace (c). For stronger attenuation, only a partial correction can be achieved.