Estimation of anisotropy parameters in Orthorhombic media

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Summary

The purpose of this study is to estimate Thomsen's parameters for P-wave, namely ϵ and δ in orthorhombic media by using the shifted hyperbola NMO equation. We propose a method for estimating the anisotropy parameters independent of the vertical velocity; in fact we also estimate vertical velocity. This method was tested on the data collected over a synthetic model with both vertical transverse isotropy (VTI) and orthorhombic symmetries. This method fails when $\epsilon = \delta$, i.e if the symmetry of the medium is elliptical.

Introduction

In this paper we will be detailing the procedure to estimate the anisotropy parameters which characterize orthorhombic media. Determination of δ (the short offset effect) is relatively easy but ϵ (long offset effect) is difficult and needs a measure of horizontal velocity, which is difficult to measure. In this study the long offset moveout information is used for ϵ estimation. Usually Dix type normal moveout (NMO) (Castle, 1994) correction at long offsets is not very accurate and it even worsens when there is anisotropy present. The Shifted hyperbola NMO (SHNMO) equation is more accurate at longer offsets than Dix NMO equation . Therefore by using the Shifted hyperbola NMO equation to correct long offset data we get a better estimation of RMS velocity (therefore better interval velocity) and so a better estimation of δ and ϵ.

The main technique used to estimate the seismic velocity is to fit a moveout (NMO) hyperbola to the traveltime curve. The main methods used to perform the velocity analysis are Normal Moveout Analysis and Shifted hyperbola NMO (SHNMO) analysis.

Shifted NMO hyperbola equation

(Castle, 1994) derives a new approximation to the NMO equation using the principles of reciprocity, finite slowness and exact constant velocity limit. For "reasonable" offsets, his approximation, termed as SHNMO, and can be written in different ways, but the easily comprehensible and implementable form can be written as:

$$t_x = t_s + \sqrt{t_0 + \frac{4x^2}{s.v_{nmo}}}, \tag{1}$$

where t_x is the traveltime at an offset (x), t_s is the *time-shift*, t_0 is the zero-offset time, v_{nmo} is the NMO velocity and s is the shift parameter. In the above equation, the "shift parameter", s, is a constant and is described as:

$$s = \frac{A_4^2}{A_2^2}, \tag{2}$$

where A_2 and A_4 are the second and fourth order moments of the velocity distribution. Although with a constant s the shifted hyperbola equation fits larger offsets better than Dix NMO formula (equation (**??**)), (Castle, 1994) showed that by varying the s with offset, one could obtain an exact fit of the traveltime curve. In equation (1) the shift parameter s is a constant. (Castle, 1994) states that for seismic data at longer offsets we can design s such that it varies with offset . The scale dependent s can be expressed as follows:

$$s(x) = \frac{1 + ax^2}{1 + bx^2} \tag{3}$$

We have modified this equation into the following form so that it is easy to implement and can be extended to larger offsets in order to account for strong non-hyperbolic moveout. This new offset dependent on offset $s(x)$ can be written as:

$$s(x) = s + ax^1 + bx^2 + cx^3 + dx^4. \tag{4}$$

By extending the *normal*-SHNMO equation, which has a constant shift into offset dependent SHNMO, we hope to get a velocity dependent on the offset. The offset dependent SHNMO can be written as:

$$t_x = t_{s(x)} + \sqrt{t_0 + \frac{4x^2}{s(x).v_{nmo}}}. \tag{5}$$

Advantages of SHNMO

The following are the advantages of SHNMO over the Dix NMO hyperbola:

- Accurate up to fourth order in offset,
- can approximate higher orders in media with "lesser" inhomogeneity,
- easier to implement than other higher order approximations,
- shift parameter provides vital information on the anisotropy and parameters

We will now discuss the procedure for estimation of anisotropy parameters in detail.

Estimation of ϵ

ϵ is the parameter which depends on the difference between both vertical and horizontal vertical phase velocities scaled with vertical velocity. ϵ can be expressed as:

$$\epsilon = \frac{v_{vert} - v_{hor}}{2v_{hor}} \tag{6}$$

As a cursory inspection of equation (6) would show that if we have an estimate of both vertical velocity (v_{vert}) and horizontal velocity (v_{hor}) we can calculate ϵ.

Implementation

The SHNMO is a non-linear equation and therefore linear inversion techniques like least square inversion fail. A random walk technique like SA inversion would serve the purpose of inverting the moveout equation for both s and v_{nmo}. The offset-traveltime information of each significant reflector is used for this inversion.

The model space to be inverted for here in this case is $m(s, v_{nmo})$. One of the advantages of this inversion technique is it gives a good control on the range of solutions in the model space and the acceptable error range.

Simulated Annealing inversion needs an initial guess of the range of model parameters in which the solution falls. First a very broad range of model values is specified as the search window along with a very large acceptable error. This range is refined at each run and also the acceptable error is reduced. This operation is repeated until the error converges at minima acceptable to the user.

Estimation of v_{vert} and v_{hor}

In equation (5) v_{nmo} is not constant and it is determined by $s(x)v_{nmo}$, therefore the NMO velocity is a function of offset. From this function we can determine zero offset velocity which equates to v_{vert} and far-offset velocity. This far offset velocity can be equated to v_{hor} if we have long enough offsets.

The estimated v_{vert} and v_{hor} are used to calculate the Thomsen's anisotropy parameter ϵ. As shown earlier ϵ can be calculated using equation (6).

Estimation of δ

The other Thomsen's parameter that needs to be calculated is δ. This parameter δ controls the depth determination and its determination is essential for depth imaging of anisotropic media.

The relation for δ in weak anisotropic media can be written as:

$$\delta = \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{33} - c_{55})}, \tag{7}$$

where c_{ij} are the Christoffel's coefficients, which are equal to $a_{ij}\gamma$; where a_{ij} is the velocity if ij direction and γ is the density.

δ can is also be written as the relation between zero-offset velocity v_0 and the v_{nmo} as :

$$v_{nmo} = v_0\sqrt{1 + 2\delta}. \tag{8}$$

The drawback of this relation is that it requires the knowledge of the v_0. we do not always have the information on the nature of v_0. Usually the value of v_0 is obtained from a VSP experiment or a checkshot value which is not usually available. This has been a major obstacle in estimating anisotropy parameters. (Alkhalifah and Tsvankin, 1995) proposed a parameter η to quantify the anisotropy in the media which is easier to estimate and can be used to get better time imaging.

Determination of δ without the knowledge of v_0

As seen in the above section, it is impossible to measure the anisotropy parameter δ without information v_0. The following method can be used to overcome this difficulty of prior information of v_0. The first step in using this method is to estimate the value of ϵ as described in the above section. This is accomplished by fitting a higher order NMO curve to the travel time data. We fit a SHNMO curve with the shift parameter s varying with offset. The next step involves in relating s to the parameter δ.

Relation between s and δ

(Siliqi and Bousqui, 2000) have worked on relating s to the parameter η. According to them the parameter η is related to the s in the following way:

$$s = 1 + 8\eta, \tag{9}$$

which was found to be not correct. We found the correct relation to be as follows:

$$s = 1 + 4\eta. \tag{10}$$

The derivation of this equation is shown in Appendix 1. Equation (10) can be written as follows:

$$s = 1 + 4\frac{\epsilon - \delta}{1 - 2\delta}. \tag{11}$$

In Figure 1 equation (11) is plotted with various values of ϵ and δ. This figure shows how s varies with δ for ϵ fixed at -0.2. Figure **??** shows how s varies with δ for ϵ fixed at -0.1 and figure 1 with ϵ fixed at -0.15. Using these curves and with a knowledge of s we can determine δ.

Orthorhombic medium

Orthorhombic media, unlike VTI media has three mutually orthogonal planes of mirror symmetry (Tsvankin,

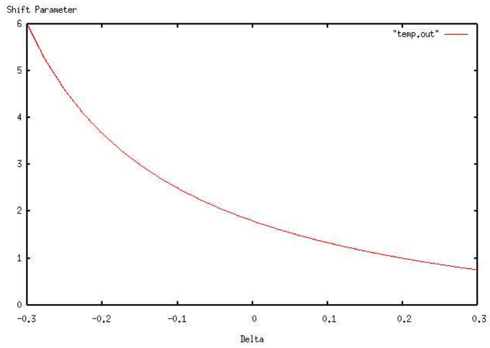

Fig. 1: The plot of s vs. δ with $\epsilon = -0.2$.

2001). In this section we detail a method which can be used to estimate the anisotropy parameters in orthorhombic media.

Nine independent stiffness coefficients are needed to describe this symmetry. The most common example of orthorhombic symmetry in real geology is that of sedimentary basins in a combination of parallel vertical fractures with vertical transverse symmetry in the background. (Bakulin et al., 2000) states that orthorhombic symmetry may be the simplest realistic symmetry for many geophysical problems.

Anisotropy parameters in Orthorhombic media

(Tsvankin, 2001) has introduced anisotropic parameters that characterize a wide range of signatures for orthorhombic anisotropy. They have used the orthorhombic anisotropy's analogy with VTI media to propose these parameters on the lines of Thomsen's parameters for VTI media.

There are *five* independent parameters which can be used to classify this media, they are: $\epsilon^{(2)}$, $\delta^{(2)}$, $\epsilon^{(1)}$, $\delta^{(1)}$, and $\delta^{(3)}$.

Estimation on a model

The method is now tested on the same model we used earlier to test the method detailed above in orthorhombic model. The lower layer now has has orthorhombic symmetry. As discussed above it has *five* parameters $(\epsilon^{(2)}, \epsilon^{(2)}, \delta^{(1)}, \delta^{(1)}$ and $\delta^{(3)})$ to characterize it. we generate seismic data over an orthorhombic media, the method detailing the procedure for generating seismic data is detailed in Elapavuluri et al.; the anisotropy analysis is performed on this data.

Model

The model is 2.5D model with a single layer; the second layer is orthorhombic. Orthorhombic media have two different directions of symmetry (Tsvankin, 2001); therefore the estimation of the parameters in orthorhombic media have to estimated in two different different directions orthogonal to each other.

Estimation of $\epsilon^{(2)}$ and $\delta^{(2)}$

The anisotropy parameters $\epsilon^{(2)}$ and $\delta^{(2)}$ are the anisotropy parameters with the x_2 axis as the symmetry axis, so in order to estimate these parameters, the traveltime data is analyzed along the x_1 axis. A CMP point at the end of the model is selected and the traveltime values are picked w.r.t to the offset. Figure shows the traveltime curve w.r.t to the offset. The inversion routine based on simulated annealing is performed on this data, and s and v_0 and v_{hor} are estimated. Using these values, $\epsilon^{(2)}$ and $\delta^{(2)}$ are calculated.

Estimation of $\epsilon^{(1)}$ and $\delta^{(1)}$

The anisotropy parameters $\epsilon^{(1)}$ and $\delta^{(1)}$ are the anisotropy parameters with x_1 axis as the symmetry axis, so in order to estimate these parameters, the traveltime data is analyzed along the x_2 axis. The same scheme used in the above section to estimate $\epsilon^{(2)}$ and $\delta^{(2)}$ is used for the estimation of these parameters. The value of v_0 remains the same, while the v_{hor} becomes c_{22} rather than c_{11}.

Estimation technique

As discussed earlier, the traveltime is picked in the x_1 direction and also x_2 direction. The Figures 2 and 3 show the moveout curves for Model1 in the these directions respectively; green curve the moveout if the medium were isotropic and the red curve for the anisotropic case. The Table 1 shows the values of s, v_0, v_{hor}, $\epsilon^{(2)}$ and $\delta^{(2)}$ in x_1 direction. The Table 2 shows the values of s, v_0, v_{hor}, $\epsilon^{(1)}$ and $\delta^{(1)}$ in x direction. These four parameters ($\epsilon^{(1)}$, $\delta^{(1)}$, $\epsilon^{(1)}$ and $\delta^{(1)}$) are used to estimate the stiffness coefficients, $(c_{11}, c_{22}, c_{33}, c_{13}$ and $c_{23})$. The only coefficient left to be calculated is c_{12}. As discussed,this stiffness coefficient can be calculated using the parameter $\delta^{(3)}$, using equation (**??**).

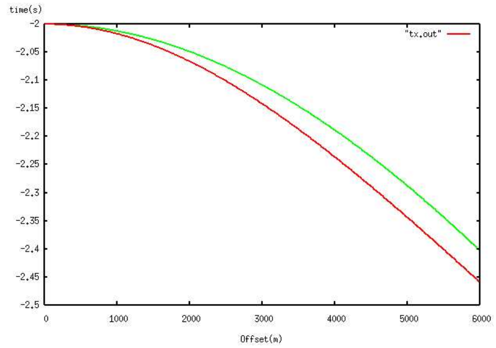

Fig. 2: Traveltime moveout in the x_2 axis.

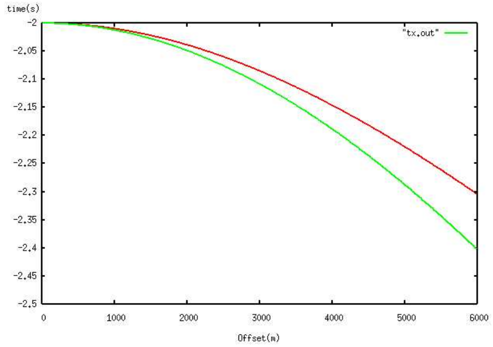

Fig. 3: Traveltime moveout in the x_1 axis.

Table 1: Comparison of estimated and model anisotropy parameters along the x_2 axis.

Model	v_0	$\epsilon^{(2)}_{model}$	$\epsilon^{(2)}_{est}$	$\delta^{(2)}_{model}$	$\delta^{(2)}_{est}$
Model 1	2000	0.2	0.1777	-0.2	-0.1888
Model 2	2000	0.1	0.087	-0.1	-0.088
Model 3	2000	0.1	0.1777	0.0	0.017

Table 2: Comparison of estimated and model anisotropy parameters along the x_1 axis.

Model	v_0	$\epsilon^{(2)}_{model}$	$\epsilon^{(2)}_{est}$	$\delta^{(2)}_{model}$	$\delta^{(2)}_{est}$
Model 1	2000	0.1	0.0768	0.0	0.017
Model 2	2000	0.2	0.189	-0.2	-0.189
Model 3	2000	0.15	0.1777	-0.1	-0.078

Conclusions

In this paper we have presented a scheme to estimate anisotropy parameters in generalized anisotropic media. We have applied this method on both VTI, which needs only 2 parameters to be characterized, and also to orthorhombic media which on the other hand needs 5 parameters. We have shown that the limitation of requiring knowledge of v_0 for the estimation of the anisotropy parameters can be overcome using the additional information obtained by using a shifted hyperbola with offset dependent shift parameter. The major limitation of this method is, it needs the moveout to be hyperbolic. In case of elliptical anisotropy, where the medium is anisotropic yet the moveout os hyperbolic, this method fails to estimate the anisotropy parametes.

This method is then tested on data generated over both VTI and orthorhombic models. This method will be tested on real data to test the effectiveness of this technique.

References

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Appendix A

In this appendix we will derive an equation to relate s and η. According to (Castle, 1994) s can be written as:

$$s = \frac{A_2^4}{A_4^4}, \tag{A-1}$$

where A_2 is the 2-order moveout and A_4 is the fourth order moveout. On the other hand, according to (Alkhalifah and Tsvankin, 1995) η can be written as

$$v_{nmo} = v_{hor}\sqrt{1+2\eta}. \tag{A-2}$$

We know that v_{nmo} is the second order moveout and is therefore equal to A_2 and in the similar fashion v_{hor} is approximately equal to the fourth order moveout, A_4. Applying these relations and equating equations A-1 and A-2 we get the following relation:

$$s = (1+2\eta)^2. \tag{A-3}$$

Expanding equation (A-3) and neglecting the higher order terms we get the final relation as:

$$s = 1 + 4\eta. \tag{A-4}$$