Seismic Depth Imaging with the Gabor Transform

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Summary

Wavefield extrapolation by spatially variable phase shift is currently a migration tool of importance. In this paper, we present a new prestack seismic migration algorithm using the Gabor transform with application to the Marmousi acoustic dataset. The imaging results show a very promising depth imaging algorithm, which is competitive with the best depth imaging algorithms. The Gabor depth imaging algorithm approximates generalized phase shift plus interpolation (GPSPI) wavefield extrapolation using the Gabor, or windowed Fourier, transform to localize the wavefield. The key to an efficient algorithm is to develop an adaptive windowing scheme that only localizes the wavefield as required by the lateral velocity variation. If there is no lateral velocity variation then no localization (windowing) is required. When velocity varies rapidly, then many, relatively narrow, windows are required for accurate wavefield extrapolation. We present the details of an adaptive windowing method that has a controlled phase error. Programs have been coded with the adaptive windowing algorithm, which substantially reduces the computational burden in wavefield extrapolation when compared to the full GPSPI integral. We will illustrate the performance of this algorithm with images from prestack depth migration of the Marmousi dataset.

Introduction

Migration with Gazdag (1978) phase shift method can only accommodate constant lateral velocity in any depth step, which is unrealistic for many practical applications, where velocity structures are often heterogeneous with strong lateral velocity fluctuations. To address lateral velocity variations in phase-shift wavefield extrapolations, phase shift plus interpolation (PSPI) was proposed by Gazdag and Sguazzero (1984) using a set of reference (laterally homogeneous) velocities to calculate the corresponding extrapolated wavefields; the final extrapolated wavefields are obtained by interpolating with specific velocities corresponding to certain lateral positions. Stoffa et al. (1990) gave an alternative extrapolation algorithm, split-step Fourier migration, dealing with lateral velocity variation while keeping the advantages of the phase-shift method, i.e., accuracy and efficiency. Other phase-shift wavefield extrapolation methods such as 'phase-screen propagator' (Wu and Huang, 1992; Roberts et al., 1997; Rousseau and de Hoop, 2001; Jin et al., 2002) also provide for accurate imaging with abrupt velocity variations in such geological settings as salt-dome environments. Margrave and Ferguson (1999) used a nonstationary phase shift (NSPS) method and a generalized phase shift plus interpolation (GPSPI) to improve migration results, where wavefield extrapolations were done totally in the Fourier domain using arbitrary velocity variations. Our wavefield extrapolation method follows Jin and Wu (1998) and approximates GPSPI with a Gabor extrapolator. We also have control over speed and accuracy of Gabor wavefield extrapolations with the help of the adaptive windowing algorithm by Ma and Margrave (2005). In the following sections, we will demonstrate the adaptive Gabor wavefield extrapolation algorithm and give some imaging results created by these algorithms.

Gabor wavefield extrapolation theory

The Gabor transform

The continuous Gabor transform pair is written as (following Margrave and Lamoureux (2001))

$$V_g s(x'_T, k_T) = \int_{\mathbb{R}} s(x_T) g(x_T - x'_T) \exp(-ix_T k_T) dx_T$$
(1)

and

$$s(x_T) = \int_{\mathbb{R}^2} V_g s(x'_T, k_T) \gamma(x_T - x'_T) \exp(ix_T k_T) \mathrm{d}k_T \mathrm{d}x'_T,$$
(2)

where x_T denotes transverse coordinates (e.g., $x_T = x$ in 1D, $x_T = (x, y)$ in 2D), $s(x_T)$ is the input signal, $V_g s(x'_T, k_T)$ is the Gabor spectrum of $s(x_T)$, $g(x_T - x'_T)$ is an analysis windowing function with its center at x'_T , $\gamma(x_T - x'_T)$ is a synthesis windowing function, and k_T is the coordinate in the wavenumber domain corresponding to x_T , \mathbb{R} denotes real domain for integrations. Equation (1) is in fact a Fourier transform of a windowed version of signal $s(x_T)$.

Equation 1 is used to calculate the Gabor spectrum of $s(x_T)$; in order to recover the original signal $s(x_T)$ from its Gabor spectrum $V_q s(x'_T, k_T)$, analysis and synthesis windows must satisfy

$$\int_{\mathbb{R}} g(x_T) \gamma(x_T) \mathrm{d}x_T = 1 \tag{3}$$

(Margrave and Lamoureux, 2001), which is called a partition of unity (POU). The analysis windows could be any kind of mathematical function. However, in our wavefield extrapolation applications, we choose functions with a localization property. In this way, we may represent our wavefield extrapolator depending on local velocities with a small error. Gaussian windows are good candidates, and we have chosen them for this paper. We also choose the synthesis window as unity, that is, we do not localize wavefields in the synthesis process.

Gabor Wavefield extrapolation

The generalized phase shift plus interpolation (GPSPI) wavefield extrapolation is formulated as (Margrave and Ferguson, 1999; Margrave et al., 2004)

$$\psi_P(x_T, z + \Delta z, \omega) = \int_{\mathbb{R}} \hat{\psi}(k_T, z, \omega) \hat{W}(k_T, x_T, \Delta z)$$

$$\cdot \exp\left(-ik_T x_T\right) dk_T, \qquad (4)$$

where

$$\hat{W}(k_T, x_T, \Delta z) = \exp\left(ik_z\left(v(x_T)\right)\Delta z\right),\tag{5}$$

$$k_z(v(x_T)) = \begin{cases} \sqrt{\frac{\omega^2}{v^2(x_T)} - k_T^2}, & \frac{\omega^2}{v^2(x_T)} > k_T^2 \\ i\sqrt{k_T^2 - \frac{\omega^2}{v^2(x_T)}}, & \frac{\omega^2}{v^2(x_T)} < k_T^2, \end{cases}$$
(6)

 Δz is the step size of extrapolation in z (vertical) direction, ω is temporal frequency and $v(x_T)$ denotes lateral velocities along a slab with thickness Δz . Equation (4) extrapolates wavefields at depth z down to depth $z + \Delta z$ in the frequency-wavenumber domain.

To develop a Gabor approximation to equation (4), we introduce the approximation

$$\hat{W}(k_T, x_T, \Delta z) \approx \sum_{j \in \mathbb{Z}} \Omega_j(x_T) S_j(x_T) \hat{W}_j(k_T, \Delta z), \quad (7)$$
$$\sum_{j \in \mathbb{Z}} \Omega_j = 1, \quad (8)$$

where Ω_j is a family of windows forming a POU (refer to equation (8), the discrete form of POU), $S_j(x_T)$ is a splitstep Fourier operator for phase correction in the Gabor imaging, $\hat{W}_j(k_T, \Delta z)$ is a wavefield extrapolator with reference velocities v_j , which are

$$S_j(x_T) = \exp\left(i\omega\Delta z \left(\frac{1}{v(x_T)} - \frac{1}{v_j}\right)\right),\tag{9}$$

$$\hat{W}_j(k_T, \Delta z) = \exp\left(ik_z(v_j)\Delta z\right) \tag{10}$$

and

$$v_j = \frac{\int_{\mathbb{R}} \Omega_j(x_T) v(x_T) \mathrm{d}x_T}{\int_{\mathbb{R}} \Omega_j(x_T) \mathrm{d}x_T},$$
(11)

respectively. Notice that in equation (10), k_z is still calculated with equation (6), using the reference velocity v_j corresponding to a specific window Ω_j (see equation (11)) instead of $v(x_T)$.

Using approximate wavefield extrapolator (7) in (4) gives

$$\psi_P(x_T, z + \Delta z, \omega) = \sum_{j \in \mathbb{Z}} \Omega_j(x_T) S_j(x_T) \int_{\mathbb{R}} \hat{\psi}(k_T, z, \omega) \\ \cdot \hat{W}_j(k_T, \Delta z) \exp\left(-ik_T x_T\right) dk_T.$$
(12)

Equation (12) specifies our Gabor wavefield extrapolator.

Adaptive Gabor wavefield extrapolation

If analysis windows in Gabor wavefield extrapolations are uniformly distributed along the lateral dimensions, we will, in most circumstances, have excessive redundancy in computation. That is, the algorithm without adaptive windowing usually calculates more windowed Fourier transforms than it requires. For example, we know that for homogeneous media, we only need one window instead of many, where the GPSPI method degenerates into Gazdag (1978) Fourier migration (phase shift with constant velocity) in laterally homogeneous media. If the lateral velocity structures in a slab are not extremely inhomogeneous, we can use fewer windowed Fourier transforms in wavefield extrapolations than we do in rapidly varying velocity models. Adaptive windowing algorithms are suggested to deal with different types of lateral velocity structures met in Gabor wavefield extrapolations.

At this time, we use the Ma and Margrave (2005) algorithm, which uses phase errors as criteria to determine the number of windows needed in wavefield extrapolations. i.e.,

$$= \left\| \arg \left(\Omega_j(x_T) \hat{W}(k_T, x_T, \Delta z) \right) - \arg \left(\Omega_j(x_T) \sum_{k \in \mathbb{Z}} S_k(x_T) \Omega_k(x_T) \hat{W}_j(k_T, \Delta z) \right) \right\|_{\infty}$$

$$j, k \in \mathbb{Z}, \qquad (13)$$

where ϵ_j is the total phase error within the j^{th} window Ω_j , $\hat{W}(k_T, x_T, \Delta z)$ and its approximation $\hat{W}_j(k_T, \Delta z)$ have been windowed by Ω_j , the reference velocity used to calculate the Gabor extrapolator $\hat{W}_j(k_T, \Delta z)$ equals to the j^{th} reference velocity v_j ; $\|\cdot\|_{\infty}$ denotes the L_{∞} norm.

Examples

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 ϵ_j

The Marmousi synthetic data set has been widely used as a benchmark for testing depth imaging algorithms. The original dimensions of the Marmousi velocity section are 10,000 m in width and 3,000 m in depth. The portion we try to image is shown in Figure 1 (c) (2000-9000 m and 3000 m in depth). In the Marmousi synthetic data set, we have 240 shot records, each of which has 96 traces, with time extending to about 2.9 seconds. For each shot record, there are 241 extrapolation steps with step size $\Delta z = 12.5$ meters.

Before discussing the imaging results, we explain the parameter used in the adaptive windowing (or partitioning) algorithm. We call this parameter 'threshold', which is used to set the threshold in terms of the relative phase error. For example, when threshold = 25%, we mean that 25% of the absolute phase related to an exact velocity is set as a tolerance value for phase errors between the phases corresponding to a reference velocity (used for extrapolation) and the exact velocity when patitioning the lateral velocity structures in each depth. To show how this windowing



(a) Partitioning the velocity slice using a phase error threshold of 35% (16 windows created)



(b) Partitioning the velocity slice using a phase error threshold of 25% (26 windows created)



(c) Slice (the red line) adapted from the Marmousi velocity (colored) model



(d) Marmousi reference velocity section created by the adaptive windowing algorithm using a phase error threshold of 35%



(e) Marmousi reference velocity section created by the adaptive windowing algorithm using a phase error threshold of 25%

Figure 1: Adaptively windowing the Marmousi velocity section for the Gabor depth imaging

algorithm works, we give two examples using different thresholds (25% and 35%) and the windows created by the adaptive windowing algorithm with phase error criterion on a velocity slice (the red line across Figure 1 (c) in depth around 1500 m) from the Marmousi velocity section (See Figure 1 (a) and (b)).



(a) The Marmousi velocity model



(b) Gabor imaging result using a phase error threshold of 35%



(c) Gabor imaging result using a phase error threshold of 25%

Figure 2: Imaging Marmousi velocity model using the Gabor transform

To see how the adaptive windowing algorithm works in the whole Marmousi velocity section, in Figure 1 (d) and (e), we show the reference velocity sections resolved by the adaptive windowing algorithm in the Gabor imaging process with two different phase error thresholds. In Figure 1 (d), a relative phase error threshold of 35% has been used. We can see the reference velocity section is very coarse compared to the exact Marmousi velocity model (see Figure1 (c)). Though the split-step Fourier corrections (Stoffa et al., 1990) can be applied to minimize phase errors in the Gabor wavefield extrapolation, we are not able to make calculations accurate enough as we carry on spatial phase shift in depths if the difference between exact and reference velocities is too large. When we use a relative phase error threshold of 25%, we find that the reference velocity section seen by the phase error windowing algorithm fits the exact velocity model quite well (see Figure 1 (e) and (c)). As a result, we may be able to imaging the Marmousi velocity structures with higher accuracy. However, as we can see from Figure 1 (a) and (b), when the phase error threshold is set smaller (meaning more accurate spatial phase shift), the number of windows used in the Gabor depth imaging increases. More windows results in lower imaging efficiency due to more Fourier transforms used in the Gabor wavefield extrapolation. With limitation from computing facilities, we have to trade off between imaging efficiency and accuracy. Our Gabor imaging algorithm gives us the freedom of choosing between them. In this paper, we use a threshold of 25% as a fairly good phase error threshold to create an acurate imaging result of the Marmousi velocity model.

To show how these phase error thresholds influence the imaging results, we give two imaging examples of the Marmousi velocity model shown in Figure 2 (b) and (c); the accurate Marmousi velocity is shown in Figure 2 (a) for comparison.

The results shown in Figure 2 (b) and (c) verify the analysis about the imaging accuracy given before this section. We can see that in Figure 2 (b), the Marmousi velocity section has been adequately imaged (though roughly). In the upper part, we have a more clear image than in the lower part of the Marmousi model. However, from the middle to the bottom, some parts of the velocity section are not properly imaged. For example, the region of the anticline enclosing the target reservoir (in depth about 2500 m with distance from 5800 m to 7500 m) is not clearly imaged in Figure 2 (b). Figure 2 (c) shows us a much better imaging result, we can see both the shallow and deep parts of the Marmousi velocity section are very clearly imaged. Especially, the target reservoir area is imaged in a great details compared to the one in Figure 2 (b).

The running time for the depth imaging result in Figure 2 (b) is about 34 hours on a PC, while the imaging result in Figure 2 (c) is obtained on the same PC using about 104 hours. Even this running time (104 hours) may not be appealing, we still have opportunities to improve the efficiency. Nevertheless, we have a new imaging method that can be used to approximate the 'exact' GPSPI imaging approach, which takes much longer computation time than this one does (examples not shown here).

Conclusions

The Gabor extrapolator is a very promising imaging tool for seismic depth migration. The Gabor imaging results have shown that we can get accurate depth images for complicated velocity structures such as the Marmousi velocity model, which is a solid basis to carry out further research and exploration of the new imaging algorithm. The Gabor extrapolator can be used to image velocity structures as accurately as we may require. Computation (imaging) speed has been highly improved when the adaptive windowing algorithm is integrated into the Gabor wavefield extrapolation.

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