Long wavelength solutions to the surface consistent equations

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SUMMARY

The surface consistent equations always have one or more singular values, depending on the configuration of the seismic survey. These singular values slow convergence, add uncertainty, and make it difficult to resolve the long wavelengths in the solution. Multigrid methods possess a greater ability to resolve long wavelength terms than Gauss-Seidel methods that are currently in use. These improved solutions are calculated at little or no additional computational cost. While total convergence is not guaranteed, multigrid methods seem to be able to universally improve the quality of surface consistent decomposition. There are some limitations we reach in solving the surface consistent equations. An attempt is made to further justify our previous conclusion (Millar and Bancroft, 2004) that some of the long wavelength drift that can plague Gauss-Seidel solutions is theoretically avoidable. In more or less the same amount of computer time using multigrid techniques we are getting more accurate synthetic solutions. We see how the quality of our solution depends on the geometry of the survey, and the role singular values play in the solution. Lastly, we explore the challenges of including of a time variant term in the equations as well.

INTRODUCTION

The surface consistent equations are a standard tool for any land seismic processing geophysicist. The underlying assumption behind the method is that strong effects on the seismic signal can be attributed to the near-surface conditions and coupling quality of a particular source or receiver. Long wavelength components of solutions to the surface consistent equations are notoriously difficult to solve for. They are considered a likely source of problems where near surface conditions change quickly and extend over large distances, such as near lakes, or sand dunes.

A large portion of the conventional commercial land processing job flow relies on the surface consistent equations (Taner and Koehler, 1981). Decomposing statics into components associated physically with a particular surface location improves our confidence that we are not arbitrarily changing the time structure of an event when applying static shifts. For AVO work, correcting for near surface absorption and other amplitude effects are both difficult and absolutely critical to the study. Surface consistent deconvolution is a robust and effective method to reject noise and whiten the spectrum of the data.

Derived in their appendix, Taner et al. (1974) predicts solutions will be indeterminate by a cubic polynomial. Our recent work on this subject suggest that while some of that error is irreducible, we can almost always resolve a very large component of the long wavelengths using iterative methods (Millar and Bancroft, 2004).

The multigrid method will often converge to within a specified error tolerance an order of magnitude (or more) faster than Gauss-Seidel methods do. Typically, solutions of no more than 10 iterations of Gauss-Seidel relaxation are performed during a commercial surface consistent decomposition.

THE EQUATIONS

We consider the problem of de-coupling separate source and receiver consistent components of a signal from a seismic trace. This could be an amplitude, a static as calculated by an auto-correlation with a model stack, or any number of features or attribute.

We assume a correction for each trace t_{ij} can be expressed as a contribution from the (both unknown) i^{th} source, S_i, and the j^{th} receiver,

$$t_{ij} = S_i + R_j. \tag{1}$$

In the case of deconvolution and amplitudes, the effect is a product,

$$t_{ij} = S_i \times R_j. \tag{2}$$

so we have to use the log of the spectrum to transform the multiplication into an addition .

$$\log t_{ij} = \log S_i + \log R_j. \tag{3}$$

Each trace contributes an equation to the linear system, which we express as a matrix operation of the form

$$\mathbf{As} = \mathbf{t}. \tag{4}$$

Here, $\mathbf{A}$ is a matrix of coefficients, $\mathbf{t}$ is a vector with all of the calculated *trace values* (static shift, amplitude etc.), and $\mathbf{s}$ is an unknown vector of the separated source and receiver consistent component of $\mathbf{t}$.

The form of $\mathbf{A}$ is a sparse rectangular matrix whose coefficients are determined by the geometry of the seismic survey. We assign a column of $\mathbf{A}$ to each unique shot, and one to each unique receiver. It has as many rows as traces, which is usually much greater than the number of columns. Its form is demonstrated best by partitioning it,

$$\mathbf{As} = [\mathbf{A}_s|\mathbf{A}_r][\frac{\mathbf{s}_s}{\mathbf{s}_r}]. \tag{5}$$

The unknowns $\mathbf{s}_s$ and $\mathbf{s}_r$ are vectors of source and receiver unknown values corresponding to equation 1. The n^{th} trace has a value of $\mathbf{t}_n$ associated with it. The n^{th} row of $\mathbf{A}_s$ is empty except for 1 in the column corresponding to the shot, and $\mathbf{A}_r$ contains only a 1 in the appropriate receiver column.

The system of equations that results from this problem is over-determined (more equations than unknowns), requiring a least squares solution.

$$\mathbf{A}^T\mathbf{A} = \mathbf{A}^T\mathbf{b}. \tag{6}$$

The classification of $\mathbf{A}^T\mathbf{A}$ would be that of a symmetric, positive indefinite matrix. It is not strictly diagonally dominant, as the sum of the off diagonals is equal to the diagonal term in each row. Trottenberg et al. (2001) implies that a necessary condition for multigrid is a property of being an "M" matrix. One of the conditions of "M", is that it be non-singular, which is not true in this case, $\mathbf{A}^T\mathbf{A}$ is not "M". We can't guarantee multigrid will converge (Wesseling, 1992) .

Singularities and geometry

Questions arise concerning the roles the singular values play, where they come from, and how they manifest in the final solution we arrive at. For the case in the previous section, involving only source and receiver terms, the matrix $\mathbf{A}^T\mathbf{A}$ is rank deficient by 1. Often it is desirable to decompose the signal into more channels, including effects consistent with the offset bin and mid-point bin. Introducing more channels that we decompose over increases the number of unknowns, but does not necessarily increase the number of independent equations that we have in the system. Cary and Lorentz (1993) describes four component surface consistent deconvolution.

Exact relationships involving the number of singular values and geometry are not clear, but a few general rules are apparent. Including mid-points can introduce more singular values into the system if there are areas of low fold. Including an offset consistent channel can cause many more singular values, as most shots will have very similar offset distributions. Rolling the data on at the edge increases the the number of unknowns in the offset term, but the rank deficiency remains the same. This seems to stabilize the solution slightly, suggesting that it may be some function of the rank and the number of singular values that controls the error.

For highly singular matrices the Gauss-Seidel and multigrid methods seem to always converge to a very similar solution, with a similar long wavelength error. This lends us to believe that there is a predictable process behind the error, and it is not random. It does have the appearance of a polynomial. This new evidence may help support the findings of Taner et al. (1974).

The picture that is emerging is that an increasingly singular operator decreases our ability to resolve long wavelengths in the surface consistent equations. There may be an acquisition footprint, or a resolution limit that exists in the geometry.

We interpret the number of singular values as measuring a level of interdependence of the equations. It makes sense that variables that are more connected to each other would resist the Gauss-Seidel relaxation, as it would imply a broader operator, with less frequency content. It could be thought of as reducing the diagonal dominance, or "M"ness of the system, to which the success of Gauss-Seidel and multigrid are tightly connected (Press et al., 1992)

The effect of introducing singular values to the equations is displayed in Figures 1 and 2. Multigrid and Gauss-Seidel methods were used to solve identical problems. The two methods were allowed to run for approximately the same time, and the results compared.

The synthetic survey we consider has 60 separate shots and 61 live geophones. Each successive shot and its live recording patch advances 4 stations. This gives 236 separate receivers, and 472 midpoints. The receivers hanging over the outside edges of the survey are chopped off. The 4 channel decomposition yields 829 unknowns, with a rank of 810, or 19 singular values. The 2 channel decomposition has 296 unknowns and is rank deficient by 1.

The trace value b is a sum of a source, receiver, offset and midpoint consistent effect (eg. static), so it fits our surface consistent model perfectly. The source and receiver effects are random between 0 and 1, and half way through the survey a receiver consistent jump of 8 is included. Two thirds through the survey, a cmp consistent jump of 3 is made, and there is a small linear drift with offset. These effects were chosen to simulate a receiver consistent effect, that would possibly mask a nearby cmp consistent event. A variety of models were tested in Millar and Bancroft (2005).

The 2 term surface consistent solution converges very quickly to a result in figure 1. The multigrid solution has significantly less error than the Gauss-Seidel solution. The Gauss-Seidel solution continues to converge to the full solution, but requires increasingly more computer time. Left of the thick line, the shot consistent solution is plotted, to the right the receiver solution is plotted.

The long wavelength components in the 4 term reduction are more difficult to reduce, due to the 18 additional singularities. These singularities greatly increase the difficulty in producing an accurate solution. As prescribed by Cary and Lorentz (1993), we correct the mid-point co-ordinate first, and force the cmp channel to be smooth. We are converging to a solution with some long wavelength error. Even so, multigrid again provides a more accurate answer to what was available with Gauss-Seidel, at no additional computer time.

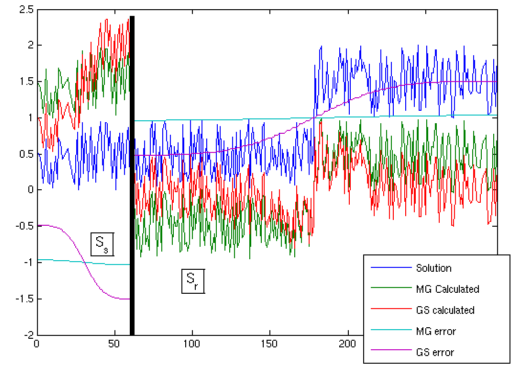

Figure 1: The calculated source and receiver solutions for the 2 term surface consistent reduction. This equation is only rank deficient by 1, so the long wavelengths are relatively easy to resolve.

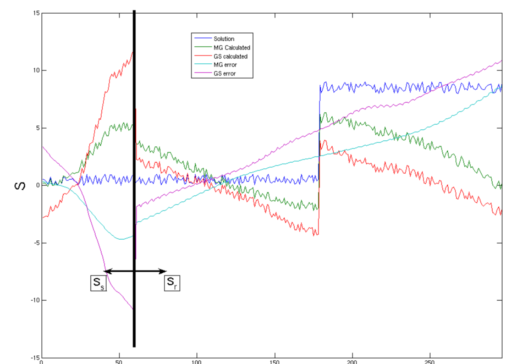

Figure 2: The calculated source and receiver solutions for the 4 term surface consistent reduction. The long wavelength error is more persistent due to multiple singular values in the offset and mid-point terms.

CONVERGENCE TIME

Suppose we were to let a Gauss-Seidel solution run until it met the same strict convergence criteria we use for multigrid. Figure 3 shows the rate of convergence of a multigrid method compared to a Gauss-Seidel method. The times are for a synthetic survey with 200 shots and 240 live geophones, with shots every 4th station. A 2 channel reduction was performed, due to its rank deficiency of only 1. This allows us to ensure out iterative methods were converging to as stable a solution as possible. As was mentioned before, the multigrid and Gauss-Seidel final solutions were very similar, however, the *defect*, δ (a measure of residual error) is plotted versus computer time in figure 3. δ is calculated as follows,

$$\delta = \mathbf{A}^T \mathbf{A} \bar{\mathbf{s}} - \mathbf{A}^T \mathbf{t}, \quad (7)$$

where $\bar{\mathbf{s}}$ is the most recent estimate of the solution. δ is a vector with as many entries as number of unknowns.

Both the multigrid and Gauss-Seidel method do appear to be converging to the full solution as predicted. For the same error tolerance the Gauss-Seidel takes an order of magnitude longer.

In order to directly compare the error on the coarse grids to the errors on the fine grid, a factor was included to normalise the error with respect to the number of grid points. The defect was multiplied by 2^{m+1}, where m is the number of grids below the finest grid spacing (ie. 3 restrictions, multiply by 16). A more appropriate way to compare errors on different grid spacing is left for future work.

Where the curve settles down into a steadily decreasing function in Figure 3, represents the time overhead associated with calculation of a multigrid solution. A result at full resolution is not obtained until after about 4 iterations of the Gauss-Seidel are performed, but the total estimated error is far lower right from the start.

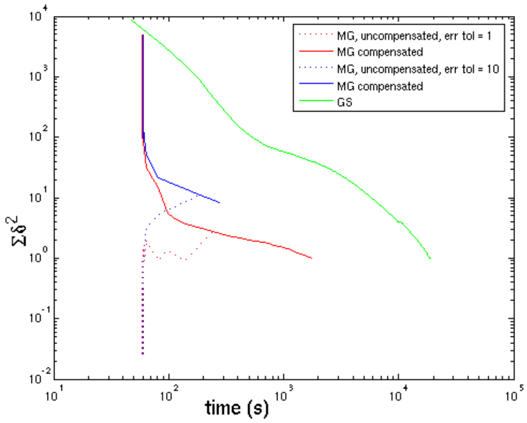

Figure 3: The y axis is sum of each value in the defect vector, squared, or $\Sigma\delta^2$, as calculated by equation 7, versus calculation time. A multigrid method with an error tolerance set to 1 and 10, and a Gauss-Seidel method are plotted.

ADDING A TIME VARIANT TERM

Adding a time variant term greatly increases the computational difficulty in the surface consistent equations. Because each row corresponding to a time channel in $\mathbf{A}^T\mathbf{A}$ contains almost all ones (nowhere near sparse), the diagonal dominance and "M" conditions are further violated. For a 60 shot survey, with 253 total receivers, and 10 time windows, we get 849 unknowns, and 825 as the rank, giving a rank deficiency of 24. Without the time window, the rank deficiency is 20, with 790 of 810 unknowns being unique.

With an overly singular matrix and a reasonable error tolerance, both the multigrid and Gauss-Seidel methods will approach an error minimum, then diverge with high amplitude low frequency instabilities. To bypass this we will need to somehow manipulate the equations in a way that preserves the original system, or add independent equations based on other data or assumptions.

CONCLUSIONS

In a synthetic setting, the multigrid method allows us to get an estimate of the long wavelength component of the solution to the surface consistent equations. While Gauss-Seidel methods do converge to the solution eventually, not in few enough iterations to be practical. While confirming this result by applying it to field data has not yet yielded significant improvement on a stack section, the numerical results are certainly encouraging.

Adding the time variant term greatly increases the difficulty of the problem. So far no method that converges to an acceptable solution

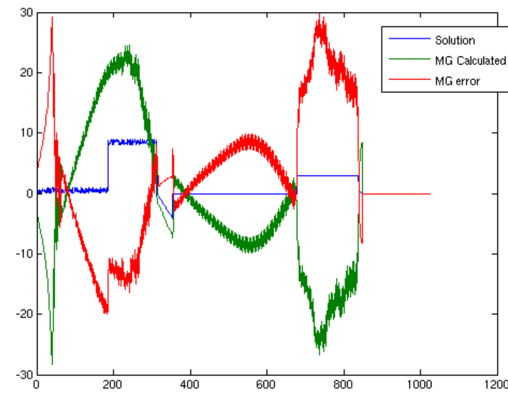

Figure 4: The multigrid method is unable to produce a reasonable long wavelength solution when a term is included to model a time variant signal. The solution continues to diverge.

has been found. While the number of unknowns does not increase by much, the operator simply is unable to remove some of the components of the error. In some settings the solution will approach the correct solution then begin to diverge. Strategies to bypass this will involve somehow reducing the number of singular values in the equations. This may be done by assigning values to a particular unknown (for instance, forcing a shot static to be equal to its up-hole time). Another option is to stagger the time windows, adding some variability from trace to trace.

While multigrid methods clearly improve solutions to the surface consistent equations in synthetic models, the end usefulness of the multigrid solution rests in whether the errors in the surface consistent model, and noise in the data dominate over the added accuracy we achieve over the Gauss-Seidel solution. Even small gains are beneficial, as the calculation time is at worst about the same.

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