Surface-consistent Gabor deconvolution

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Summary

The Gabor deconvolution aims at the simultaneous elimination of the wavelet and the attenuation effects, both endowed with the minimum phase character. In the absence of an accurate estimation of Q, the phase component of the Gabor deconvolution operator is designed with the help of the Hilbert transform. In the presence of noise the computation of the phase by the Hilbert transform may introduce serious distortions in the Gabor deconvolved data. A more robust implementation of the Gabor deconvolution method, with respect to the presence of either random or coherent noise, can be obtained through the use of the surface consistency assumption. A surface-consistent nonstationary convolutional model is formulated as an essential element for a surface-consistent Gabor deconvolution method.

Introduction

A generally accepted model for the seismic trace is to consider it as a convolution of the earth seismic response with a wavelet. In turn, this wavelet can be regarded as the convolution of several effects: source signature, recording filter, earth filter, surface reflections and geophone response (e.g. Robinson, 1980). Deconvolution is the process of removing the wavelet from the seismic trace to estimate the earth seismic response, which is composed of primaries and multiple reflections. The application of deconvolution to seismic processing relies on the fulfillment of a set of assumptions on which the convolutional model is based: stationarity, minimum phase wavelet, and white additive noise.

In presence of inelastic attenuation, the stationary assumption is not valid. A nonstationary convolutional model (e. g. Margrave et al., 2005) is formulated using the constant-Q theory and the mathematical operation called nonstationary deconvolution (Margrave, 1998). The Gabor deconvolution method is a nonstationary extension of the Wiener deconvolution method, based on the nonstationary convolutional model.

Minimum phase, the second assumption of the convolutional model, is an essential concept in Gabor deconvolution. Besides the minimum-phase character associated with the source wavelet generated by an explosive source, the constant-Q theory gives strong arguments to consider that the attenuation earth filter is also endowed with a minimum-phase character (e.g. Futterman, 1962).

In signal theory a minimum phase wavelet is also defined as a causal stable wavelet with a causal stable inverse, in which the term ‘stable’ is associated with a precise physical meaning: finite energy. For this special kind of wavelets, the mathematical theory gives an extraordinary, interesting and useful result: in a minimum phase wavelet its phase spectrum is equal to the Hilbert transform of the logarithm of its amplitude spectrum.

Although the constant Q theory for attenuation finds an analytical expression to compute the Hilbert transform of the attenuation function (e.g. Aki and Richards, 2001), the explicit dependence on Q of this analytical expression makes it inappropriate for the cases when either a poor estimation of Q, or no estimation at all is available. In these cases the estimation of the phase through the Hilbert transform of the logarithm of the amplitude spectrum seems to be a more suitable alternative.

One of the drawbacks of using the digital implementation of the Hilbert transform to compute the phase is its high sensitivity to noise. This problem can be overcome by recurring to the surface consistency assumption, thus taking advantage of the redundancy of the seismic data to attenuate distorting effects in the design of the deconvolution operators.

The performance of a minimum phase, surface-consistent Gabor deconvolution method in the presence of high levels of both coherent and incoherent noise is examined in this paper. A synthetic seismic dataset, courtesy of Geo-X, is used to illustrate the problem and its surface-consistent solution.

The surface-consistent convolutional model

In this model the earth’s effect on a seismogram are classified into those caused by the near surface and those caused by the subsurface. In practice the near surface effects are associated with the source and the receiver coordinates, whereas subsurface effects are those which vary as a function of midpoint and offset.

The nonstationary convolutional model for the seismic trace made up of primaries (e.g. Margrave et al., 2005), can be formulated in the mixed time-frequency domain as

$$\sigma(\omega) = w_s(\omega)\left[\alpha(\omega,\tau) \otimes \rho(\tau)\right]w_r(\omega) \qquad (1)$$

where ω is frequency, $\sigma(\omega)$ is the Fourier transform of the seismic trace, $w_s(\omega)$ and $w_r(\omega)$ are the Fourier transform of the source and receiver wavelets respectively (including the near surface attenuation effects), $\alpha\ (\omega,\tau)$ represents the subsurface attenuation effects and $\rho\ (\tau)$ is the reflectivity.

The symbol $\otimes$ stands for the nonstationary convolution operation between $\alpha(\omega,\tau)$ and $\rho(\tau)$, as defined by Margrave (1998). The expression in brackets can also be considered as an anti-standard pseudo-differential operator (e.g. Kohn and Nirenberg, 1965).

A first approach to a surface-consistent nonstationary convolutional extension of equation (1) can be obtained by expressing the dependence of each term on the source *(s)*, receiver *(r)* or midpoint *(x)* coordinates as

$$\sigma(\omega) = w_s(\omega,s)\left[\alpha(\omega,\tau,x)\otimes\rho(\tau,x)\right]w_r(\omega,r)\,. \qquad (2)$$

A first approach to a surface-consistent Gabor deconvolution algorithm

When the single-channel Gabor deconvolution method (e.g. Margrave et al., 2005) is applied to the $\sigma_{ijk}(t)$ trace, estimations of the amplitude component of the i^{th} midpoint subsurface attenuation operator, $\alpha(\omega,\tau,x_i)$, the j^{th} source wavelet, $w_s(\omega,s_j)$, and the k^{th} receiver wavelet, $w_r(\omega,r_k)$, are obtained. The amplitude component of the surface-consistent Gabor deconvolution operator, $\left|\theta_{ijk}(\omega,\tau)\right|$, for the trace $\sigma_{ijk}(t)$ is given by

$$\left|\theta_{ijk}(\omega,\tau)\right| = A_i * \left(w_s\right)_j .* \left(w_r\right)_k\,, \qquad (3)$$

where (*) denotes matrix multiplication, (.*) element by element matrix multiplication, and the matrix A_i and the vectors $(w_s)_j$ and $(w_r)_k$ are defined by,

$$A_i = \frac{\sum_{l=1}^{L_i}\left|\alpha(\omega,\tau,x_l\right|_i}{L_i}\,, \qquad (3a)$$

$$\left(w_s\right)_j = \frac{\sum_{m=1}^{M_j}\left|w_s(\omega,s_m)\right|_j}{M_j}\,, \qquad (3b)$$

$$\left(w_r\right)_k = \frac{\sum_{n=1}^{N_k}\left|w_r(\omega,r_n)\right|_k}{N_k}\,, \qquad (3c)$$

where L_i, M_j and N_k are the CMP, source and receiver fold respectively.

As it is assumed that $\theta_{ijk}(\omega,\tau)$ is a minimum-phase function, its phase component is estimated from its amplitude spectrum using the Hilbert transform as

$$\varphi_{ijk}(\omega,\tau) = \int_B \frac{\ln\left|\theta_{ijk}(\omega',\tau)\right|}{\omega-\omega'}\,d\omega'\,, \qquad (4)$$

where B denotes the available spectral band. It is not hard to see why the computation of phase through equation (4) could find troubles in the presence of noise. For any frequency, the phase is found as an integral over all the frequencies, thus the presence of noise at a particular frequency will affect the phase at any other frequency.

Finally the Gabor spectrum of the reflectivity, $G\rho_{ijk}(\omega,\tau)_{est}$ is estimated in the Gabor domain as

$$Gr_{ijk}(\omega,\tau)_{est} = \frac{G\sigma_{ijk}(\omega,\tau)}{\theta_{ijk}(\omega,\tau)}\,, \qquad (5)$$

where $G\sigma_{ijk}(\omega,\tau)$ is the Gabor transform of the seismic trace.

Examples

The synthetic dataset used in the shown examples belongs to Geo-X and is similar to the data used in the tests shown in Perz et al. (2005), the difference is the added noise. It was generated using the reflectivity series from a real well log, applying forward NMO and forward Q filtering with Q = 40, and then convolving each trace with an additional "surface-consistent", minimum phase wavelet. Each wavelet comprises two terms, a minimum phase source term and a minimum phase receiver term. Finally groundroll and random noise, with strength varying from trace to trace in such a way as to mimic acquisition in a windy day, was added to each trace. Figure 1 shows one of the 80 raw shots of the dataset; figures 2 and 3 show a stack of the raw data.

After the application of minimum phase, single-channel Gabor deconvolution to the raw shots (Figure 4), the computation of the phase through equation (4) is dramatically distorted. The stack of the deconvolved data is shown in Figure 5, clearly showing how the original flat reflectors between 1300 and 1600 ms have been transformed into fake complex structures.

The application of the minimum phase, surface-consistent Gabor deconvolution to the raw shots is much less sensitive to the presence of noise as can be seen in the stack of the deconvolved shots shown in Figure 7. The reflectors between 1300 and 1600 ms do not show the fake complexity introduced by the single-channel Gabor deconvolution.

Is worth to add that this synthetic dataset is a rough test for the method, considering that Q=40 for two-seconds-long seismic traces represents a high level of attenuation. Moreover, the signal to noise ratio of the raw shots is rather low, as can be observed in Figure 2. In practice, part of the noise can be removed safely by careful filtering procedures before the deconvolution is applied. In this way, the example shown here can be considered as an extreme situation.

Figure 8 shows a comparison of a portion of the stacks for the raw data, the surface-consistent Gabor deconvolved data, and the ideal output (the original clean non attenuated data). It can be observed that a positive drift remains after the deconvolution is applied; the removal of this drift is not addressed in this paper. A discussion of the drift removal in Gabor deconvolution is presented in Montana and Margrave (2005).

Conclusions

Estimation of the phase using the digital Hilbert transform is highly sensitive to the presence of noise in the seismic data. The inclusion of the surface-consistent assumption in the Gabor deconvolution method allows extending it to a more robust surface-consistent version of the method.

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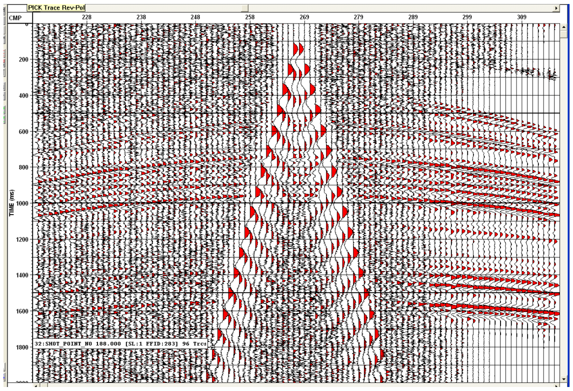

Figure 1. Raw shot of the synthetic dataset used for testing the method.

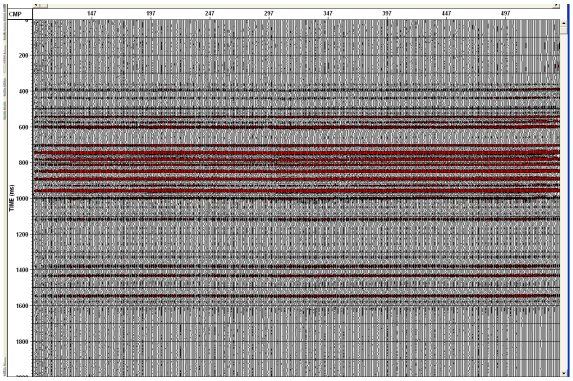

Figure 2. Stack of the raw synthetic data. The length of the traces is 2 sec, but the zone between 800 and 1600 ms will be shown in the next figures.

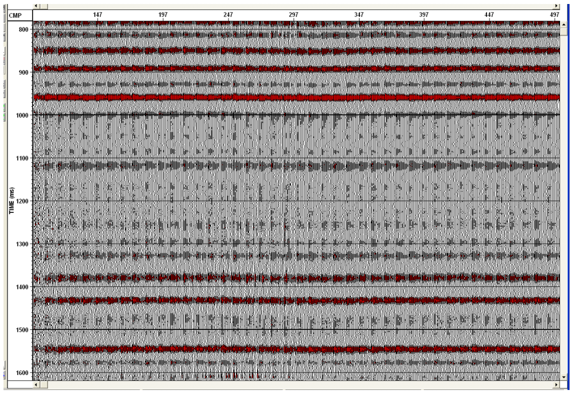

Figure 3. Same as in figure 2, but only the area of interest between 800 and 1600 ms is shown.

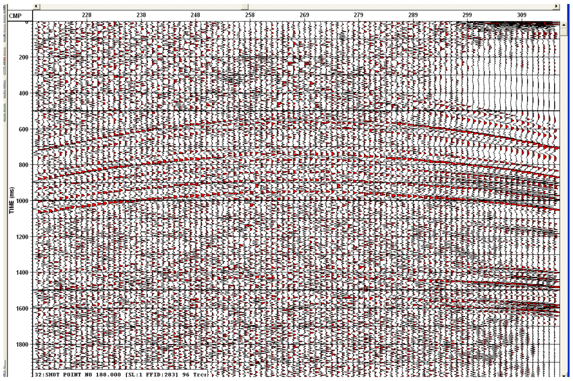

Figure 4. Same shot as in Figure 1, after minimum phase, single-channel Gabor deconvolution. The maximum frequency has been limited to 100 Hz.

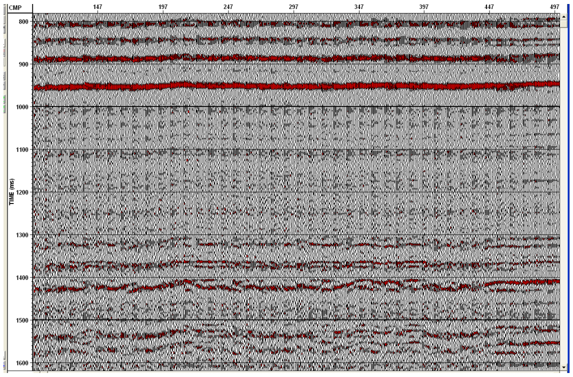

Figure 5. Stack after minimum phase, single-channel Gabor deconvolution was applied to the pre-stack data. The maximum frequency has been limited to 100 Hz.

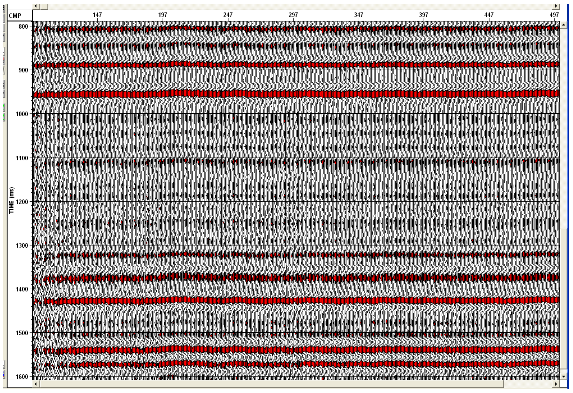

Figure 6. Stack after minimum phase, surface-consistent Gabor deconvolution. The maximum frequency has been limited to 100 Hz.

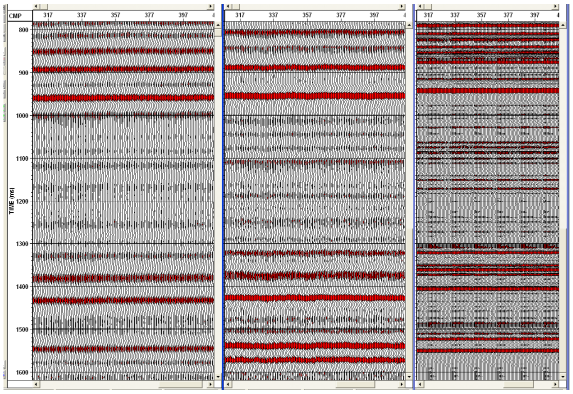

Figure 7. A comparison among the stack of the raw data (left), the stack after minimum phase, surface-consistent Gabor deconvolution (center) and the ideal output (right). The maximum frequency has been limited to 100 Hz.