Fundamental parameters of spherical-wave reflection coefficients

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Summary

Spherical-wave reflection coefficients for a two-layer system depend on more parameters than do their corresponding plane-wave analogues. The additional parameters to be specified are depth, overburden velocity, and any parameters required to define the wavelet. For a Rayleigh wavelet it has been shown analytically that the additional parameters can be reduced to a set of two. For other wavelets numerical investigations can be used to explore the possibility of similar simplification. Ricker wavelet reflection coefficients are shown to depend on only one additional parameter, and the Ormsby wavelet on two.

Introduction

A plane-wave reflection coefficient (PWRC) is a solution to the Zoeppritz equations and is usually described as depending on six earth parameters and a direction parameter (Aki & Richards, 1980). A common choice consists of the following:

α_1 : P-wave velocity of upper layer
α_2 : P-wave velocity of lower layer
β_1 : S-wave velocity of upper layer
β_2 : S-wave velocity of lower layer
ρ_1 : density of upper layer
ρ_2 : density of lower layer
θ_i : angle of incidence ($= \sin^{-1}(\alpha_1 p)$, where p is the ray parameter)

It can be shown however that the six earth parameters can be replaced by three velocity ratios and a density ratio. For amplitude-versus-offset (AVO) studies, a convenient set are the following:

$\Delta\alpha/\alpha = (\alpha_2 - \alpha_1) / [(\alpha_1 + \alpha_2) / 2]$
$\Delta\beta/\beta = (\beta_2 - \beta_1) / [(\beta_1 + \beta_2) / 2]$
$\Delta\rho/\rho = (\rho_2 - \rho_1) / [(\rho_1 + \rho_2) / 2]$
$\beta/\alpha = (\beta_1 + \beta_2) / (\alpha_1 + \alpha_2)$

In moving to a normalized spherical-wave reflection coefficient (SWRC), in addition to the four ratios above one also requires z (the depth), α_1 (the overburden velocity, defined above), and any wavelet parameters. An Ormsby wavelet for instance would add four frequency parameters, given as f_1/f_2-f_3/f_4.
It is often assumed that spherical-wave effects are only important in the near surface region. However it has been demonstrated that near critical angles an SWRC can differ significantly from a PWRC even at considerable depth (Haase, 2004). This is illustrated in Figure 1. (Here, as in other figures in this abstract, all spherical divergence effects have been normalized out.)

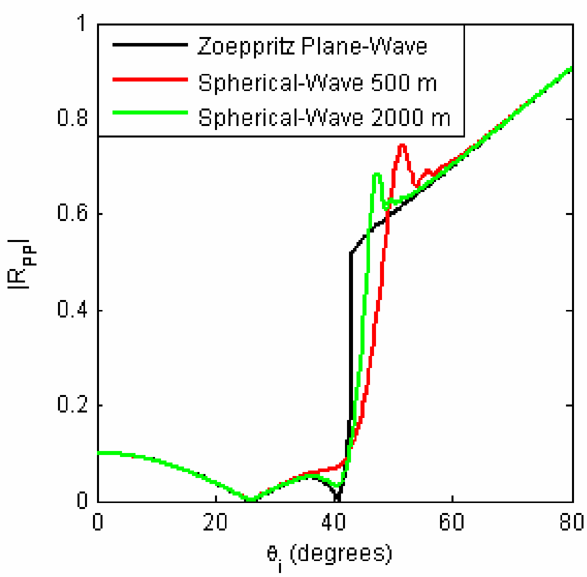

Figure 1: Class I AVO curves of PP reflection coefficients, R_{PP}. The critical incidence angle is at 43°. The black line represents the plane-wave Zoeppritz curve, and the colored lines represent normalized spherical-wave curves with an Ormsby wavelet. Elastic and wavelet parameters are given in Haase (2004). R_{PP} is the plane wave reflection coefficient and θ_i is the angle of incidence.

AVO inversion techniques have recently been extended to the supercritical regime (Downton and Ursenbach, 2005). This was carried out with plane-wave reflection coefficients, and in view of Figure 1, it is of considerable interest to develop methods which simplify the calculation of spherical-wave reflection curves.
It was previously shown that employing a wavelet of a particular exponential form can simplify calculation of the SWRC by permitting an analytic integration over frequency (Ursenbach, Haase and Downton, 2005). This wavelet has been discussed in the literature and is referred to as the Rayleigh wavelet (Hubral and Tygel, 1989), whose zero-phase form is written

$$w^{\text{Rayleigh}}(f) = f^n \exp(-nf/f_0), \quad n = 0,1,2,... \qquad (1)$$

where $w^{\text{Rayleigh}}(f)$ reaches its maximum at f_0. The additional parameters for the Rayleigh SWRC thus consist of n, f_0, z and α_1. However, it was shown that the last three of these enter only through the combination $S \equiv \alpha_1/(zf_0)$. Thus only two additional variables, n and S, are required to convert a PWRC into a Rayleigh SWRC.

The Rayleigh wavelet possesses desirable properties, but has not yet been widely adopted in exploration geophysics. The most commonly used wavelets at present are the Ricker and Ormsby wavelets. It is of interest in spherical-wave studies to know if simplified sets of parameters exist for these other common wavelets. This abstract reports on a study to determine such parameters if they exist.

Numerical Studies

In the case of the Rayleigh wavelet, it was shown analytically that the normalized SWRC depends only on n and S (and on the earth parameter ratios). This was accomplished by first integrating analytically over frequency and then dividing by the exact SWRC in the case that $R_{\text{PP}} = 1$. (See Ursenbach, Haase & Downton, 2005, for details.) This yielded an expression for the SWRC as a weighted integral of the PWRC:

$$R_{\text{PP}}^{\text{sph}}(\theta_i) = \int_0^\infty R_{\text{PP}}^{\text{pw}}\left(\theta; \frac{\Delta\alpha}{\alpha}, \frac{\Delta\beta}{\beta}, \frac{\Delta\rho}{\rho}, \frac{\beta}{\alpha}\right) \times W_n(\theta; \theta_i, S) \tan\theta \, d(\sin\theta). \quad (2)$$

This expression can be evaluated efficiently to obtain a spherical-wave reflection coefficient as an integral over a plane-wave curve times the weighting function W_n. It is clear from this expression which parameters are required to determine the SWRC. Because analytical integrations over frequency cannot be carried out for other wavelets, the question of fundamental parameters is explored through numerical computation.

The Ricker wavelet has only a single parameter, f_0, and is given as

$$w^{\text{Ricker}}(f) = f^2 \exp[-(f/f_0)^2]\,. \quad (3)$$

In Figure 2 it is shown to have an SWRC which depends essentially on only one additional parameter, S:

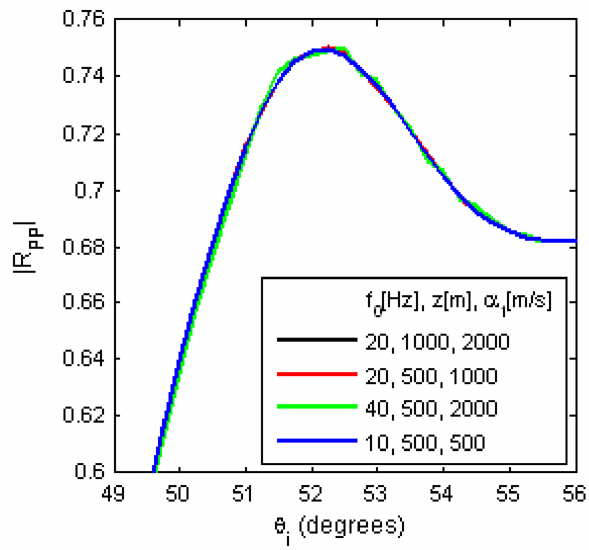

Figure 2: A close-up near the critical angle of four Ricker wavelet SWRC curves which each possess a value of S = 10. The differences are negligible for practical AVO applications. Even lesser differences are present outside of the close-up region.

It is instructive to look at the differences between the lines in Figure 2, and these are shown in Figure 3:

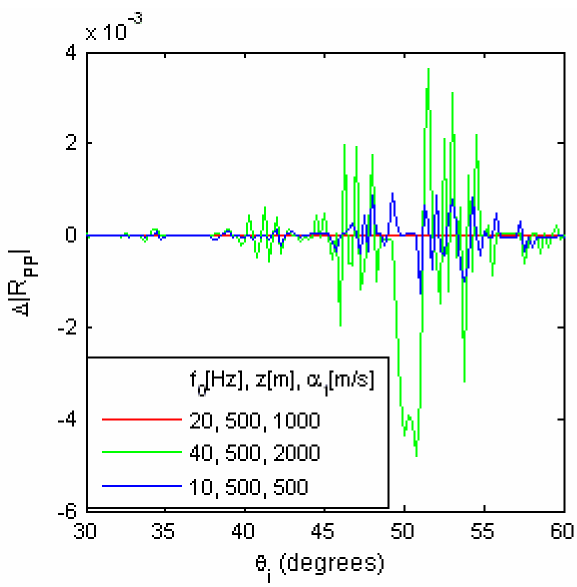

Figure 3: This figure contains the last three lines of Figure 2, but with the first line subtracted from each. The black and red curves of Figure 2, which share the same value of α_1/z, are identical to within numerical precision. The other differences are of the order of 10^{-3}, and appear to be due to numerical noise.

This demonstrates amply that the Ricker wavelet spherical reflection coefficients are, for practical purposes, if not exactly, dependent only on earth parameters and S.

The Ormsby wavelet on the other hand depends on four parameters, but it can be shown numerically that its SWRC depends primarily on only two additional variables. To begin, Figure 4 displays three Ormsby wavelets with the same average upper and lower band edges but with differing taper definitions. One is a 5/15-80/100 wavelet with the usual linear taper, another is the same but with a cosine taper,

$$w^{\text{Ormsby}}(f)=\begin{cases}0, & f<5\text{Hz}\\ \left[\cos\dfrac{(15\text{Hz}-f)\pi}{10\text{Hz}}+1\right]\Big/2, & 5\text{Hz}<f<15\text{Hz}\\ 1, & 15\text{Hz}<f<80\text{Hz}\\ \left[\cos\dfrac{(f-80\text{Hz})\pi}{20\text{Hz}}+1\right]\Big/2, & 80\text{Hz}<f<100\text{Hz}\\ 0, & f>100\text{Hz}\end{cases}$$

and the last is a boxcar taper, as shown in Figure 4:

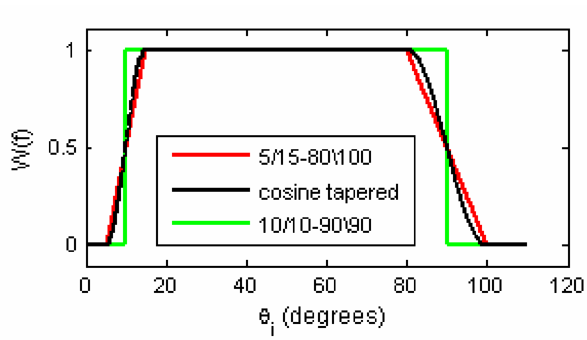

Figure 4: Three Ormsby wavelets with the same average lower and upper band edges (10 Hz and 90 Hz) but with different taper definitions.

In Figure 5 we compare three SWRC curves which result from the wavelets of Figure 4. All are for 500m depth with a 2000 m/s overburden. The three curves are nearly identical everywhere, so a close-up view is given near the critical angle, where differences are the largest.

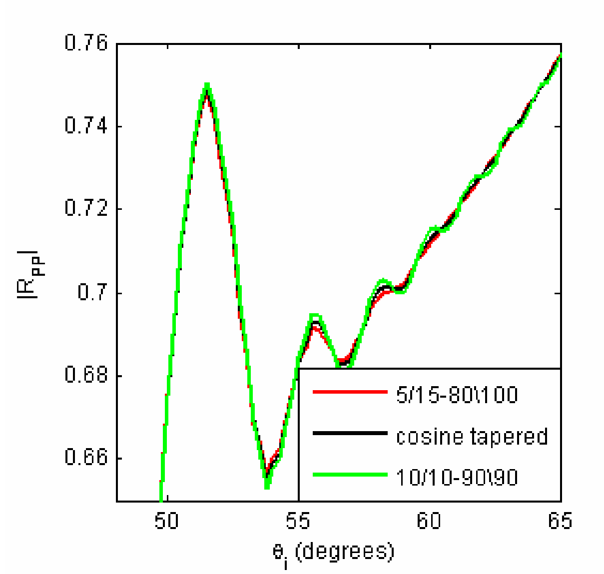

Figure 5: A close-up near the critical angle of three Ormsby wavelet curves which are identical except for their taper definitions. The red curve is for a conventional 5/15-80/100 Ormsby wavelet with linear taper, the black curve is the same but with a cosine taper, and the green line is for a 10/10-90/90 box filter wavelet. The differences are negligible for practical AVO applications. No visible differences are present outside of the close-up region.

This effectively reduces to two the number of degrees of freedom of the Ormsby wavelet that are important here. Experimentation has further revealed that these variables can be defined as $S_a \equiv \alpha_1/(zf_a)$, where f_a is the average frequency of the wavelet, and $B \equiv (f_3+f_4)/(f_1+f_2)$, the ratio of upper and lower band edges. The efficacy of these two quantities in defining the Ormsby SWRC curve is shown in Figure 6. Four curves are shown with differing values of z, α_1 and wavelet parameters, but which all have a common value of S_a. All curves agree over most of their range except near the critical angle. In addition, though, the curves are divided into two pairs of curves each of which is characterized by a value of B. Curves with the same value of B overlap strongly even near the critical angle. Thus the Ormsby SWRC is effectively defined by S_a and B (and the earth parameters).

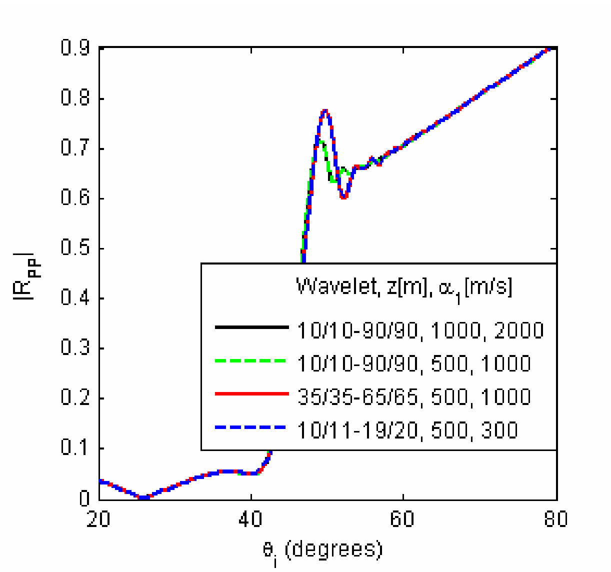

Figure 6: Comparison of four Ormsby wavelet spherical wave reflection coefficients all with S_a = 1/25. All are virtually identical except near the critical angle. However, the two curves with values of B = 9 overlap even near the critical angle, as do the curves with B = 1.86. These two parameters thus effectively define the reflection coefficient curves for Ormsby wavelets.

Conclusions

Identifying fundamental parameters simplifies considerably the study of spherical wave reflection coefficients by reducing the parameter space to more manageable proportions. It is expected that the results obtained here for Ricker and Ormsby wavelets could be readily duplicated for other wavelets of interest as well.

References

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Acknowledgments

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