Estimating an accurate RMS velocity for locating a microseismic event

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Summary

The location and clock-time of a microseismic event can be computed analytically using the Apollonius method that requires the first arrival clock-times at four known receiver locations. The accuracy of the source location is dependent on the accuracy of the velocity. that is assumed to be known and constant. The sensitivity of the velocity is demonstrated for the analytic solution. An initial estimate of the velocity can be improved by using the Apollonius method with the traveltimes of five receivers.

Introduction

The Apollonius method computes an analytic solution for the location of a microseismic source from the clock-times at four arbitrarily located receivers. Given accurate clock-times and velocity, the solution is accurate to the resolution capabilities of the computer. Errors in the measurement of the clock-times, or the velocity, introduce errors in the estimate. This paper addresses the sensitivity of the velocity, and presents a method to update or improve its accuracy.

The sensitivity of the Apollonius method for locating the source locating the source is presented using four receivers and a variable velocity. This method is then used with five receivers to improve the initial estimate of the velocity. The method is iterative, and when using a noise free model, the velocity will converge to 0.1% accuracies in four iterations or to machine accuracy in eight iterations.

The method uses the first arrival clock-times of the P- or S-waves at five receiver locations and assumes the velocity of the medium is constant or applicable to a constant RMS velocity. The method uses five groups of four receivers, each with an analytic solution.

Theory and/or Method

The four unknowns of a microseismic source at a location (x_0, y_0, z_0) and its source clock-time (t_0) can be computed from an analytic solution, Bancroft and Du (2007), that is referred to as the Apollonius method. This method assumes the receiver locations and clock-times are known accurately, and that the velocity is constant, or in a medium that can assume a constant RMS velocity. The sensitivity of the source location, relative to a small change in the velocity, was evaluated using a simple model of four receivers, a known source location, and a known velocity. A clock-time of the source was chosen and added to the traveltimes from the source to the receivers. The receiver clock-times were then used to compute the source location.

The sensitivity of the source location, relative to the accuracy of the velocity, was demonstrated by varying the known velocity in 5% increments over a range from 50% to 150%, and plotting the estimated source locations as illustrated in Figure 1. The geometry in normalized units was:

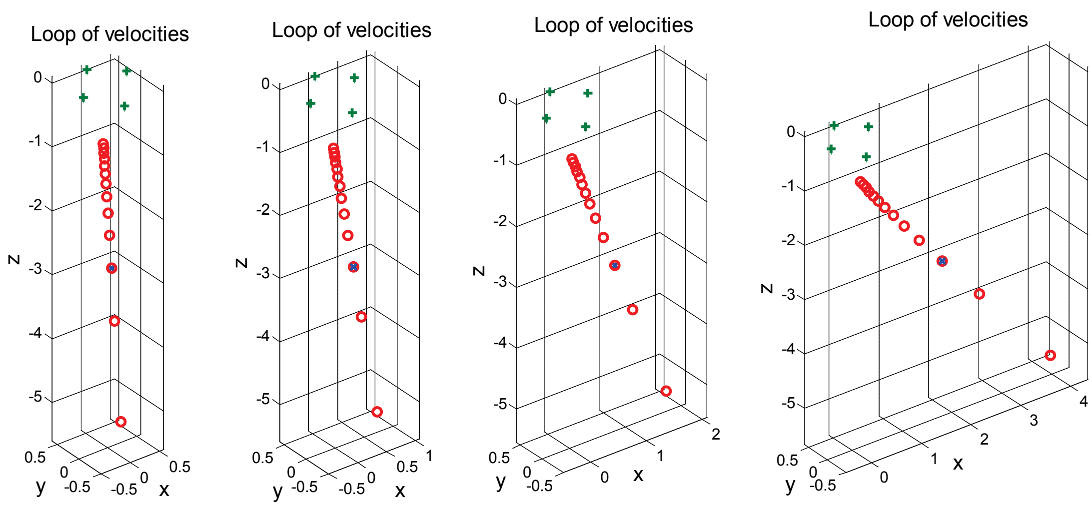
```
x0 = 0.25;  y0 =  0.25; z0 = -3.0;   % Defined location of source

x1 =  0.4;  y1 =  0.10; z1 = +0.10;  % Four given receiver locations
x2 =  0.1;  y2 =  0.60; z2 =  0.00;
x3 = -0.5;  y3 = -0.10; z3 = +0.10;
x4 = -0.1;  y4 = -0.50; z4 =  0.00;
```

This figure contains four images in which the source location was shifted with an increasing displacement in the *x* direction of 0.25, 0.5, 1.0, and 2.0. The *y* component and the depth *z* remained constant.

The results in Figure 1 show results of the velocities varying from 50% to 110%. When the velocities exceed 110%, the depth of the source locations become very large and will distort the display, or may even fail to compute. This instability increases with increased displacement in *x*.

The estimated source locations tend to have a linear direction that increases with depth as the velocity increases. For the geometry in Figure 1, the depth locations for the 95% velocity are -2.4419, -2.4409, -2.4395, and -2.4268, which represents a depth error close to 20%. The depth locations for a 105% velocity produce a depth error approaching 30%.

This method only describes the error in the source location when the velocity varies, and cannot tell which velocity is correct.



a) x = 0.25 b) x = 0.5 c) x = 1.0, d) x = 2.0

Figure 1 The distribution of estimated source locations when the velocity is varied from 50% to 120%. The receiver locations are green "+", the defined source location by blue "*x*", and the estimated source locations by a red "*o*".

The velocity of the medium may not be known accurately, so a method is presented that uses five receivers to estimate to improve the accuracy of the velocity V.

Examples

Consider a model of five receivers with clock-times t_1, t_2, t_3, t_4, and t_5 can be separated into five groups of four receivers with times (t_1, t_2, t_3, t_4), (t_1, t_2, t_3, t_5), (t_1, t_2, t_4, t_5), (t_1, t_3, t_4, t_5)), and (t_2, t_3, t_4, t_5). The clock-times of the five groups are used to compute an analytic estimate of the source location using the initial estimate of the velocity V_0. Each analytic solution provides an independent estimate of the source location and of the source clock times $t_{0,\ 1\text{-}5}$. These five solutions are then averaged to get an improved estimate of the source location ($x_{0\text{-}1}$, $y_{0\text{-}1}$, $z_{0\text{-}1}$) and clock-time ($t_{0\text{-}1}$). Using this new estimate of the source location, traveltimes are computed to the five receivers ($t_{01\text{-}1}$, $t_{02\text{-}1}$, $t_{03\text{-}1}$, $t_{04\text{-}1}$, $t_{05\text{-}1}$), then added to the estimated source clock-time ($t_{0\text{-}1}$), to get an estimate of the clock-times at each receiver ($t_{1\text{-}1}$, $t_{2\text{-}1}$, $t_{3\text{-}1}$, $t_{4\text{-}1}$, $t_{5\text{-}1}$). These new times with the original receiver clock-times provides a correction for the velocity that will be described later.

Initially, the velocities were varied over a range from 0.7 to 1.3 of the actual velocities, and the errors of the raypaths computed. The clock-time of the event was arbitrarily set to minus two, t_0 = -2, and then the parameters estimated for the range of velocities. The parameters for the source location are:

```
Velocity = 1.0
x0 = 0.00, y0 = 0.20, z0 =-3.00 , Clock t0 =-2.00

x1 = 0.40, y1 = 0.10, z1 = 0.10 , Clock t1 = 1.13
x2 = 0.10, y2 = 0.60, z2 = 0.00 , Clock t2 = 1.03
x3 =-0.50, y3 =-0.10, z3 = 0.10 , Clock t3 = 1.15
x4 =-0.10, y4 =-0.50, z4 = 0.00 , Clock t4 = 1.08
x5 = 0.00, y5 = 0.00, z5 = 0.00 , Clock t5 = 1.01
```

Figure 2 displays the estimates of the source clock-times $t_{0,\ 1\text{-}5}$ from each of the analytic solutions when the velocity is varied from 0.7 to 1.3. All solutions pass through the correct solution the t_0 = -2.0, when the velocity is correct at 1.0. Solutions 3 and 5 are smooth continuous lines, however 1, 2, and 5 are not smooth indicating the Apollonius solution is having problems. However, within a velocity range of 0.95, and 1.05, all curves are smooth. For the given geometry, velocities within this range will converge rapidly to the defined value.

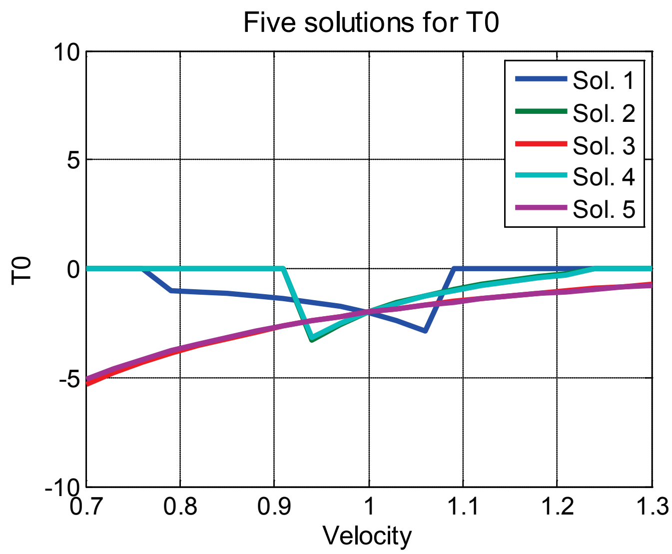


FIG. 2 The estimated time t0 for each of the five different combinations of receivers over the velocity range 0.7 to 1.3.

Plots of the *x*, *y*, and *z* locations are displayed in Figure 3 for the same range of velocities given above. Note that the accuracy of the *x*, and *y* locations have little positioning error when compared to the errors in the depth *z*.

Note the convergence of *x*, *y*, and *z* to the defined values at the correct velocity. Also notice the range of some of the estimates, and how they do not all converge toward the defined values for the whole range of velocities. Once again, this configuration of source and receiver locations will converge simply between the velocity range from 0.95 to 1.05. Other configuration may have a greater or smaller range of convergence.

A method was developed to make use of the error in the traveltimes $t_{0,\ 1\text{-}5}$. A small change in the initial velocity will produce a small change of the t_0 times. If all these times converge, then a simple Newton Raphson iteration will converge to the common value for the velocity. This will occur in the above figure for velocities between 0.95 and 1.05. It is possible to broaden the range of convergence by using an intelligent voting system that chooses, for example, three of the five t_0s that tend to converge.

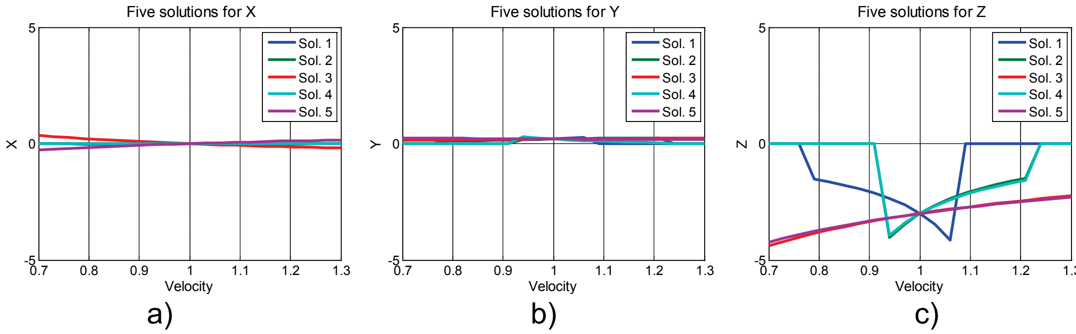


FIG. 3 The estimated locations a) x, b) y, and c) z

The defined parameters and iteration values follow for an iterative test follow. The velocity started at 0.9 and converged to less than 1% percent in 5 iterations, and less than 0.00001% error in8 iterations.

Another test iterated the parameters to the correct solution. The defined parameters and iteration values follow. The velocity started at 0.9 and converged to less than 1% percent in 5 iterations, and less than 0.00001% error in 8 iterations.

```
Defined source values t0 = 2
x0 =  0     y0 =  0.2   z0 = -3
x1 =  0.4   y1 =  0.1   z1 =  0.1
x2 =  0.1   y2 =  0.6   z2 =  0
x3 = -0.5   y3 = -0.1   z3 =  0.1
x4 = -0.1   y4 = -0.5   z4 =  0

Ivel = 1  V = 0.9     %V = -10%              error  = -21.8108%
Ivel = 2  V = 0.909   %V = -9.1%             error  = -20.5277%
Ivel = 3  V = 1.053   %V = 5.299%            error  =  24.1346%
Ivel = 4  V = 0.97518 %V = -2.4819%          error  =  -7.3763%
Ivel = 5  V = 0.9934  %V = -0.6605%          error  =  -2.1419%
Ivel = 6  V = 1.0008  %V = 0.084839%         error  =   0.28565%
Ivel = 7  V = 0.99997 %V = -0.0028635%       error  =  -0.0095983%
Ivel = 8  V = 1       %V = -1.2377e-005%     error  =  -4.1492e-005%
Ivel = 9  V = 1       %V = 1.8059e-009%      error  =   6.0554e-009%
Ivel = 10 V = 1       %V = -4.2188e-013%     error  =   5.7339e-014%
Ivel = 11 V = 1       %V = -4.4409e-013%     error  =   5.7339e-014%
```

Conclusions

The sensitivity of estimating the location of a microseismic event for varying velocities was demonstrated using modelled data. The sensitivity of an analytic solution that used the first arrival times at four arbitrarily located receivers was demonstrated. This analytic solution could not identify the correct velocity when used alone. However, this method was able to find the correct velocity when used with five arbitrarily located receivers by evaluating the estimated source clock time.

Acknowledgements

I wish to thank the sponsors of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES) for supporting this research.

References

Bancroft, J. C., and Du, X., 2007, Traveltime Computations for Locating the Source of Micro Seismic Events and for Forming Gridded Traveltime Maps, EAGE 69th Conference.