Interpolation of aliased seismic data in the curvelet domain

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Summary

We propose a robust interpolation scheme for aliased regularly sampled seismic data that uses the curvelet transform. In a first pass, the curvelet transform is used to compute the curvelet coefficients of the aliased seismic data. The aforementioned coefficients are divided into two groups of scales: alias-free and alias-contaminated scales. The alias-free curvelet coefficients are upscaled to estimate a mask function that is used to constrain the inversion of the alias-contaminated scale coefficients. The mask function is incorporated into the inversion via a minimum norm least squares algorithm that determines the curvelet coefficients of the desired alias free data. Once the alias-free coefficients are determined, the curvelet synthesis operator is used to reconstruct seismograms at new spatial positions. The proposed method can be used to reconstruct both regularly and irregularly sampled seismic data. A synthetic data example is used to illustrate the performance of the proposed curvelet interpolation method.

Introduction

Interpolation and reconstruction of seismic data has become an important topic for the seismic processing community. It is often the case that logistic and economic constraints dictate the spatial sampling of seismic surveys. Wave fields are continuous, in other words, seismic energy reaches the surface of the earth everywhere in our area of study. The process of acquisition records a finite number of spatial samples of the continuous wave field generated by a finite number of sources. The latter leads to a regular or irregular distribution of sources and receivers. Many important techniques for removing coherent noise and imaging the earth interior have stringent sampling requirements which are often not met in real surveys. In order to avoid information losses, the data should be sampled according to the Nyquist criterion. When this criterion is not honored, reconstruction can be used to recover the data to a denser distribution of sources and receivers and mimic a properly sampled survey.

Spitz (1991) introduced beyond-alias seismic trace interpolation methods using prediction filters. These methods operate in the frequency-space (f-x) domain. In both cases, low frequency data components in a regular spatial grid are used to estimate the prediction filters needed to interpolate high frequency data components. An equivalent interpolation method in the frequency-wavenumber (f-k) domain was introduced by Gulunay (2003) and often referred as f-k interpolation. The main contribution of this paper is the introduction of a strategy that utilizes the curvelet transform to interpolate regularly sampled aliased seismic data. It is important to stress that the curvelet transform has been used by Hennenfent and Herrmann (2007), to interpolate seismic data. In their articles, they reported the difficulty of interpolating regularly sampled aliased data with the curvelet transform and therefore, proposed random sampling strategies to circumvent the aliasing problem. This paper, however, proposes a new methodology which successfully eliminates the requirement of randomization to avoid aliasing. We create a mask function from the alias-free curvelet scales (low frequencies) to constrain the interpolation of alias-contaminated scales (high frequencies).

Theory

The curvelet transform is a local and directional decomposition of an image (data) into harmonic scales (Candes and Donoho, 2004). The curvelet transform aims to find the contribution from each point of data in the t-x domain to isolated directional windows in the f-k domain. Using matrix notation, the curvelet transform $\mathbf{C}$ can be represented as follows

$$\mathbf{c} = \mathbf{C}\mathbf{m} \qquad (1)$$

where it is understood that, in our notation, the discrete set of coefficients computed by the curvelet transform are represented via the vector $\mathbf{c}$. Similarly, the t-x discrete data are represented by the vector $\mathbf{m}$ and the transform via the matrix $\mathbf{C}$. One needs to stress, however, that the coefficients and the data are not stored in vector format. Moreover, the curvelet transform is not implemented via an explicit matrix multiplication. The latter is just a notation that permits us to use the simple language of linear algebra to solve our reconstruction problem. The inverse curvelet transform (our synthesis operator) is represented as the adjoint of equation 1

$$\mathbf{m} \approx \mathbf{C}^{H}\mathbf{c} \qquad (2)$$

It is important to notice that the operator $\mathbf{C}^{H}$ is a good approximation to the inverse of $\mathbf{C}$. Due to this property, in mathematical terms, curvelets are said to constitute a tight frame (Candes and Donoho, 2004). Figure 1a shows a synthetic seismic data in the t-x domain. Figures 1b and 1c show the f-k domain and curvelet representations of data in figure 1a, respectively. This is not the traditional way of displaying the coefficients of the curvelet transform. Because the number of coefficients at a given scale and direction is variable, each patch of coefficients for a given scale and direction was scaled to fit a matrix of size 50 × 50.

We now turn our attention to the problem of reconstructing seismic data using the curvelet transform. For this purpose, we denote $\mathbf{m}$ our desired interpolated data in the t-x domain. In addition, the available traces are indicated by $\mathbf{d}$. The available traces and the desired data are connected via a sampling operator $\mathbf{G}$ (Liu and Sacchi, 2004)

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{n}, \qquad (3)$$

where we have also incorporated the term $\mathbf{n}$ to include additive noise. Now, the curvelet adjoint operator can be used to represent the desired data. The alias-free desired data will be represented via

$$\mathbf{m} = \mathbf{C}^{H}\mathbf{W}\mathbf{c}, \qquad (4)$$

where we have introduced a mask function $\mathbf{W}$ that serves to preserve the subset of alias-free curvelet coefficients. A stable solution can be found by minimizing the following cost function

$$J = \left\| \mathbf{d} - \mathbf{G}\mathbf{C}^{H}\mathbf{W}\mathbf{c} \right\|_2^2 + \mu^2 \left\| \mathbf{c} \right\|_2^2. \qquad (5)$$

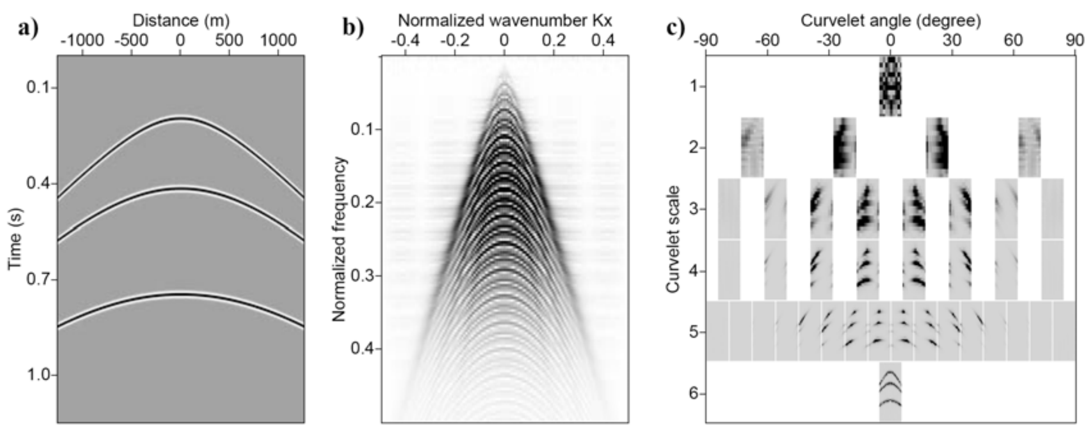

Figure 1: a) Data in the t-x domain. b) Data in the f-k domain. c) Data in the curvelet domain.

We propose a strategy where $\mathbf{W}$ is bootstrapped from the data. The curvelet coefficients are divided into two groups according to their scale: alias-free and alias-contaminated scales. Let us define the indices j and l that indicate scale and angle, respectively. Furthermore, the parameter j_a indicates the index of the maximum alias-free scale. With this definition in mind, the mask function for the alias-free scales can be computed as follows

$$[\mathbf{W}]_{j,l} = \begin{cases} 0 & \left|[\mathbf{c}]_{j,l}\right| < \lambda_j \\ 1 & \left|[\mathbf{c}]_{j,l}\right| \geq \lambda_j \end{cases} \quad \mathbf{if} \quad j \leq j_a, \tag{6}$$

where parameter λ_i is a user defined threshold value for scale j. The mask function for alias-contaminated scales is calculated use the following algorithm

$$[\mathbf{W}]_{j,l} = \Gamma\,[\mathbf{W}]_{j-1,\bar{l}} \quad \mathbf{if} \quad j \leq j_a, \tag{7}$$

where in the expression above Γ denotes the nearest neighbor operator that is needed to upscale the mask function from scale $j-1$ to j and $\bar{l}$ indicates the directionality index closest to l.

Example

In order to examine the performance of our beyond alias curvelet interpolation, we have created a synthetic seismic section with three hyperbolic events (Figure 2a). Figure 2b shows the f-k spectrum of the data prior to decimation. Next, we decimate the original data by a factor of 4 (eliminating 3 traces between each pair of traces) to obtain the data in Figure 2c. The f-k spectrum of the decimated data is shown in Figure 2d. Figures 2e and 2f show the reconstructed data using the curvelet interpolation in the t-x and f-k domains, respectively. Figure 2g shows the difference between the original data in Figure 2a and the interpolated data in Figure 2e data.

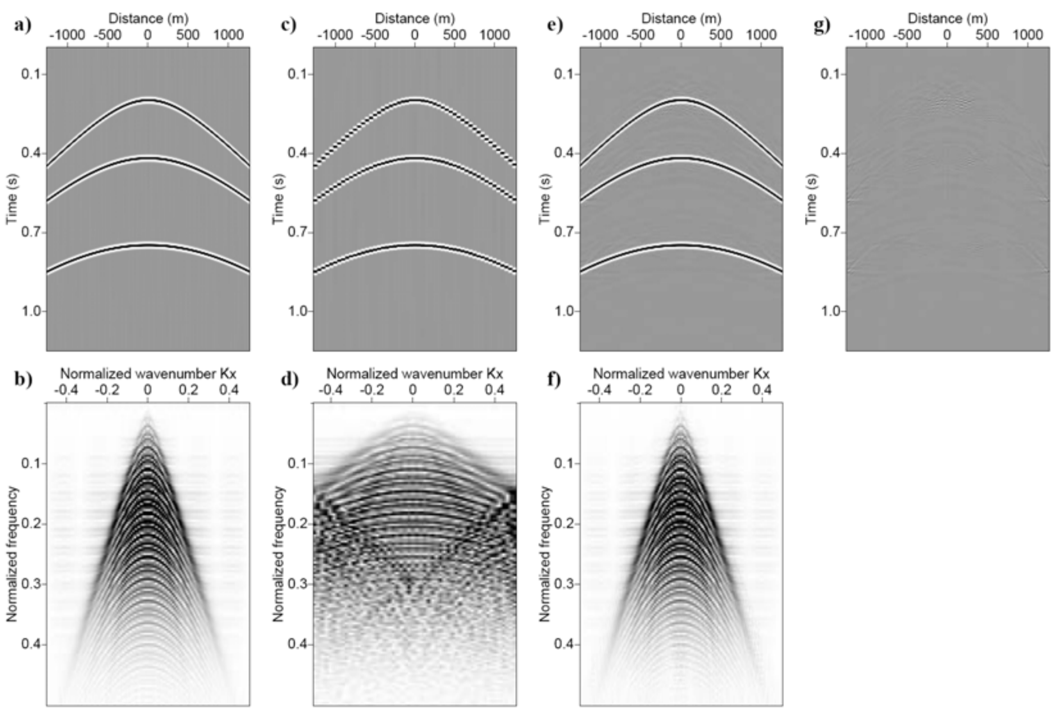

Figure 2. Synthetic example showing the curvelet interpolation method. a) Original data. c) Data decimated by a factor of 4. e) Interpolated data using curvelets. g) The difference between the original and the interpolated data. b), d), and f) are the f-k spectra of (a), (c), and (e), respectively.

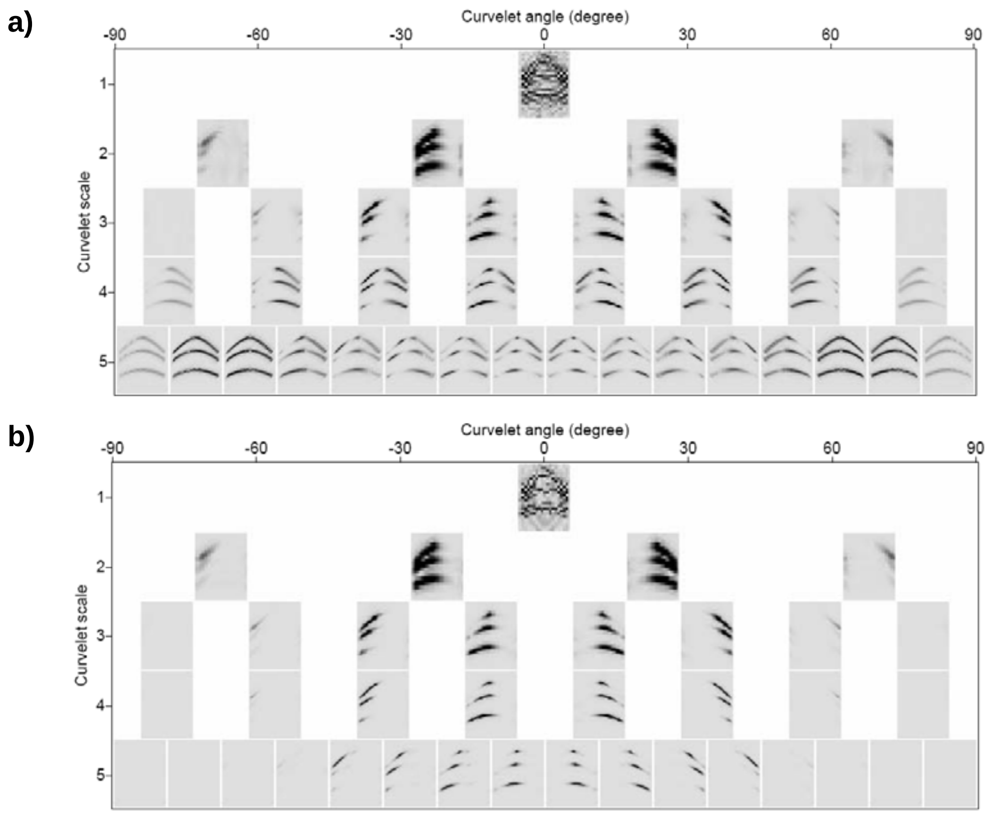

Figure 3. a) Curvelet representation of decimated data after zero-padding. B) curvelet representation of interpolated data.

Figure 3a shows the curvelet representation of decimated data in Figure 2c after interlacing 7 zero traces between each available trace. Scales 1-3 are free of alias while the alias energy emerges in scales 4 and 5. Figure 3b depicts the curvelet representation of the interpolated data which is interpolated using the proposed method in this paper.

Conclusions

In this paper, we propose a novel method for interpolation of aliased seismic data using the curvelet transform. We have shown that spatially aliased data can be represented in the curvelet domain by two types of coefficients. Those that belong to coarser scales and that have not been affected by spatial sampling and those at finer scales that have been contaminated by alias. We assume that the seismic signal is expected to have similar local dips in lower and higher scales. This assumption is used to design a mask function that allows us to filter out aliased coefficients.

Acknowledgements

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References

Candes, E. J. and D. L. Donoho, 2004, New tight frames of curvelets and optimal representations of objects with piecewise-c2 singularities: Comm. on Pure and Appl. Math., 57, 219-266.
Gulunay, N., 2003, Seismic trace interpolation in the Fourier transform domain: Geophysics, 68, 355-369.
Hennenfent, G. and F. J. Herrmann, 2007, Random sampling: New insights into the reconstruction of coarsely sampled wavefields: SEG, Expanded Abstracts, 26, 2575 -2579.
Liu, B. and M. Sacchi, 2004, Minimum weighted norm interpolation of seismic records: Geophysics, 69, 1560-1568.
Spitz, S., 1991, Seismic trace interpolation in the F-X domain: Geophysics, 56, 785–794.