

# 3D R & T coefficients for a dipping interface: A plain wave domain approach.

Ritesh K. Sharma and Robert J. Ferguson, University of Calgary, rksharma@ucalgary.ca

## Summary

We derive reflection (R) and transmission (T) coefficients in the plane wave domain (PWD). By deriving R & T coefficients in the PWD for 3D media, the determination of dip and azimuth of interface are avoided, and, thereby, we avoid ray tracing and exposure to caustics especially in anisotropic media. Classical R & T coefficients in plane wave coordinates are worked out for reflectors aligned with the computational grid. For non-aligned reflectors, those with dip and azimuth, computation of effective R & T coefficients is not straight forward, for this the coordinate system must be rotated. To do this, a normal for each individual plane wave based on local velocity and vector cross product of this normal with the normal to reflector are computed. This cross product yields a ray parameter that presently is used to compute corresponding R & T coefficients for a given plane wave. The importance of this approach is the automatic adaptation of R & T coefficients expression to a special case of dipping interface. Another importance of R & T coefficients in the PWD is their use in Rayleigh Sommerfeld Modeling (RSM) of seismic data. Since reflected and transmitted wave amplitudes depend on R & T coefficients. These coefficients play an important role in the wavefield extrapolation. Further, monochromatic wavefield extrapolation can be done in the PWD with the guaranty of reduced computational time. Thus, R & T coefficients in the PWD can play an important role in RSM also. In-line traces and cross-line traces are required in order to model the plane wave inputs. The problem associated with data acquisition is revealed here by changing the number of cross line traces.

## Introduction

Historically, the calculation of R & T coefficients for plane waves on, both, a free surface and a welded contact interface, was obtained by Zoeppritz (Borejko, 1996). This work was elaborated upon by Aki and Richards (Aki, 1980). The assumption of 2D plane waves ensures us to discuss two separate groups of waves, the coupled P and SV waves, and the SH waves (Slawinski, 2003). Further, assuming isotropy, the standard '2D' formulas can be used for any plane reflector regarding its 3D orientation (Krebes, 2008). Presently, only SH wave is considered for simplicity. The analytic expressions of R and T coefficients for three dimensional plane waves in elastic media were given by Borejko (Borejko, 1996). Generalized ray integral representations of pertinent waves were used in that paper. A generalized ray integral representation of SH wave for dipping structure was given by Ziegler and Pao (Pao, 1984). The importance of R & T coefficients in the PWD for RSM has been discussed by author in another paper (Sharma, 2009). The following section shows how to express R & T coefficients in the PWD.

## Theory

The analytic expression of R & T coefficients are known in terms of angle (Kennett, 2001). The angle of incidence is the angle that of the incident and scattered plane make with the normal to the plane reflector. The plane of incidence can be represented by the unit normal vector to the plane wave in the propagation direction and can be computed as (Ferguson, 2008)

$$\hat{p} = \frac{p_1 \hat{i} + p_2 \hat{j} + q \hat{k}}{\sqrt{p_1^2 + p_2^2 + q^2}} \quad (1)$$

where  $p_1, p_2$  are the input plane wave parameters,  $q$  is the vertical slowness in the incident medium and  $p_1, p_2$ , and  $q$  are coupled according to relation derived from the dispersion relation as

$$q = \frac{1}{v} \sqrt{1 - (vp_1)^2 - (vp_2)^2}, \quad (2)$$

here  $v$  is the velocity of a wave in the incident medium. The unit normal associated with the reflecting surface can be computed as

$$\hat{a} = \sin\theta \cos\phi \hat{i} + \sin\theta \sin\phi \hat{j} + \cos\theta \hat{k}, \quad (3)$$

where  $\theta$  is the dip and  $\phi$  is the azimuth of the normal to the interface. These two unit normals are shown in Figure 1. Following simple vector calculus, the cross product of these normal vectors implicitly determines the angle of incidence,  $\theta_I$ , as

$$\sin\theta_I = |\hat{p} \times \hat{a}|, \quad (4)$$

The sine is related to the effective ray parameter,  $p_I$ , along the interface as

$$p_I = \frac{\sin\theta_I}{v} = |\hat{p} \times \hat{a}| \sqrt{p_1^2 + p_2^2 + q^2}. \quad (5)$$

Thus, the angle of incidence is obtained according to equation (5). This value is substituted now in the known analytic expression to obtain R & T coefficients in the PWD and can be written as

$$R_{SH} = \frac{\rho_1 v_1^2 q_1 - \rho_2 v_2^2 q_2}{\rho_1 v_1^2 q_1 + \rho_2 v_2^2 q_2}, \quad (6)$$

and

$$T_{SH} = \frac{2\rho_1 v_1^2 q_1}{\rho_1 v_1^2 q_1 + \rho_2 v_2^2 q_2}, \quad (7)$$

where  $\rho, v$  are the density and velocity, respectively. The incident and refracted medium are indicated by subscript 1 and 2, respectively. The vertical slowness  $q_1$  is determined by

$$q_1(\vec{p}) = \frac{1}{v_1} \sqrt{1 - (v_1 p_I)^2}, \quad (8)$$

and  $q_2$  is described as

$$q_2(\vec{p}) = \frac{1}{v_2} \sqrt{1 - (v_2 p_I)^2}. \quad (9)$$

### Dipping interface problem:

The above expressions for R & T coefficients can be used for a special case of the dipping interface problem. In this case the normal to the interface would be different from the normal of horizontal and can be computed from equation (3). Normal to the interface lies in the plane of propagation is assumed here. This constraint is applied on the equation (3). This assumption ensures that SH wave is still decoupled from P and SV waves (Sten, 2002). Now, the ray

parameter for each individual plane wave is computed according to equation (5) and used in equations (6) and (7) in order to obtain R and T coefficients for dipping interface, respectively. R and T coefficients for in-line and cross-line slices can be obtained using  $p_2=0$ ,  $p_1=0$ , respectively, with different values of  $q_1$  and  $q_2$  from equations (8) and (9). Recalling equation (5) for  $p_2=0$ , the effective ray parameter is the same as horizontal slowness  $p_1$  in the 2D case, and for  $p_1=0$  it would be  $p_2$ .

## Examples

To explore how R & T coefficients change with  $p_1$  and  $p_2$ , an example in which  $v_1 = 1500$  m/s and  $v_2 = 2500$  m/s with constant density across the reflector is considered. Figure 2 shows the 3D real and imaginary part of R & T coefficients, respectively. Presently, 512 in-line and cross-line traces are used for a particular frequency, 40Hz. Symmetrical behavior of R & T coefficients with respect to  $p_1$ ,  $p_2=0$  is shown in this Figure. Figure 3 shows the in-line and the cross-line slice of R&T coefficients. In this case, the obtained results for in-line and cross-line slices follow the theoretical behavior along slowness axes. After the critical slowness (it is 0.0004m/s in the present case), due to the imaginary nature of vertical slowness, the complex behavior of R & T coefficients is demonstrated in this case. Following the theory, there will be a distortion of the reflected and transmitted pulses at  $p_1 > 1/v_2$  [4] as depicted in Figure 3. Figure 4 shows the real and imaginary part of the 3D R and T coefficients when the interface is dipping. In-line and cross-line slices of the R & T coefficients are shown in Figure 5. The region in between negative and positive critical slowness is shifted, and the former symmetrical behavior about zero slowness no longer holds here in this Figure. The reflected and transmitted amplitudes are not getting values 1 and 2 respectively at positive critical slowness here. The reason for this discrepancy will be revealed in next cases. 3D R & T coefficients can be obtained in the similar way with same in-line traces as previously but different cross-line traces (128, 8) for horizontal interfaces. In-line slices of R & T coefficients for both cases are the same as shown in Figure 3. Cross-line slices of R & T coefficients for both cases are shown in Figure 6. According to this Figure the reflected and transmitted amplitudes are holding the values as per theory at and near to zero slowness. Presently, the obtained reflected and transmitted amplitudes at critical slowness are deviating from expected results. The absence of a sample point with calculation at critical slowness is the responsible for this. The sample rate depends on the number of traces inversely. Interpolation is used to get the values at all points between two sample points. According to theory, there is large difference between the values of the reflected and transmitted amplitude at the sample points on either side of the critical slowness. Interpolation is not sensitive to these kinds of abrupt changes. Thus, the obtained results differ from the expected one. The shifting of R & T along slowness axes leads to no more symmetry about zero slowness as depicted in these slices for dipping interface. The inconsistency in the reflected and transmitted amplitudes at positive and negative Nyquist has been registered in both kinds of slices. The main reason for this inconsistency is the program implementation in which negative Nyquist is used as per theory but positive Nyquist is taken as the difference of positive Nyquist and sample rate. With a coarse sample rate, the difference between positive and negative Nyquist becomes considerable and gives distorted results.

## Conclusions

3D R & T coefficients have been obtained in the PWD by using an effective ray parameter approach. First, R & T coefficients have been obtained for the reflector aligned with the computational grid. It has been shown that obtaining R & T coefficients for the reflector non-aligned with computational grid is little bit tricky. The importance of this approach is the automatic adaptation of R and T expressions to the special dipping interface case. The power of

this is that no ray tracing is required. The obtained R & T coefficients for the same number of in-line traces but different number of cross-line traces deviate from the expected ones and give the distorted picture of subsurface even for isotropic media. This reveals the problems associated with data acquisition and force us to acquire data correctly.

## Acknowledgements

I would like to thank my supervisor, R.J.Ferguson, for many discussions. I also thank G.Margrave and L. Lines for their comments on the work. This work was not possible without help of students and supporting staff of CREWES. Finally I thank the CREWES sponsors for financial support.

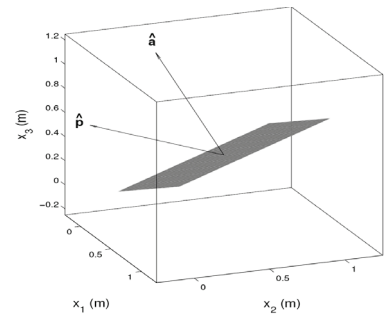


FIG. 1. Schematic representation of unit normals to plane wave and tilted interface. The angle of incidence  $\theta_I$  is the angle between normal  $\hat{p}$  and  $\hat{a}$ .

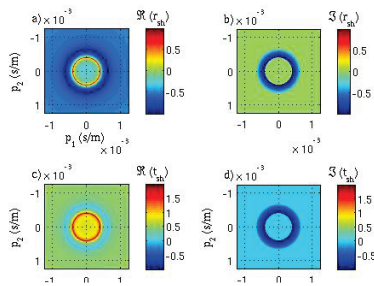


FIG. 2: Horizontal interface

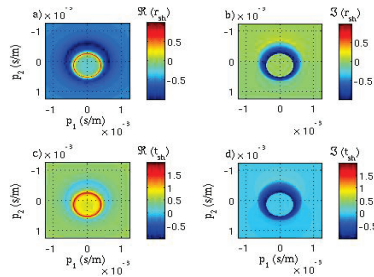


FIG. 4. a) Real part of the R coefficient. b) Imaginary part of the R coefficient. c) Real part of the T coefficient. d) Imaginary part of the T coefficient for a dipping interface

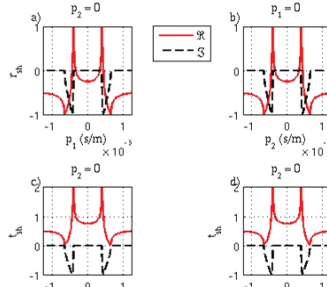


FIG. 3: Horizontal interface

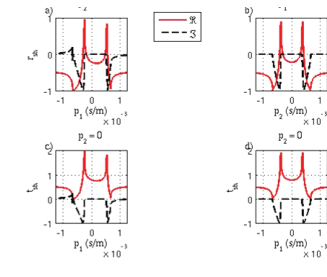


FIG. 5. a) In-line slices of the R coefficient. b) Cross line slice of the R coefficient. c) In-line slice of the T coefficients. d) Cross-line slice of the T coefficient for a dipping interface

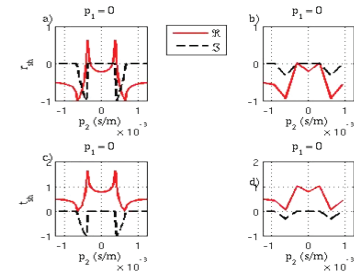


FIG. 6. Cross-line slices of the R coefficient a) with 128 traces b) with 8 traces. Cross-line slices of the T coefficient c) with 128 traces d) with 8 traces for a horizontal interface.

## References

1. Aki, K., and Richards, P.G., 1980, Quantitative Seismology Theory and Methods: W.H.Freeman and Co., San Francisco.
2. Borejko, P., 1996, R and T coefficients for 3D plane waves in elastic media: Wave Motion, 24.
3. Ferguson, R.J., and Margrave, G.F., 2008, 3D anisotropic phase shift operators: CREWES Research Report, 20, 2-4.
4. Kennett, B.L.N., 2001, The Seismic Wavefield: Cambridge
5. Krebs, E.S., 2008, Seismic Theory and Methods.
6. Pao, Y.H., Ziegler, F., and Borejko, Y.S.W., 1984, Generalized ray integral representation of transit SH in a layered half space with dipping structure. ACTA Mechanica
7. Sharma, R.K., and Ferguson, R.J., 2009, R & T coefficients for SH wave in plane wave domain. CREWES Research Report, 21.
8. Slawinski, M.A., 2003, Seismic Waves and Rays in Elastic media: Pegamon.
9. Sten, S., and Wyssession, M., 2002, An Introduction to Seismology, Earthquakes, and