G023

Some Formulas for AVF-AVA Inversion of Reflections from Absorptive Targets

K.A. Innanen* (University of Calgary)

SUMMARY

Recent reports of frequency-dependent seismic data anomalies, detected near low-Q hydrocarbon zones, have led several researchers to suggest that the spectral imprint of an absorptive reflection coefficient is being observed. By expanding various mathematical forms for an absorptive reflection coefficient about small parameter contrasts and incidence angles, and examining them frequency by frequency (AVF), or angle by angle (AVA), simultaneous variations in wavespeed and Q, may be separately estimated.

Introduction

In the recent geophysical literature there are reports of seismic anomalies attributable to strong absorptive-dispersive reflection coefficients[1]. Frequency variations in these reflection coefficients could be put to use determining relative changes in the attenuation properties of geophysical targets, for instance fluid-bearing reservoirs. In fact, absorptive inverse scattering procedures have been shown to employ precisely this type of amplitude information (Innanen and Weglein, 2007; Innanen and Lira, 2009). Here we pursue this idea more fully, deriving direct formulas for the estimation of target absorptive medium properties, both in linearized forms and with non-linear corrections, given absorptive reflection coefficients as input. The methodology we use is a modification of the framework developed by Zhang and Weglein (2009a,b) for direct acoustic and elastic target identification.

An absorptive reflection coefficient can be analyzed by considering its characteristic variations with frequency (AVF), or angle (AVA). We will consider both, but practically, AVF may be the better route. Absorptive variability has been reported to be observable only at large angles when synthetic data are examined at a single frequency over a suite of incidence angles (Samec and Blangy, 1992). Whereas, field data variability associated with low-Q, fluid filled reservoirs has been attributed to "strongly frequency dependent" reflection coefficients (Odebeatu et al., 2006), and this latter observation appears to be supported by other recent discussions (Chapman et al., 2006; Lines et al., 2008; Ren et al., 2009).

We consider the problem of separately estimating the influence of absorptive medium parameters on measured reflection coefficients. The key features of the problem are captured by examining a simple, absorptive reflection coefficient due to a contrast in wavespeed, and a simultaneous contrast in a nearly constant quality factor Q. We express the reflection coefficient in terms of a variety of different plane-wave variables, expand it about small parameter contrasts and incidence angles, and directly invert these series to determine the properties of the target. The medium of incidence is here assumed to be known, and non-attenuating. This corresponds roughly to problems in which either attenuation is negligible in a characterized overburden above an absorptive target, or in which attenuation is present above the target but has been corrected (Q-compensated).

Absorptive wave equations

Consider a wavefield P_0, propagating in a 2D, homogeneous, source-free, non-absorptive medium (medium 0), and another, P_1, propagating in an absorptive medium (medium 1), according to

$$\left[\nabla^2 + \frac{\omega^2}{c_0^2}\right] P_0(x,z,\omega) = 0, \quad \text{and} \quad \left[\nabla^2 + \frac{\omega^2}{c_1^2}\left(1+\frac{F(\omega)}{Q_1}\right)^2\right] P_1(x,z,\omega) = 0, \tag{1}$$

respectively. Here $F(\omega) = i/2 - (1/\pi)\log(\omega/\omega_r)$, where ω_r is a reference frequency. This is consistent with the nearly-constant Q model reviewed by Aki and Richards (2002). Fourier transforming equations (1), we have

$$k_x^2 + k_z^2 = \omega^2/c_0^2, \qquad k_x^2 + k_z'^2 = (\omega^2/c_1^2)[1+F(\omega)/Q_1]^2, \tag{2}$$

the first of which implies a range of possible relationships based on plane wave geometry, e.g., $k_z = (\omega\cos\theta)/c_0$, where θ represents the plane wave angle measured away from the direction of positive z.

Absorptive reflection coefficients

If P_0 is incident upon a plane boundary separating medium 0 from medium 1, continuity of the field (e.g., pressure) across the interface requires that there be a reflection coefficient $R = (k_z - k_z')/(k_z + k_z')$ which we anticipate to be expressive of the an-acoustic properties of the target medium in some hopefully useful

[1] An-elastic reflection coefficients have been widely discussed in theory; see for instance White (1965); Borcherdt (2009).

ways. At oblique incidence, there is some room for choice in studying $R(\theta)$ in terms of the quantities k_x, k_z, and ω. For instance, if frequency ω is a parameter, by equation (2) we have

$$R_\omega(\theta) = \frac{c_1 \cos\theta - c_0 \left[1 + Q_1^{-1} F(\omega)\right] \sqrt{1 - \frac{c_1^2}{c_0^2} \left[1 + Q_1^{-1} F(\omega)\right]^{-2} \sin^2\theta}}{c_1 \cos\theta + c_0 \left[1 + Q_1^{-1} F(\omega)\right] \sqrt{1 - \frac{c_1^2}{c_0^2} \left[1 + Q_1^{-1} F(\omega)\right]^{-2} \sin^2\theta}}. \tag{3}$$

The variations of R with ω, θ, etc. provides the information by which parameters are determined.

Series expansions of R

Quantities embedded in R that are, from a geophysical point of view, sometimes small, but not always, include angle of incidence, the relative change in wavespeed (from c_0 to c_1), and the inverse quality factor Q^{-1} (from 0 to Q_1^{-1}). As measures of the latter two we define

$$a_c = 1 - c_0^2/c_1^2, \qquad a_Q = 1/Q_1, \tag{4}$$

and as a measure of the first, we consider R as a function of $\sin^2\theta$. The forthcoming inversion scheme relies on series expansions of R in orders of these quantities. Dispersion brings an additional level of complexity in the θ dependence beyond the acoustic case. If ω is a parameter R expands as

$$\begin{aligned} R_\omega(\theta) = &\left[\left(\frac{1}{4}a_c - \frac{1}{2}F(\omega)a_Q\right) + \left(\frac{1}{8}a_c^2 + \frac{1}{4}F^2(\omega)a_Q^2\right) + ...\right]\left(\sin^2\theta\right)^0 \\ &+ \left[\left(\frac{1}{4}a_c - \frac{1}{2}F(\omega)a_Q\right) + \left(\frac{1}{4}a_c^2 - \frac{1}{2}F(\omega)a_c a_Q + \frac{3}{4}F^2(\omega)a_Q^2\right) + ...\right]\left(\sin^2\theta\right)^1 + ..., \end{aligned} \tag{5}$$

but if k_z is chosen to be a parameter, we have instead the more complicated θ dependence

$$\begin{aligned} R_{kz}(\theta) = &\left[\left(\frac{1}{4}a_c - \frac{1}{2}F_{kz}(\theta)a_Q\right) + \left(\frac{1}{8}a_c^2 + \frac{1}{4}F_{kz}^2(\theta)a_Q^2\right) + ...\right]\left(\sin^2\theta\right)^0 \\ &+ \left[\left(\frac{1}{4}a_c - \frac{1}{2}F_{kz}(\theta)a_Q\right) + \left(\frac{1}{4}a_c^2 - \frac{1}{2}F_{kz}(\theta)a_c a_Q + \frac{3}{4}F_{kz}^2(\theta)a_Q^2\right) + ...\right]\left(\sin^2\theta\right)^1 + ..., \end{aligned} \tag{6}$$

where $F_{kz}(\theta) = i/2 - (1/\pi)\log(k_z c_0/\omega_r \cos\theta)$. It is possible to express the θ-variability of F in powers of $\sin^2\theta$ also, but this is not necessary for the forthcoming inversion.

Absorptive AVF inversion

The forward problem we consider is the calculation of R over a range of the variables k_x, ω (i.e., the Fourier conjugates of offset and time respectively, which can be transformed to k_z, θ), given a_c and a_Q. The inverse problem is the determination of a_c and a_Q (and thereby target c and Q values) from values of R over a range of one of these variables. We assume exact data, and thus to determine 2 parameters we require 2 values of R; clearly for any practical application of these ideas all available data would be incorporated via some suitable regression.

We first treat the AVF inverse problem by considering a wave field impinging at normal incidence on a plane contrast in c and Q; setting $\theta = 0$ in equation (5), the requisite R has the form

$$R_\omega = \left(\frac{1}{4}a_c - \frac{1}{2}F(\omega)a_Q\right) + \left(\frac{1}{8}a_c^2 + \frac{1}{4}F^2(\omega)a_Q^2\right) + ... \ . \tag{7}$$

This expression is inverted by forming inverse series $a_c = a_{c1} + a_{c2} + ...$, and $a_Q = a_{Q1} + a_{Q2} + ...$, substituting them into equation (7), equating like orders, and summing the sequentially-determined components of a_c and a_Q. Using R at two angular frequencies, ω_1 and ω_2, and for convenience denoting $F(\omega_i) \equiv F_i$ and $R_{\omega_i} \equiv R_i$, we obtain

$$a_Q = -2\left(\frac{R_1 - R_2}{F_1 - F_2}\right) + 2(F_1 + F_2)\left(\frac{R_1 - R_2}{F_1 - F_2}\right)^2 + ..., \tag{8}$$

and

$$a_c = -4\frac{F_2R_1 - F_1R_2}{F_1 - F_2} - 8\frac{(F_2R_1 - F_1R_2)^2}{(F_1 - F_2)^2} - 4\frac{F_1F_2(F_2 - 1)}{(F_1 - F_2)^3}(R_1 - R_2)^2 + ... \quad . \tag{9}$$

Truncating these expressions at first order produces the (absorptive) inverse Born approximation. We emphasize that it is the contrast in dispersive properties at the target that drives the inversion: by inspection of equation (8) we see that only if the reflection coefficient varies with frequency, does $a_Q \neq 0$.

In Figure 1 we illustrate the use of these formulas to invert for c_1 and Q_1, using R at normal incidence for an acoustic medium (c_0 =1500m/s) overlying an absorptive medium (c_1 =1800m/s, Q_1 =10). Target medium properties are determined using R values at pairs of frequencies $f_1 = \omega_1/2\pi$, $f_2 = \omega_2/2\pi$ ranging from 2-120 Hz. In Figure 1, top left, the target Q value is recovered to first order for each frequency pair; bottom left, to second order. In Figure 1 top right, the target wavespeed value is recovered to first order; bottom right, to second order. In addition to the significant increase in accuracy from first to second order, we note that the spurious variation of the linear Q inversion result with experimental variables (in this case frequency), a characteristic of the inverse Born approximation, diminishes as order increases. Proximal frequencies evidently affect the conditioning of the second order recovery of c_1.

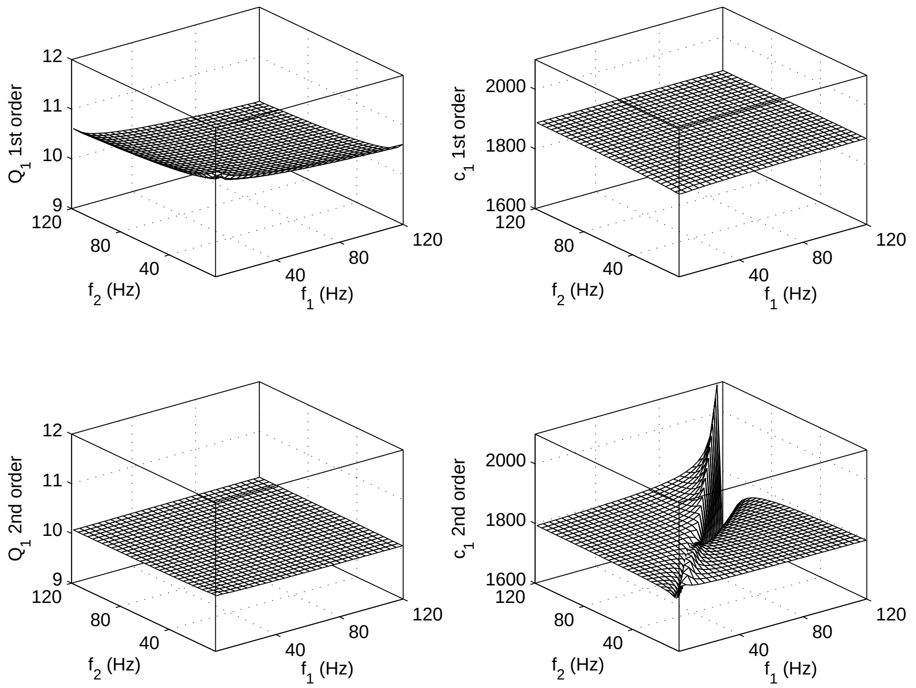

Figure 1 *Target medium properties recovered over a range of frequency pairs. Exact model values are* $Q_1 = 10$, $c_1 = 1800$. *All c/Q values in rank deficient cases (i.e., where* $f_1 = f_2$*) are interpolated over.*

Absorptive AVA inversion

We have thus far made use of the frequency dependence of R to separately determine c and Q. Variations in a_c and a_Q may also be determined by examining the angle dependence of the reflection coefficient, although this simply makes implicit rather than explicit use of the same frequency dependence. This, then, amounts to AVA inversion tuned to the particular problem of absorption. We begin with the expansion of R in equation (6). Again forming inverse series for a_c and a_Q, substituting, and equating like orders, we obtain the formulas

$$\begin{aligned} a_c &= a_{c1} + a_{c2} + ..., \\ a_Q &= a_{Q1} + a_{Q2} + ..., \end{aligned} \tag{10}$$

where, for convenience defining $R_i \equiv R_{kz}(\theta_i)$, $F_i \equiv F_{kz}(\theta_i)$, $T_i \equiv \cos^2\theta_i + 2\sin^6\theta_i$, $G_i \equiv -1/8 - 1/4\sin^2\theta_i - 5/16\sin^4\theta_i$, $H_i \equiv 1/2\sin^2\theta_i + 3/4\sin^4\theta_i$, $J_i \equiv -1/4 - 3/4\sin^2\theta_i - \sin^4\theta_i$, the solution is constructed via

$$\begin{aligned} a_{Q1} &= -2(F_1 - F_2)^{-1}(R_1T_1 - R_2T_2), \\ a_{c1} &= -4(F_1 - F_2)^{-1}(R_1T_1F_2 - R_2T_2F_1), \end{aligned} \tag{11}$$

and

$$\begin{aligned} a_{Q2} &= -2(F_1 - F_2)^{-1}[T_1(G_1a_{c1}^2 + H_1a_{c1}a_{Q1} + J_1a_{Q1}^2) - T_2(G_2a_{c1}^2 + H_2a_{c1}a_{Q1} + J_2a_{Q1}^2)], \\ a_{c2} &= -4(F_1 - F_2)^{-1}[T_1(G_1a_{c1}^2 + H_1a_{c1}a_{Q1} + J_1a_{Q1}^2)F_2 - T_2(G_2a_{c1}^2 + H_2a_{c1}a_{Q1} + J_2a_{Q1}^2)F_1], \end{aligned}$$

etc.

Conclusions

By way of response to reports of strongly absorptive (i.e., dispersive) reflection coefficients being measured at low-Q reservoir targets, we present several formulas for direct determination of relative contrasts in wavespeed and Q from R at several frequencies or incidence angles. These formulas carry out direct linear and non-linear AVF and/or AVA inversion. Numerical examples suggest that for a contrast in Q^{-1} from 0 to 0.1, the second order correction leads to an increase in accuracy in estimating target Q of roughly 5%. Non-linear velocity inversion (c_1 with second order correction) appears to be the least well-conditioned of the cases investigated.

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