Sensitivity of locating of a microseismic event when using analytic solutions and the first arrival times

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Summary

Analytic solutions for estimating the location of a microseismic event using first arrival times are presented. The solutions are based on the Apollonius method which fails when the four receivers are coplanar or collinear. Two analytic methods are presented for four coplanar receivers at the corners of a square, and for three collinear and equally spaced receivers. These analytic solutions are assumed to part of a larger system of receivers composed of a square grid of receivers or equally spaced receivers in a well. An additional method is presented that is based on the intersection of vectors defined from the center of the receivers to the analytic solutions. The sensitivity in estimating the source location is illustrated with receivers in a vertical well.

Introduction

The Apollonius method is an analytic solution that directly computes a microseismic source time and location using the first arrival times at four arbitrarily located receivers Bancroft and Du (2007), Bancroft et.al (2009)'. This solution fails if the receivers are coplanar or collinear, and may produce poor results when the source is located at specific locations relative to the receiver locations. Simpler solutions are presented for four coplanar receivers at the corners of a square, and for collinear receivers.

We assume that the receivers used in the analytic solutions are part of a larger grid system, such as many receivers on the surface, or in a well. Consider sixteen receivers on a 4x4 grid with a separation distance h. There will be nine hxh squares, four $2h$x$2h$ squares, and one $3h$x$3h$ square on the perimeter. Each of these squares can be used to estimate a source location (x_0, y_0, z_0).

Three collinear receivers in a vertical can only compute a radial offset r_0, depth z_0, and the source time t_0. When estimating the three variables (r, z, t_0), only the first arrival times of three equally spaced collinear receivers are required. A vertical array of six receivers spaced with an interval h will produce five possible receiver groups with interval h, four with interval $2h$, three with interval $3h$, and two with interval $4h$. These fifteen groups will each produce an estimate of the source location.

The above estimate will produce results that match the machine accuracy when there is no error in the estimate of the arrival times or the locations of the receivers. We will assume that the velocity and location of the receivers are known exactly, but there is a noise error in the estimated arrival times (jitter) at the receivers. We then estimate the error in the source location for different levels of jitter.

The final estimate of the source location is typically the mean of each component (x_0, y_0, z_0) or (r_0, z_0) from the many possible combinations of receivers in large arrays of receivers. The standard deviation (SD) provides an estimate of the accuracy of the solution. An alternate method computes vectors to each of the initial estimated source locations and the intersection of these vectors provides a new estimate of the overall solution.

Theory and Method

The Apollonius solutions are based on geometric constructions, have relatively simple algebraic operations (+, -, * and /), and require only one square-root. However, part of the computation requires a division that can go to zero if the receivers are co-planar or co-linear. We are able to overcome the Apollonius restriction for two specific geometries; one for a planar surface with four receivers on a square grid, and the other for three equally spaced co-linear receivers. The four receivers on a square grid may be applicable to a large array of receivers on the surface, while the three collinear receivers are applicable to receivers in a well.

The traveltime equations for raypaths between a source at (x_0, y_0, z_0) and four arbitrarily located receivers at (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), and (x_4, y_4, z_4) are:

$$\begin{aligned}
(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2&=v^2(t_1-t_0)^2\\
(x_2-x_0)^2+(y_2-y_0)^2+(z_2-z_0)^2&=v^2(t_2-t_0)^2\\
(x_3-x_0)^2+(y_3-y_0)^2+(z_3-z_0)^2&=v^2(t_3-t_0)^2\\
(x_4-x_0)^2+(y_4-y_0)^2+(z_4-z_0)^2&=v^2(t_4-t_0)^2,
\end{aligned}\qquad(1)$$

where v is the (constant) velocity, t_0 is the clock-time of the source event and t_1, t_2, t_3, and t_4, are the clock-times of the event at the corresponding receivers. These equations are the starting point of the Apollonius and following solutions.

We restrict the four receivers to be located on a square grid with one point at the origin (0, 0, 0) and the other three points separated by the distance h, i.e. (h, 0, 0), (0, h, 0) and

(h, h, 0) all located on the surface with $z_{i=1\text{-}4} = 0$. In a similar manner, we restrict three equally spaced receivers (0, h, $2h$) with source time t_0, radial offset r_0, depth z_0, and define $t_h = h/v$. The corresponding solutions 'Bancroft (2009)' are contained in equation 2 below.

$$
\begin{aligned}
t_0 &= \frac{t_1^2 - t_2^2 - t_3^2 + t_4^2}{2\left(t_1 - t_2 - t_3 + t_4\right)} \\
x_0 &= \frac{v^2\left[2t_0\left(t_2 - t_1\right) - \left(t_2^2 - t_1^2\right)\right] + h^2}{2h} \\
y_0 &= \frac{v^2\left[2t_0\left(t_3 - t_1\right) - \left(t_3^2 - t_1^2\right)\right] + h^2}{2h} \\
z_0 &= \pm sqrt\left[v^2\left(t_1 - t_0\right)^2 - \left(x_0^2 + y_0^2\right)\right]
\end{aligned}
\quad . \qquad (2)
$$

and

$$
\begin{aligned}
t_0 &= \frac{t_1^2 - 2t_2^2 + t_3^2 - 2t_h^2}{2\left(t_1 - 2t_2 + t_3\right)} \\
z_0 &= \frac{1}{2h}\left[2t_0 v^2\left(t_2 - t_1\right) + v^2\left(t_1^2 - t_2^2\right) + h^2 + 2z_1 h\right] \\
r_0 &= sqrt\left[v^2\left(t_1 - t_0\right)^2 - \left(z_1 - z_0\right)^2\right]
\end{aligned}
\quad . \qquad (3)
$$

Examples

The first example shows the Apollonius solution for four receivers arbitrarily located near the surface, and the source located at a depth that matches the spread of the receivers. A source clock-time was chosen, and the clock-times at the receivers calculated. Gausian random jitter was added to the clock-time of the receivers, then using only the receiver locations and their clock-times, a source clock-time and its location were estimated. This procedure was repeated one-hundred times with a jitter of 0.1 ms, and the mean and SD of the source location estimated. In Figure 1, the receivers are a green "x", the source location a blue "+", and the red circles are the 100 estimated locations.

Figure 2 shows the distribution of estimated source locations using four coplanar receivers on a square grid. Jitter with a SD of 1.0 ms was added to the clock-times.

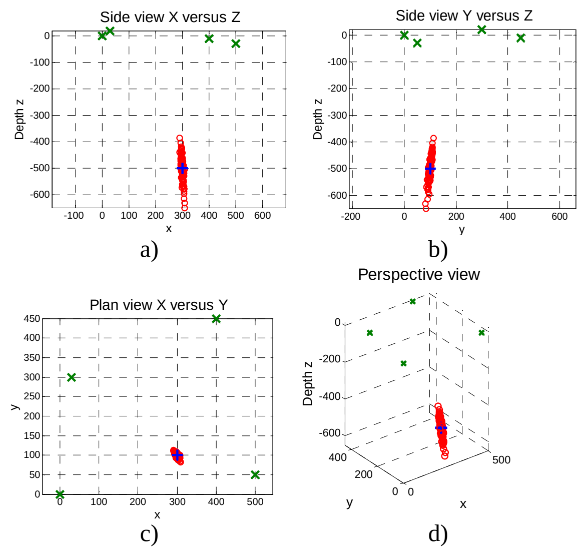


Figure 1: Four views of an Apollonius solution with four receivers "x" near the surface, the known source "+" and the estimated source locations "*o*". The standard deviation of the noise was 1 ms.

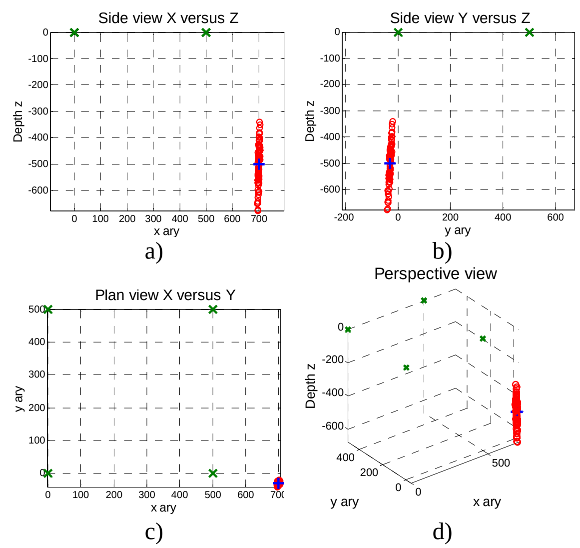
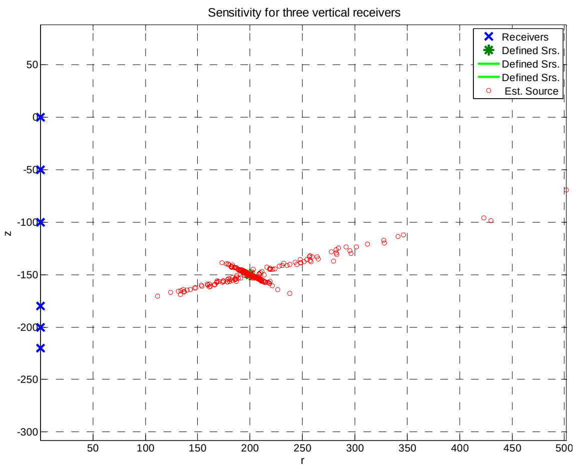
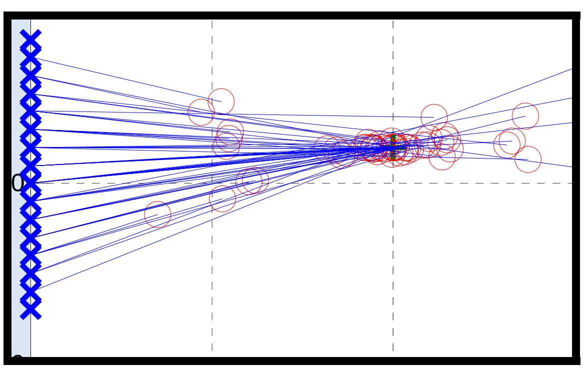


Figure 2 Four views of estimated source solutions when using a coplanar square array of four receivers on the surface.

Two sets of three collinear, equally spaced receivers in a vertical well are simulated in Figure 3a. The results are in a 2D radial plane as only the radial distance and depth from the well can be estimated. The receivers are located a zero offset with the upper receivers at depths of 0, 50 and 100m and the lower receivers at depths of 180, 200, and 220m. The source is located at offset 200 m and depth of 150 m. A source location and the traveltime to the three receivers were defined and a jitter with a SD of 0.1 ms was added to each traveltime. Using only the receiver geometry and the three receiver traveltimes, the source location was estimated. This process was repeated for 100 trials and each of the solution is a red circle in Figure 3a. The mean and SD of the corresponding source locations are displayed in Table 1.



a)



b)

Figure 3 Estimated locations for two sets of three equally spaced receivers. The upper set is separated by 50 m, and the lower set by 20 m.

The distribution of the source locations in Figure 3a illustrate an additional method for estimating the source location. The distributions of the solutions tend to be linear from the center receiver to the true source location. In this case, two vectors are defined from the two sets of the 100 solutions using the least squares method, and their intersection provides an additional solution.

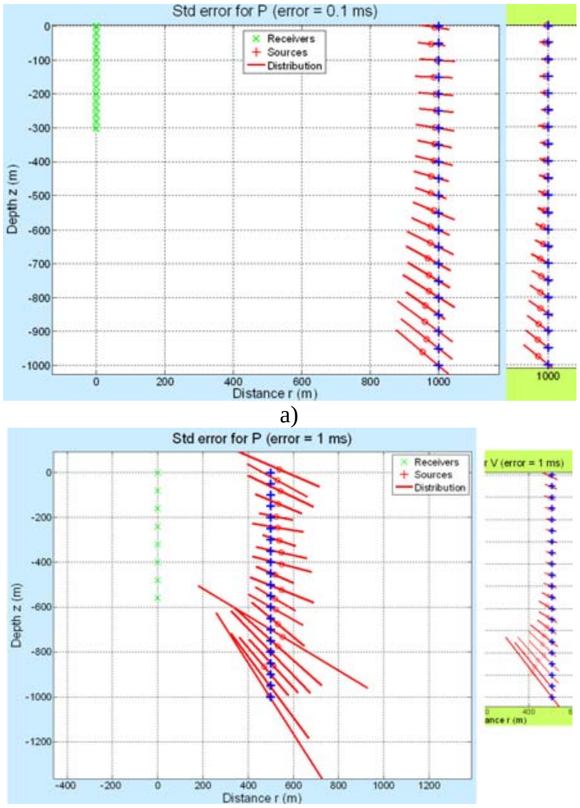
Table 1: Comparison of estimated locations and their standard deviations.

	Radial location r_0	Radial SD	Depth z0	Depth SD
True	200	0	150	
Upper receivers	200.26	11.20	150.24	5.26
Lower receivers	217.60	67.60	145.42	17.03

In practical applications, we don't have the 100 solution for each combination of three receivers, but we may have a vertical array of receivers. For example, sixteen vertical equally spaced receivers will produce 56 combinations of three equally spaced receivers.

We now have two methods for finding the estimated source location and it accuracy. The first method (*P*) computes the mean and SD of the radial and depth estimates from these 56 combinations. The second method (*V*) computes the least squares intersection of the 56 vectors defined from each initial solution to the corresponding center receiver as illustrated in Figure 3b.

Both the *P* and *V* solutions are illustrated with a vertical array of sixteen vertical equally spaced receivers that extend from the surface to a depth of 300 meters. Twenty one source locations are located at a 1000 m distance from the receivers, and extend from the surface to a depth of 1000 m. The 56 combinations of three equally spaced receivers are used to estimate the mean and SD of each source location. The results are shown in Figure 4a where a jitter of 0.1 ms is added to the clock times of the receivers. The true location is defined by the blue "+" sign, the estimated source location by the red circle, and the length of the SD by the red line. The main part of the figure shows the *P* solution and the attachment on the right shows the *V* solution. Figure 4b shows the results when eight receivers extend to a depth of 560 m, the sources are at a distance of 500 m from the receivers, and extend to a depth of 1000 m. The jitter is much larger at 1.0 ms. In these two examples, the *V* solution provides a more accurate result.



a)

b)

Figure 4 Estimated source locations for a vertical array of receivers.

Conclusions

Three analytic methods for locating microseismic events were presented. Each method only uses the location and first arrival clock-times of the receivers. The first method was based on the Apollonius solution and used four receivers in an arbitrary configuration. This method fails if the receivers are collinear or coplanar, so two additional methods were presented; four coplanar receivers on a square grid, and three collinear, equally spaced, receivers. The medium was assumed to have a constant velocity or a geometry where RMS type velocities could be used. Each method has an analytic solution that returns the exact solution in the absence of noise.

Jitter was added to the receiver clock-times as part of a sensitivity analysis to evaluate the accuracies that can be expected from the three or four receiver configurations.

Typical microseismic projects use large arrays of receivers. It is assumed that these methods would be used as part of a much larger process where selected receiver combinations are used to provide initial source locations. A typical application of a vertical well with equally spaced receivers will use many combinations of three equally spaced receivers. The source location is estimated from the mean of the analytic solutions. Examples show that a larger separation of the receivers produces more accurate results.

An additional method for computing the source location used the intersection of vectors that are defined from the center of the receivers to each of the analytic source locations, and was found to produce more accurate results.

These first arrival time methods work for both *P*- and *S*-waves. The error in picking the first arrival times may have a relative picking error in the range of 0.1 to 1.0 ms.

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