# A unified method for interpolation and de-noising of seismic records in the *f-k* domain

*Mostafa Naghizadeh*

**SUMMARY**

A unified approach for de-noising and interpolation of seismic data in the frequency-wavenumber (*f-k*) domain is introduced. First an angular search in the *f-k* domain is carried out to identify a sparse number of dominant dips. Then, an angular mask function is designed based on the identified dominant dips. The mask function is utilized with the least-squares fitting principle for optimal de-noising or interpolation of data. Synthetic and real data examples are provided to examine the performance of the proposed method.

**INTRODUCTION**

Noise elimination and interpolation of seismic data are important topics of research in seismic data processing community. Several important signal processing techniques have been utilized for de-noising and interpolation purposes. For instance, the prediction filters are used by Canales (1984) and Spitz (1991) in the frequency-space (*f-x*) domain for de-noising and interpolation of data, respectively. Other methods such as projection filters (Soubaras, 1994), Singular Value Decomposition (Trickett, 2003), Cadzow de-noising (Cadzow and Ogino, 1981; Trickett and Burroughs, 2009), and Singular Spectrum Analysis (Oropeza and Sacchi, 2009) have also been used for random noise attenuation in the *f-x* domain. All of the *f-x* de-noising methods are based on the assumption that the spatial signals at each single frequency are composed of a sum of a limited number of complex harmonics.

The Fourier transform plays a substantial role in most de-noising and interpolation methods. A frequently used domain for seismic data de-noising is the frequency-wavenumber (*f-k*) domain. One of the prominent attributes of the *f-k* domain is the separation of signals according to their dip. This property makes the *f-k* domain a suitable option for ground-roll elimination due to the distinctive dip information of ground-roll and reflection seismic data. The most commonly used *f-k* domain de-noising method is the dip filtering method. The spatial interpolation of seismic records can also be interpreted as a noise elimination problem in the *f-k* domain. Depending on the sampling function, noise introduced by decimation of data in the *f-k* domain can be considered as incoherent (for random spatial sampling) or coherent (for regular spatial sampling).

The *f-x* domain seismic-trace interpolation methods have been proposed by Spitz (1991) and Porsani (1999). These methods utilize the low frequency portion of data for a robust and alias-free interpolation of high frequencies. Gulunay (2003) have introduced the *f-k* equivalent of *f-x* interpolation methods by creating a mask function from low frequencies. In both *f-x* and *f-k* domain interpolation methods, the given frequencies are interpolated independently. Recently, Curry (2009) has introduced an *f-k* interpolation method using a Fourier-radial adaptive thresholding strategy in an attempt to utilize the continuity of events along the frequency axis. This article introduces another *f-k* domain method which utilizes information from all desired frequencies for robust interpolation or de-noising of seismic data.

The proposed *f-k* method in this paper consists of three steps. The first step is an angular search for a range of dips, over all of the frequencies, to identify the dominant energy dips. The origin of angular rays is located on the origin of the *f-k* domain. Next, a mask function is designed on the *f-k* domain over the dominant energy dips. Finally, the components of the *f-k* domain data which fall under the mask function, are fitted to the available samples in the *t-x* domain by a least-squares algorithm. The proposed methodology will be referred to, hereafter, as the Interpolation and De-noising using Dominant Dips (IDDD) method. Synthetic and real data examples are provided to illustrate the performance of IDDD method.

**THEORY**

**Identifying dominant dips in the *f-k* domain**

Suppose $d(t,x)$ is the data in the *t-x* domain and $D(\omega,k)$ its *f-k* representation. The first step of the proposed method is an angular search over a range of dips in the *f-k* domain to identify dominant dips in the data. The origin of the angular rays is located on the origin of the f-k domain, $(\omega,k) = (0,0)$. Due to the symmetrical property of the frequency axis in the f-k domain, we only use the positive frequencies to explain the methodology. Also, for theoretical simplicity, we use the concept of normalized frequencies and wavenumbers. Normalized frequency and wavenumber axes are obtained by considering $\Delta t = 1$ and $\Delta x = 1$, respectively. This leads to the ranges $0 < \omega < 0.5$ for frequencies and $-0.5 < k < 0.5$ for wavenumbers. A map of dominant dips is produced by summation along the angular rays

$$M(p) = \sum_{n=1}^{N_\omega} D(\omega_n, k = p.\omega_n - \lfloor \frac{p+1}{2} \rfloor), \tag{1}$$

where, $p$ is the slope of summation path in the f-k domain. The parameter $n$ represents the index of frequency and can include any interval of frequencies. The operator $\lfloor . \rfloor$ denotes truncation to the closest small integer value. Notice that this operator allows the ray path to wrap around the frequency axis in order to account for aliased data in the f-k domain. Figure 1 shows a schematic representation of angular rays for dips equal to -1, 0, 1, 2, and 4.

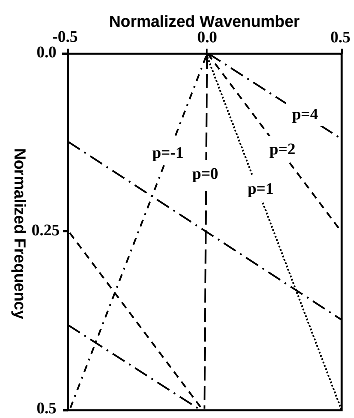


Figure 1: Graphical representation of angular search for dominat dips in the *f-k* domain.

The peak values in the function $M(p)$ are indicators of the dips or slopes with dominant energy. A slope in the function $M(P)$ was considered a peak value if it was bigger than its two neighboring values. Several peak values can be identified in $M(P)$ but we might need to only keep those with the largest values. This can be achieved in two ways. The first approach is to keep the $L$ highest values. The alternative is to set a threshold value and keep all of the peak values which are larger than the chosen threshold value. Regardless of the adapted criteria, let's assume that we have been able to identify $L$ slopes with the peak values as $p_1, p_2, \ldots, p_L$.

**Building a mask function after identifying dominant dips**

After identifying the dominant dips of function $M(P)$, we deploy straight lines along the correspondent angles of the dominant dips in the *f-k* domain. The goal here is to transfer the dominant dips $p_1, p_2, \ldots, p_L$ into a 2D mask function with the size of the original data in the *f-k* domain. Initiating $H$ matrix with zeros, the expression

$$H(\omega_n, k = p_j.\omega_n - \lfloor \frac{p_j+1}{2} \rfloor) = 1, \quad \left\{ \begin{array}{l} n = 1,2,\ldots,N_\omega, \\ j = 1,2,\ldots,L. \end{array} \right. \tag{2}$$

deploys values of 1 along the desired angles in the *f-k* domain. Convolving $H$ with a 1D box car function, $B(1, L_b)$, gives

$$\mathbf{W}(\omega, k) = H(\omega, k) * B, \tag{3}$$

where $L_b$ is the length of box function and $*$ represents convolution. Expression 3 widens the mask function along the wavenumber axis. This helps to take into account the uncertainties involved in the prediction of dominant dips. The designed mask function can serve to eliminate the unwanted artifacts (which fall under the zero values of the mask function) and preserve the original signal.

**Least-squares de-noising and interpolation using the mask function**

The mask function, $\Delta(\omega, k)$, is one for the signal portion of noisy data and zero for elsewhere in the *f-k* domain. The mask function can be deployed inside a least-squares fitting algorithm for optimal interpolation or de-noising of the data. A stable and unique solution can be found by minimizing the following cost function (Tikhonov and Goncharsky, 1987)

$$J = ||\mathbf{d} - \mathbf{T}\mathbf{F}^H\mathbf{W}\mathbf{D}||_2^2 + \mu^2||\mathbf{D}||_2^2. \tag{4}$$

where $\mathbf{F}^H$ is the inverse Fourier transform and $\mathbf{T}$ is the sampling matrix which maps the fully sampled desired seismic data to the available samples. For de-noising problems, the sampling matrix is equal to the identity matrix $\mathbf{T} = \mathbf{I}$. The minimum of the cost function $J$ can be computed using the method of conjugate gradients (Hestenes and Stiefel, 1952). For de-noising purposes one iteration of conjugate gradients suffices. In the case of data interpolation one needs to allow more, typically 8-10, iterations of conjugate gradients.

## EXAMPLES

**Synthetic de-noising example**

In order to examine the performance of the IDDD method, a synthetic seismic section with 3 linear events was created. Figures 2a and 2b show the original synthetic data in the *t-x* and *f-k* domains, respectively. The original data was contaminated with random noise, with Signal to Noise Ratio (SNR) equal to one, to obtain the noisy data in Figure 3a. Figure 3b shows the de-noised data using the IDDD method. Figures 3c and 3d show the *f-k* panel of data in Figures 3a and 3b, respectively. Figure 4 shows the plot of $M(p)$ function computed from Figure 3c for the dip range $-8 \le p \le 8$. One can clearly detect that there are distinctive peaks in Figure 4 that indicate the dip information of three linear events in the original data. Figure 5 shows the mask function built using the three dominant dips in Figure 4. It is clear that this mask function accurately matches with the *f-k* panel of the original data in Figure 2b. Figure 6a shows the de-noised data using Canales (1984) *f-x* de-noising method. Figure 6b shows the *f-k* panel of Figure 6a. In contrast, the Canales *f-x* de-noising method is separately applied to each frequency. Therefore, in the presence of strong noise, it actually struggles to de-noise most of the frequencies. This shortcoming of the Canales *f-x* de-noising method is effectively overcome by using all frequencies to find the dominant dips in the data.

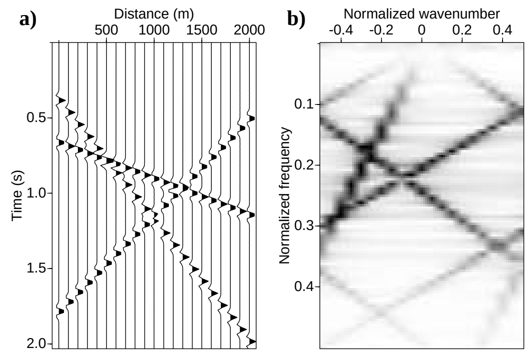


Figure 2: Synthetic seismic section with three liear events a) Original data. b) The *f-k* panel of (a).

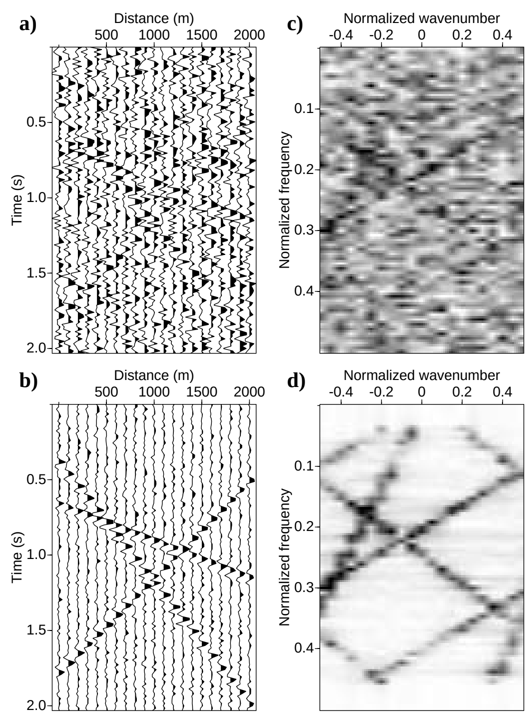


Figure 3: a) Noisy data obtained from original data in Figure 2a by adding random noise (SNR=1). b) De-noised data using IDDD method. c) and d) are the *f-k* panel of (a) and (b), respectively.

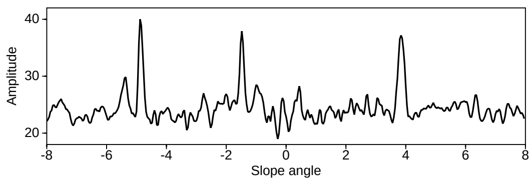


Figure 4: The distribution of energy for dip range $-8 \leq p \leq 8$, computed from *f-k* domain of noisy data (Figure 3b).

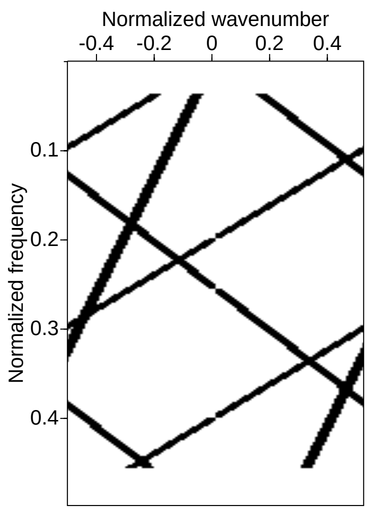


Figure 5: The *f-k* domain mask function obtained from the dominant dips in Figure 4.

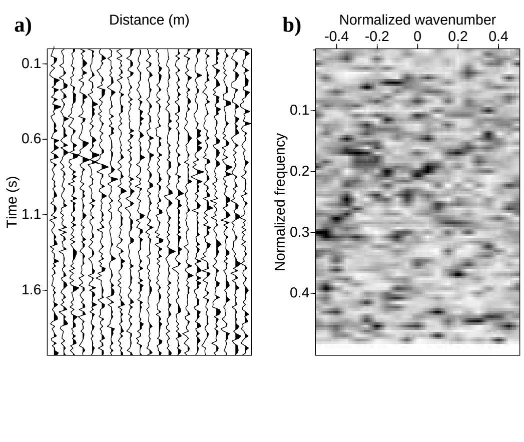


Figure 6: a) De-noised data using Canales *f-x* method. b) The *f-k* panel of (a).

### Synthetic interpolation example

The IDDD method can be used, without any changes, to interpolate seismic data. Figure 7a shows a section of seismic data after randomly eliminating almost 75% of the traces. Figure 7c shows the *f-k* representation of the section with missing traces. Figure 7b shows the reconstructed data using the IDDD method. Figure 7d shows the *f-k* representation of the reconstructed data in Figure 7b. Despite the high percentage of missing traces, the dominant dips of the original data have been successfully restored.

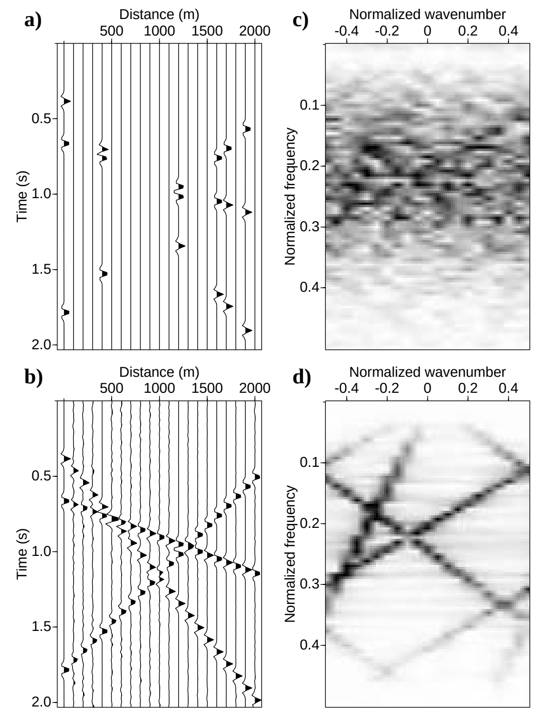


Figure 7: a) Data with missing traces. b) Reconstructed data using IDDD method. c) and d) are the *f-k* panel of (a) and (b), respectively.

The IDDD method can also be utilized to de-alias seismic data in the spatial direction. Figure 8a shows the section of seismic data after interleaving 4 zero traces between each pair of traces in Figure 3a. Figure 8c shows the *f-k* panel of data in Figure 8a. It is clear that the regular interleaving of zero traces has produced replicas of the original spectrum of data. Figure 8b and 8d show the interpolated data using the proposed method in this paper and its *f-k* panel, respectively. Note that in this example we have searched for the dominant dips in the range $-1 \leq p \leq 1$.

### Real data de-noising example

A real shot gather (Figure 9a) contaminated with random and coherent (ground-roll) noise was selected to examine the performance of IDDD method. Figure 9b shows the de-noised section using the IDDD method. The aim was to keep the reflections and remove the ground-roll and random noise. The dip range was set to the interval $-0.5 \leq p \leq 0.5$. This dip range guarantees the exclusion of ground-roll which resides in high dip ranges. Figure 9c shows the results of ordinary *f-k* dip filtering in the range $-0.5 \leq p \leq 0.5$. It is clear that the IDDD method has produced a cleaner section with less spurious events than the ordinary *f-k* dip filtering method.

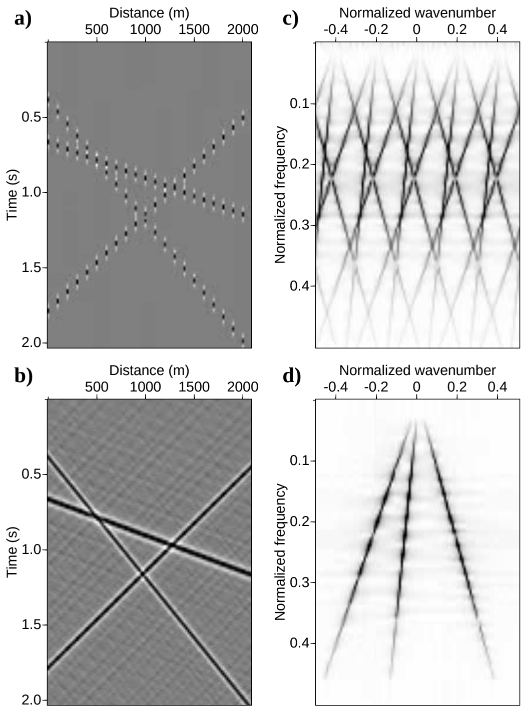


Figure 8: a) Data with missing traces. This section is obtained by adding 4 zero traces between each of traces in Figure 2a. c) Reconstructed data using IDDD method. b) and d) are the *f-k* panel of (a) and (c), respectively.

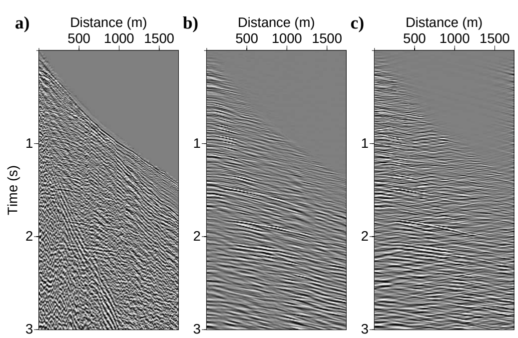


Figure 9: a) A real shot gather with random and coherent noise (ground-roll). b) De-noised data using IDDD method. c) De-noised data using traditional *f-k* dip filtering.

## DISCUSSION

One of the common assumptions for designing interpolation or de-noising methods is the linearity of seismic events (events with constant dips) in the *t-x* domain. This assumption is usually validated for non-linear seismic events by analyzing the data in small spatial windows. The presence of linear events in the *t-x* domain implies the concentration of energy, at a limited number of wavenumbers, at each given frequency in the *f-k* domain. This is the cornerstone for most of the de-noising (Canales, 1984; Soubaras, 1994; Trickett, 2003) methods which aim to eliminate the noise by preserving the limited number of wavenumbers with highest energies at each frequency. The same principle is applied for interpolation methods (Spitz, 1991; Porsani, 1999; Gulunay, 2003; Naghizadeh and Sacchi, 2007, 2009) except that information is extracted from low frequencies. All of the aforementioned methods tend to ignore another important property of linear events. In the *f-k* domain linear events start from the origin (zero frequency and zero wavenumber) and spread according to their associated dips. This property states that one can look for a sparse number of dips by summing over all frequencies in order to bring more resolution to the de-noising and interpolation methods. The IDDD method amply exploits this property by summing along angular rays associated with a range of dips.

The IDDD method not only allows extraction of information from low to high frequencies but also from high to low frequencies. This, of course, is feasible if the stringent condition of dealing with linear events is satisfied. Also, the IDDD method offers more flexiblity in choosing the band of frequencies to estimate the dominant dips. Therefore, By using any band of frequencies, one can estimate the location of a desired signal on other frequencies.

The IDDD method uses a least-squares fitting algorithm to directly match the dominant dips in the *f-k* domain to the data in the *t-x* domain. This leads to an amplitude-preserved and artifact-free de-noised, or interpolated, seismic section. The direct mapping between the *t-x* and *f-k* domains avoids introducing noisy features which are akin to *f-x* methods. However, the strict condition of the linear events makes the IDDD method intolerant to any curved or dispersed events. Therefore, it is safer to apply the IDDD method on very small spatial and time windows.

## CONCLUSIONS

In this article, a new *f-k* domain interpolation and de-noising method was introduced. In the first step, the proposed method entails an angular search in the *f-k* domain to determine the dips with dominant energy. Next, the identified dominant dips were used to design a mask function in the *f-k* domain. Finally, a least-squares fitting routine was deployed to map the Fourier coefficients under the mask function to the data in *t-x* domain. The proposed method is considered as a generalization of the *f-x* and *f-k* interpolation methods in order to fully exploit the information from all frequencies rather the just using the low frequencies. The proposed method shows effective performances on synthetic and real data examples.

## ACKNOWLEDGMENTS

I thank the sponsors of the Signal Analysis and Imaging Group (SAIG) at the University of Alberta. I would also like to thank Dr. Mauricio Sacchi for his inspiring discussions and useful comments.

## EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

## REFERENCES


Cadzow, J. A., and K. Ogino, 1981, Two-dimensional spectral estimation: IEEE Transactions on Acoustics, Speech, and Signal Processing, **29**, no. 3, 396–401, doi:10.1109/TASSP.1981.1163582.

Canales, L. L., 1984, Random noise reduction: SEG Expanded Abstracts, **3**, no. 1, 525–529, doi:10.1190/1.1894168.

Curry, W., 2009, Interpolation with fourier-radial adaptive thresholding: SEG Expanded Abstracts, **29**, 3259–3263.

Gülünay, N., 2003, Seismic trace interpolation in the Fourier transform domain: Geophysics, **68**, 355–369, doi:10.1190/1.1543221.

Hestenes, M. R., and E. Stiefel, 1952, Methods of conjugate gradients for solving linear systems: Journal of Research of the National Bureau of Standards, **49**, 409–436.

Naghizadeh, M., and M. D. Sacchi, 2007, Multistep autoregressive reconstruction of seismic records: Geophysics, **72**, no. 6, V111–V118, doi:10.1190/1.2771685.

Naghizadeh, M., and M. D. Sacchi, 2009, f-x adaptive seismic-trace interpolation: Geophysics, **74**, no. 1, V9–V16, doi:10.1190/1.3008547.

Oropeza, V. E., and M. D. Sacchi, 2009, Multifrequency singular spectrum analysis: SEG Expanded Abstracts, **29**, 3193–3197.

Porsani, M., 1999, Seismic trace interpolation using half-step prediction filters: Geophysics, **64**, 1461–1467, doi:10.1190/1.1444650.

Soubaras, R., 1994, Signal-preserving random noise attenuation by the F-X projection: SEG: Expanded Abstracts, **13**, no. 1, 1576–1579, doi:10.1190/1.1822843.

Spitz, S., 1991, Seismic trace interpolation in the F-X domain: Geophysics, **56**, 785–794, doi:10.1190/1.1443096.

Tikhonov, A. N. and A. V. Goncharsky, 1987, Ill-posed problems in the natural sciences: MIR Publisher.

Trickett, S. R., 2003, F-xy eigenimage noise suppression: Geophysics, **68**, 751–759, doi:10.1190/1.1567245.

Trickett, S. R., and L. Burroughs, 2009, Prestack rank-reducing noise suppression: theory: SEG Expanded Abstracts, **29**, 3332–3336.