

## Microseismic hypocenter location using nonlinear optimization

Joe Wong\*, Lejia Han, and John C. Bancroft, CREWES, University of Calgary

### Summary

Nonlinear optimization methods (or inversion) were investigated for analyzing synthetic microseismic arrival times. Two direct search techniques, the genetic algorithm and pattern search, were used to find the layered-earth velocity values from P-wave arrival times from a simulated perforation shot. For locating microseismic hypocenters, the gradient-based Levenberg-Marquardt algorithm was used to invert reduced arrival times from borehole and surface receiver arrays. Both categories of nonlinear optimization method, direct search and gradient-based, were effective for inverting arrival times to the required model parameters. Our experience suggests that the direct search methods, in particular pattern search, are simpler and faster in this application, i.e., inverting microseismic arrival time data to obtain layer either velocities or hypocenter coordinates.

### Introduction

Unknown model parameters in fitting geophysical survey results can be found by minimizing the misfit between observed and calculated arrival times using non-linear optimization schemes (generally called inversion techniques by geophysicists). The misfit or objective function to be minimized must be parameterized by an input vector of the variables to be found. Optimization techniques fall in two categories: *gradient-based*, and *direct-search*. An example of gradient-based techniques is the Levenberg-Marquardt algorithm (Levenberg, 1944; Marquardt, 1967). Examples of direct search techniques are the pattern search method and the genetics algorithm. These algorithms are available in the utility program *optintool* which is bundled in the MATLAB (2009) Optimization Toolbox.

Gradient-based optimization methods such as the Levenberg-Marquardt (LM) algorithm often can be trapped in local minima when the objective function is a complex nonlinear equation involving many variables. The more variables there are, the greater the likelihood for the existence of local minima, saddle points, or long narrow data valleys. Using a gradient method to find the global minimum in the objective function for such cases tends to be problematic.

Alternatives to gradient based methods exist in the form of sophisticated global search techniques such as the genetic algorithm (GA) or pattern search (PS). These direct search methods are described in the literature; see, for example, Whitley (1997), and Kolda et al. (2003). The advantages of direct search techniques are:

- They are less prone to being trapped in local minima.
- They require no calculation of partial derivatives of the objective function.
- Available implementations of the algorithms are easy to use.

We tested these nonlinear optimization techniques for their effectiveness in solving two problems related to microseismic monitoring. The first problem is estimating the velocity values in an earth model knowing the location of a perforation shot source and the arrival times at a receiver array. The second problem is locating the microseismic hypocenter knowing the arrival times at the receiver array as well as the velocity model. We performed the tests using synthetic arrival times calculated by ray-tracing through a horizontally-layered velocity model.

The left panel on Figure 1 is a section view showing the layered-earth velocity model with a microseismic source in a treatment well and an array of geophones in a vertical observation well. The geometry in cylindrical coordinates has azimuthal symmetry about the observation well or the microseismic source. Assuming P-wave velocities, Snell's Law ray-tracing from the source to the geophone array gives a set of first-arrival times as a function of depth in the observation well. Interpolation then can be used to find the arrival time to any geophone with coordinates  $(x_g, y_g, z_g)$  after determining its radial distance from the source. This procedure for finding arrival times through a non-uniform velocity structure is known as the shooting method.

### Velocity calibration using direct search methods

In the analysis of a real-world microseismic dataset, an essential first step is the calibration of the velocity model. We tested velocity calibration using direct-search inversion of arrival times from a simulated casing perforation shot. The "observed" arrival times were generated by ray-tracing through the four-layer velocity model of Figure 1; example arrival times are plotted on the right side of Figure 1.

The observed arrival times, the geophone coordinates, and the source coordinates are known quantities. The depths of the layer boundaries are fixed and also known (for example, from gamma-ray logs). We now assume that the velocity values are unknown, and we must find estimates for them given the known data. To do this, we calculated arrival times for different velocity values, and defined an objective function whose parameters are the layer velocities, and whose output are the root-mean-square (RMS) difference

## Hypocenter location by nonlinear optimization

between calculated arrival times and observed arrival times. The velocity model is calibrated when we find a set of velocity values that minimizes the objective function within some small value. This task is an instance of nonlinear optimization.

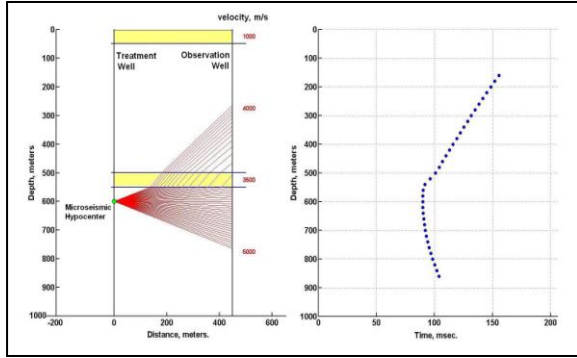


Figure 1: Left: ray-tracing to obtain first arrival times between a microseismic source and an array of geophones in a vertical borehole. Note the refraction through the low-velocity zone. Right: the associated arrival times.

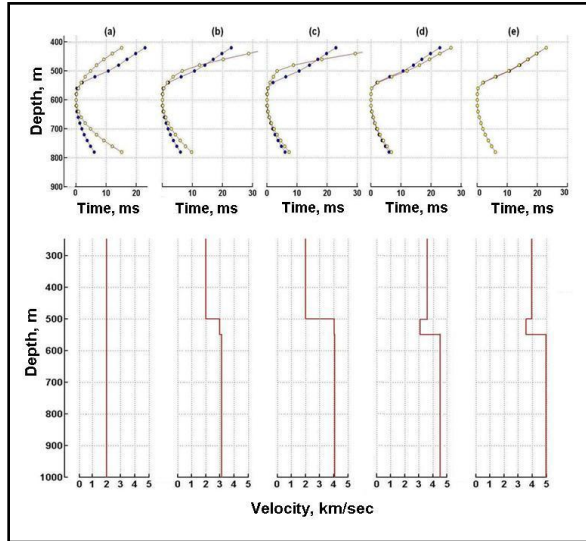


Figure 2: Calibration of layer velocities using GA optimization. Blue dots on the top panel are observed times from a casing perforation shot; yellow dots are the calculated times for the velocity profiles on the bottom panel. Results for (a) an initial guess; (b) after 10 generations; (c) after 20 generations; (d) after 40 generations; (e) after 69 generations, a good fit is found

We selected both GA and PS options in the MATLAB *optimtool* utility to minimize the objective function based on the first arrival times from 19 geophones separated by 20m.

To accelerate the search, reasonable lower and upper bounds for the velocities  $[v_1, v_2, v_3, v_4]$  were set at [1000, 2000, 2000, 2000] and [1000, 6000, 6000, 6000] m/s. Since the overburden velocity played no role in determining arrival times for this particular source-receiver geometry, we fixed it by setting its lower and upper bounds at 1000m/s. Both techniques are iterative, returning velocity values at each iteration (or “generation”) that decrease the objective function gradually. The final velocity values approached the global minimum within a user-determined small tolerance.

Figure 2 shows the results of velocity calibration using GA optimization. The initial assumed flat velocity profile on Figure 2(a) evolved to the final profile on Figure 2(e). At that point, the calculated arrival times gave almost a perfect fit to the observed arrival times. Table 1 summarizes the progress of velocity calibration using GA.

Table 1: Genetics algorithm for calibrating layer velocities (m/s). The number of times that the objective function was evaluated after each listed generation is enclosed between parentheses.

Generation	$v_1$	$v_2$	$v_3$	$v_4$	Error, ms
Initial guess	1000	2000	2000	2000	31.8
10 (200)	1000	2005	3001	3116	29.43
20 (400)	1000	2000	4029	4087	11.03
69 (1390)	1000	3961	3551	4997	.0414
True values	1000	4000	3500	5000	0

Table 2: Pattern search for calibrating layer velocities (m/s). The number of times that the objective function was evaluated after each listed iteration is enclosed between parentheses.

Iterations	$v_1$	$v_2$	$v_3$	$v_4$	Error, ms
Initial guess	1000	2000	2000	2000	69.4
10 (16)	1000	5952	2000	5476	4.31
20 (58)	1000	3482	3976	4982	0.47
55 (220)	1000	3995	3513	4997	0.02
True values	1000	4000	3500	5000	0

The results and progress of velocity calibration using PS optimization are summarized on Table 2. The final results were very similar to those from calibration using GA. However, Tables 1 and 2 show that, compared to GA, PS converged to acceptable estimates for the velocities more

## Hypocenter location by nonlinear optimization

quickly, i.e., after fewer forward calculations of the objective function.

### Hypocenter location using reduced arrival times

In hypocenter location, the assumed known quantities are the observed arrival times, the geophone coordinates, and the depth and velocities in the layered-earth model. The problem now is to estimate the coordinates of the microseismic source from these known quantities. Arrival time data from ray-tracing from a hypothetical source to the geophones through the geometry of Figure 1 was used to test nonlinear inversion for this purpose. For the particular nonlinear optimization method, we chose the gradient based Levenberg-Marquardt (LM) algorithm, so as to evaluate its performance vis-à-vis direct search methods.

For an array in a single observation well, travel times alone cannot give the horizontal coordinates  $(x_s, y_s)$  for the source, but they can yield the cylindrical coordinates  $(r_s, z_s)$  where  $r_s$  is the radial distance from the observation well. In order to get the coordinates  $(x_s, y_s)$ , we must have triaxial geophones, and use the x- and y-components with hodogram analysis to get azimuth angles to the source from the geophones. Combining the azimuths with  $r_s$  then gives estimates for  $(x_s, y_s)$ . Details for hodogram/azimuth analysis can be found in Han et al. (2009). The current discussion is limited to finding  $(r_s, z_s)$  using inversion of first-arrival times.

The actual time of occurrence  $t_0$  of a microseismic event is unknown, and is an extra parameter that must be found if true first-arrival times are used as the basis for the inversion scheme. Instead, we used the reduced arrival times  $t_{obs}(i) - \min(t_{obs}(i))$  and  $t_{cal}(i) - \min(t_{cal}(i))$  for the inversion. This simple adjustment means that arrival-time moveouts rather than absolute arrival times are the basis for locating the hypocenter, eliminating the need to know the event time  $t_0$ . The moveouts must be large enough so that they exceed any time-picking errors. They will contain sufficient geometric information for locating the hypocenter if the angles subtended by the geophone arrays relative to the source span a range of  $\pm 20$  degrees or more.

For our test, the input data were arrival times from a microseism located in the velocity model of Figure 1 at  $(x_s, y_s, z_s) = (0, 0, 600\text{m})$ . The times were observed at 36 geophones separated by 20m in a vertical observation well located at  $r=520\text{m}$ . Figure 3 and Table 3 summarize the results of using the LM method with reduced arrival times to locate a hypocenter. As is shown on Figure 3(b), after 200 iterations, the calculated reduced arrival times (yellow dots) matched the observed reduced arrival times (blue dots) almost perfectly.

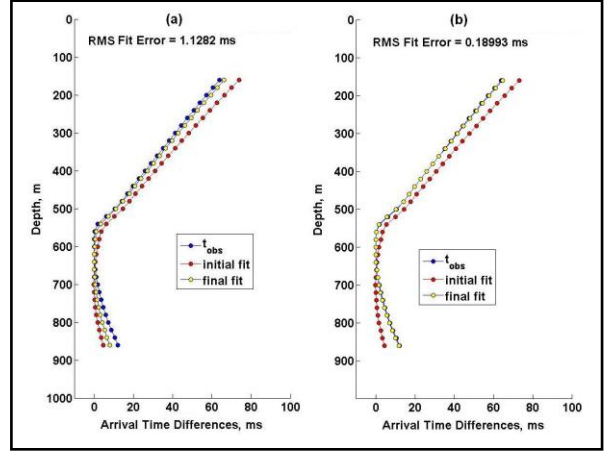


Figure 3: Reduced arrival times for the borehole array after (a) 30, and (b) 200 iterations of the LM optimization algorithm. The RMS difference between observed times and the calculated times after 200 iterations is about 0.19ms.

Table 3: Summary of modified LM inversion results for reduced arrival times from geophones deployed in a well..

Iterations	$x_s$ (m)	$y_s$ (m)	$z_s$ (m)	Error, ms
Initial guess	540	-20	700	3.14
30	581	-61	641	1.13
200	514	-6.5	606	0.19
True values	520	0	600	0

### Hypocenter location using times from surface arrays

Figure 4 shows an array of 36 geophones located on the surface of a layered earth. The geophones spacing is 50m, and the array spans distances of about 250 meters in both the x and y directions. The array is approximately centered about the treatment well. A microseismic hypocenter located in the vicinity of the well at a depth of 600m is shown in green. In a real-world monitoring situation, hundreds of geophones may be deployed, either along lines radiating from the treatment well, or on a rectangular grid (Chambers et al., 2008; Lakings et al., 2006). The shooting method was used to generate a set of “observed” arrival times  $t_{obs}$  assuming the velocity structure on Figure 5. We then assumed that the source coordinates are unknown, and used the LM algorithm with the reduced arrival times to estimate them through inversion.

## Hypocenter location by nonlinear optimization

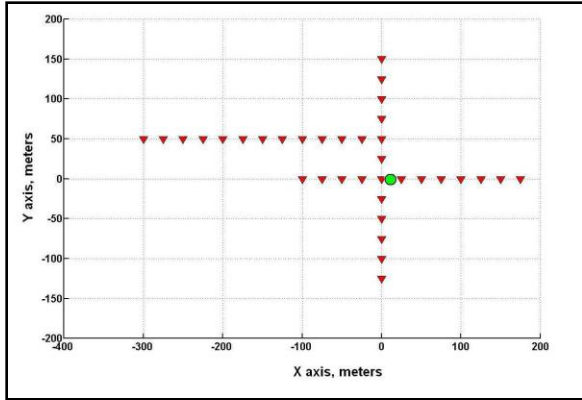


Figure 4: Plan view of a surface array of geophones for microseismic monitoring. The subsurface microseismic source, located at a depth of 600m, is shown in green.

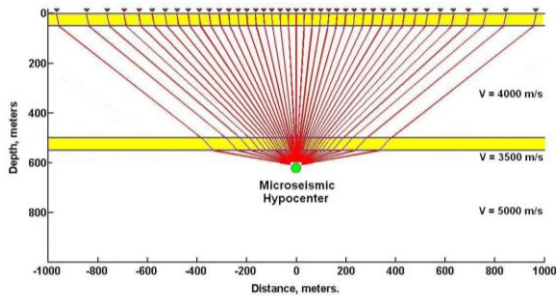


Figure 5: Ray-tracing from a microseism propagating into a surface array of geophones. Low-velocity zones are shown in yellow; the overburden velocity is 1000m/s

Table 4: Summary of LM inversion results for surface array data

Iterations	$x_s$ (m)	$y_s$ (m)	$z_s$ (m)	Error, ms
Initial guess	100	-155	900	6.31
17	21.6	-36	731	1.10
30	12.3	-0.5	613	0.05
True values	10	0	600	0

The LM inversion results are summarized on Table 4 and Figure 6. Figure 6(b) shows that after 30 iterations, the inversion procedure returned source coordinates for which the calculated arrival times matched the observed arrival times within a very small RMS difference.

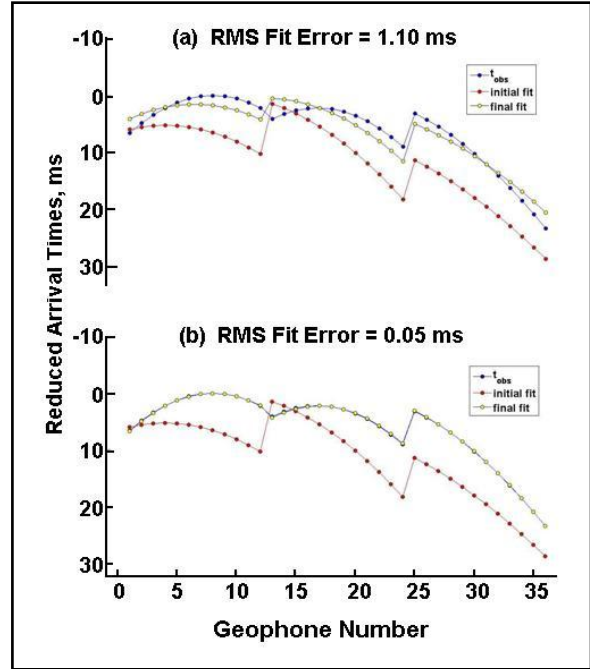


Figure 6: LM inversion of reduced arrival times for the surface array. Blue dots are the observed times; red dots are calculated times from the initial guess for the source coordinates; yellow dots are calculated times from the current source coordinates; results after (a) 17 iterations, and (b) after 30 iterations.

## Conclusion

The genetics algorithm (GA), pattern search (PS), and the Levenberg-Marquardt (LM) algorithm were used successfully to invert reduced arrival times for velocity calibration and hypocenter location. Of the two direct search methods, PS was faster and more efficient than GA. LM inversion is able to locate hypocenter coordinates, but it suffers from issues common to all gradient-based optimization methods. It can get trapped in local minima far from the desired true minimum, and if the objective function exhibits “data valleys”, saddle points, and inflection points (which are all characterized by zero or near-zero gradients), convergence to a global minimum can be extremely slow.

Based on our experience, we conclude that pattern search likely is the best choice for both microseismic hypocenter location and velocity calibration.

## Acknowledgements

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