Non-welded contact interface with media

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Outline

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- Long wavelength assumption

  -Estimate moduli for the linear slip interface
- Group theory

  -Fractured media decompose into fracture medium and background medium
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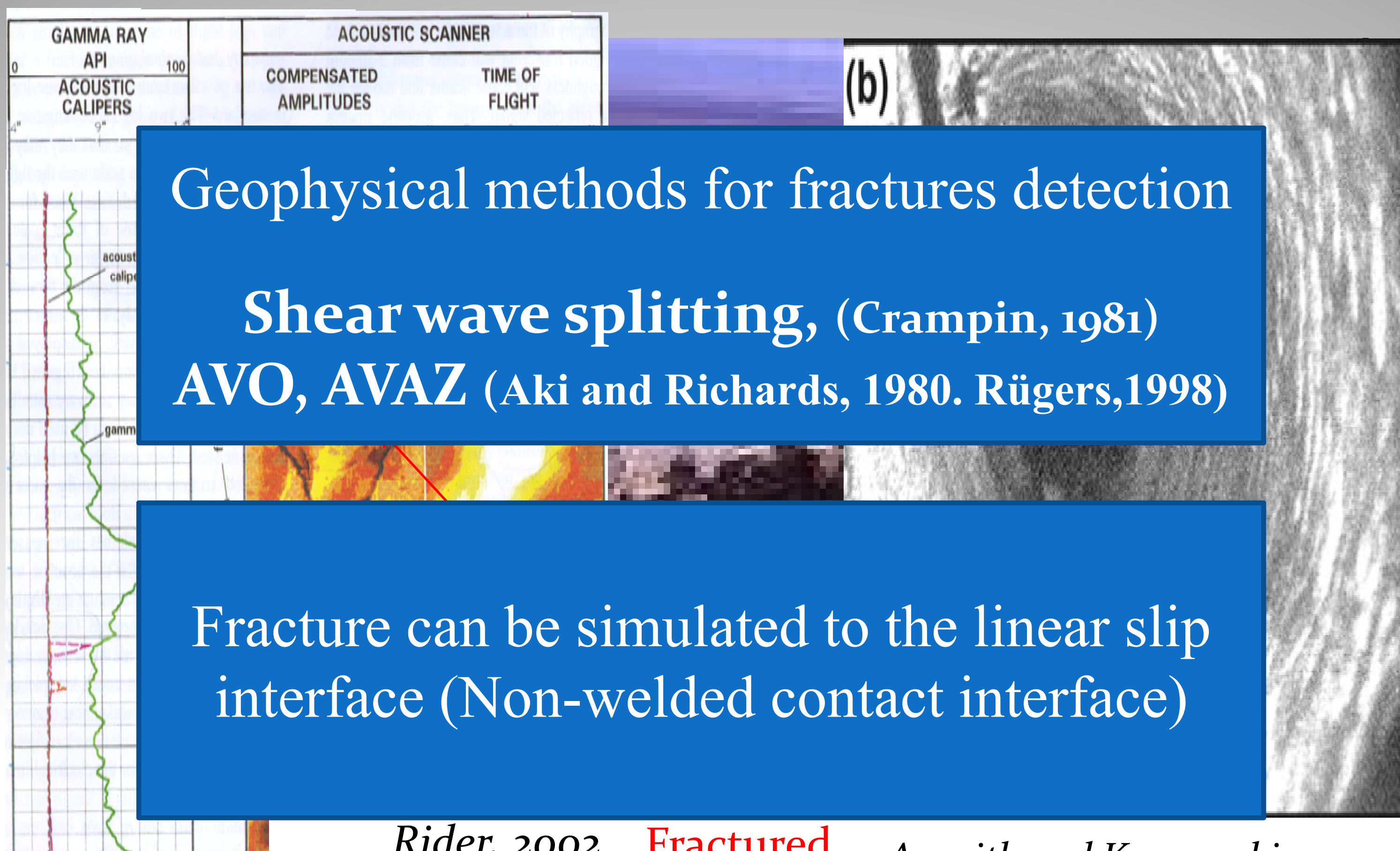
Introduction



Geophysical methods for fractures detection

Shear wave splitting, **(Crampin, 1981)**
AVO, AVAZ **(Aki and Richards, 1980. Rügers,1998)**

Fracture can be simulated to the linear slip interface (Non-welded contact interface)

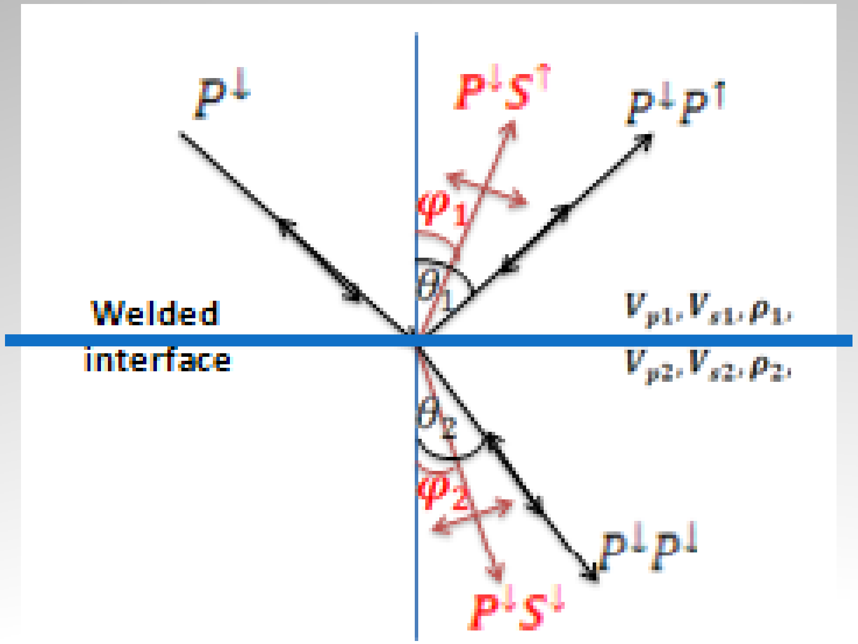
Rider, 2002   Fractured media   *Asquith and Krygowski, 2004*

Introduction

- Schoenberg developed the theory of non-welded contact interface. (1980)
- Pyrak-Nolte has confirmed non-welded contact interface theory by laboratory measurements (1990)
- Chaisri and Krebes obtained the solutions of the reflection and transmission coefficients for non-welded contact interface embedded in isotropic media. (2000)
- Slawinski and Krebes modeled SH wave propagation in non-welded contact media. (2002)
- Hood,1989, 1991; Carcione,1996; Hsu, 1993; Coates, 1995.

Mechanism of the interface contacts with media

A welded contact interface



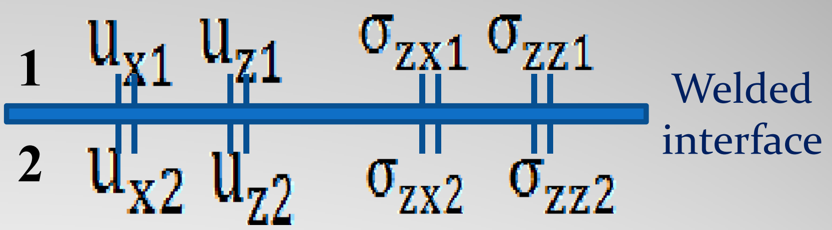
AVO and AVAZ detected fractures using amplitude variation versus offset and azimuth (Aki and Richards, 1980, Rügers, 1998)

Mechanism of the interface contacts with media

A welded contact interface boundary conditions (P-SV)

1) **The continuity displacements**

$$u_{x1} = u_{x2} \qquad u_{z1} = u_{z2}$$



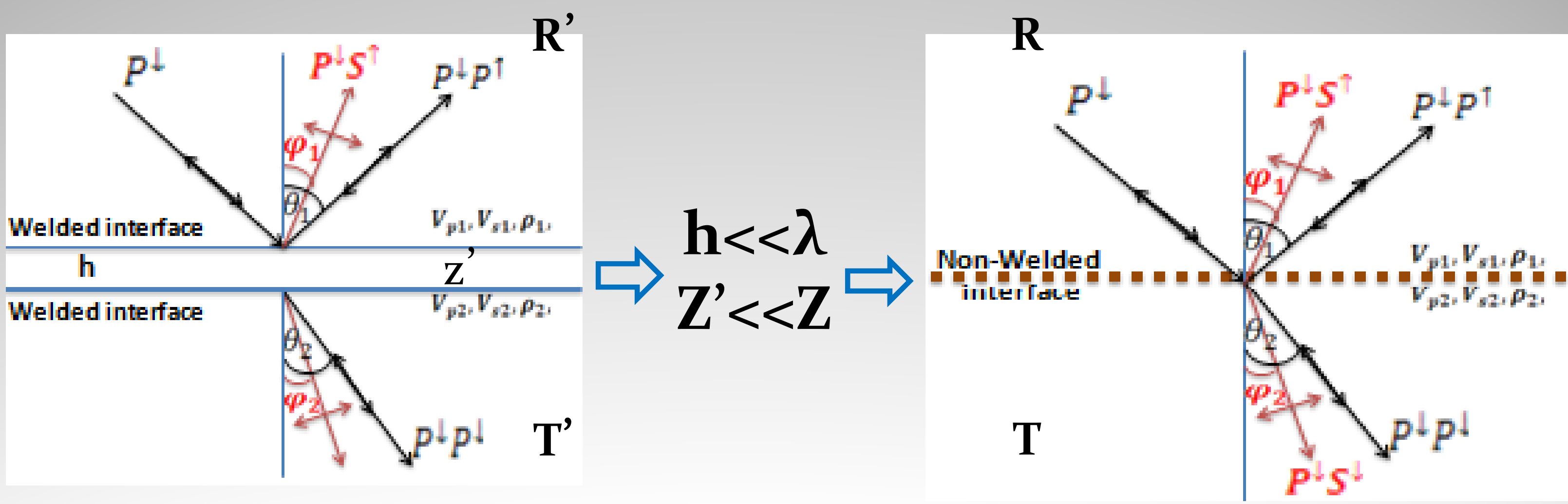
2) **The continuity stresses**

$$\sigma_{zx1} = \sigma_{zx2} \qquad \sigma_{zz1} = \sigma_{zz2}$$

$$\sigma_{zx} = \mu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \qquad \sigma_{zz} = \lambda\frac{\partial u_x}{\partial x} + (\lambda + 2\mu)\frac{\partial u_z}{\partial z}$$

Mechanism of the interface contacts with media

A non-welded contact interface



Pyrak-Nolte (1990) has confirmed non-welded contact interface theory by laboratory measurements.

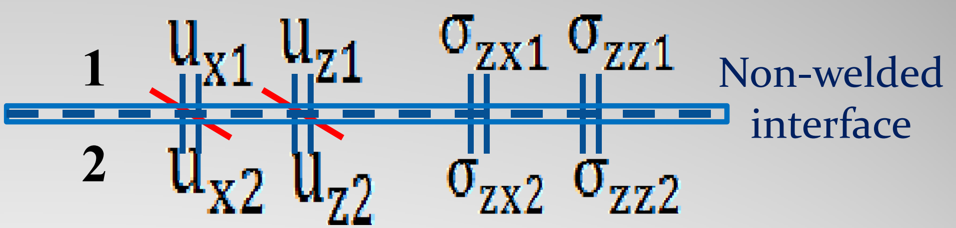
Mechanism of the interface contacts with media

An non-welded interface contact boundary conditions (P-SV)

1) The discontinuity displacements

$$u_{x1} - u_{x2} = S_x \sigma_{zx}$$

$$u_{z1} - u_{z2} = S_z \sigma_{zz}$$



2) The continuity stresses

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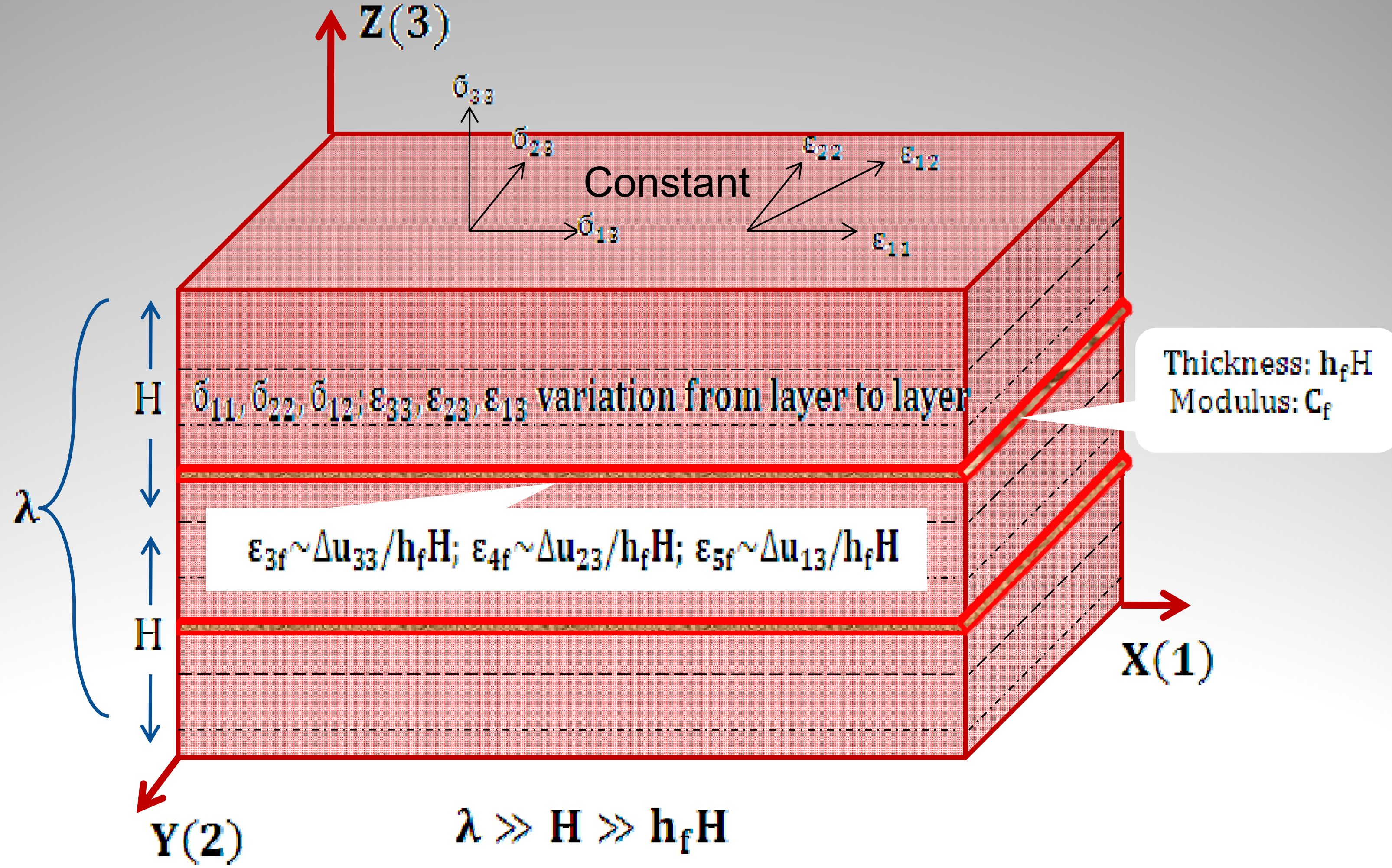
Long wavelength assumption

Estimate elastic moduli for the linear slip interface

- Finely layered medium behaves approximately as TI medium
- Some Strains of fine layer can be expressed in terms of thickness-weighted average
- An equivalent TI media can replace the fractured media (the linear slip interface embedded in background medium)

Long wavelength assumption

Estimate elastic moduli for the linear slip interface

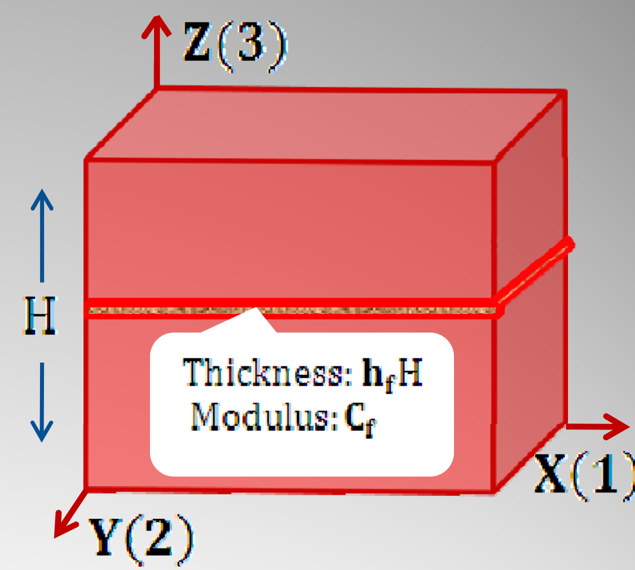


Long wavelength assumption

Estimate elastic moduli for the linear slip interface

Elastic moduli:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix}=\begin{bmatrix}C11 & C12 & C13 & C14 & C15 & C16\\ C21 & C22 & C23 & C24 & C25 & C26\\ C31 & C32 & C33 & C34 & C35 & C36\\ C41 & C42 & C43 & C44 & C45 & C46\\ C51 & C52 & C53 & C54 & C55 & C56\\ C61 & C62 & C63 & C64 & C65 & C66\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}$$



The stress tractions across the fracture:

$$\sigma=\begin{bmatrix}\sigma_3\\ \sigma_4\\ \sigma_5\end{bmatrix}=\left(h_f\begin{bmatrix}C13 & C23 & C36\\ C14 & C24 & C46\\ C15 & C25 & C56\end{bmatrix}\cancel{\begin{bmatrix}\varepsilon 1\\ \varepsilon 2\\ \varepsilon 6\end{bmatrix}}+\begin{bmatrix}C33 & C34 & C35\\ C34 & C44 & C45\\ C35 & C54 & C55\end{bmatrix}\begin{bmatrix}\varepsilon_{3f}\\ \varepsilon_{4f}\\ \varepsilon_{5f}\end{bmatrix}\right)$$

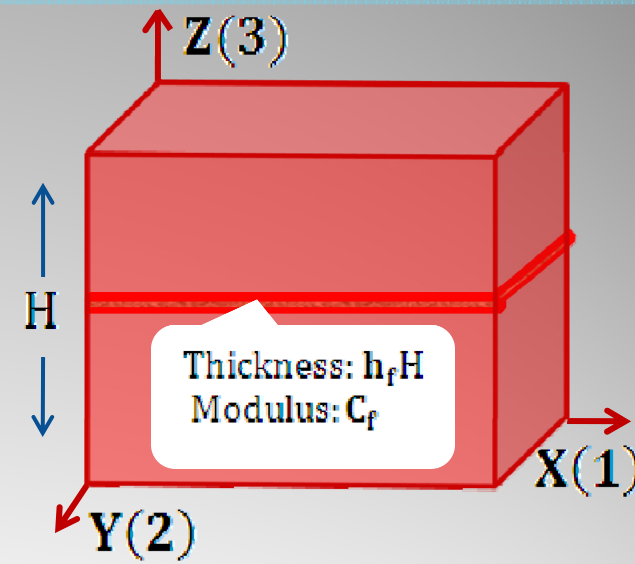
Long wavelength assumption
Strains are same as background

$$h_f \rightarrow 0 \qquad \begin{bmatrix}\varepsilon_{3f}\\ \varepsilon_{4f}\\ \varepsilon_{5f}\end{bmatrix}=\begin{bmatrix}\Delta U_{3f}/H\\ \Delta U_{4f}/H\\ \Delta U_{5f}/H\end{bmatrix}$$

Long wavelength assumption

Estimate elastic moduli for the linear slip interface

The relationship of stresses and strains in fracture system (linear slip interface)



$$\begin{bmatrix} \Delta U_{3f}/H \\ \Delta U_{4f}/H \\ \Delta U_{5f}/H \end{bmatrix} = Z \begin{bmatrix} \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{bmatrix}$$

The transverse isotropic fracture system (linear slip interface) compliance matrix

$$Z = \begin{bmatrix} Z_N & 0 & 0 \\ 0 & Z_T & 0 \\ 0 & 0 & Z_T \end{bmatrix}$$

Group theory

Fractured media decomposed into fracture medium and background medium

- elastic moduli be mapped to the elements of group
- fractured media are originated by fracture sum background media
- fractured media can decomposed into background medium and fracture.

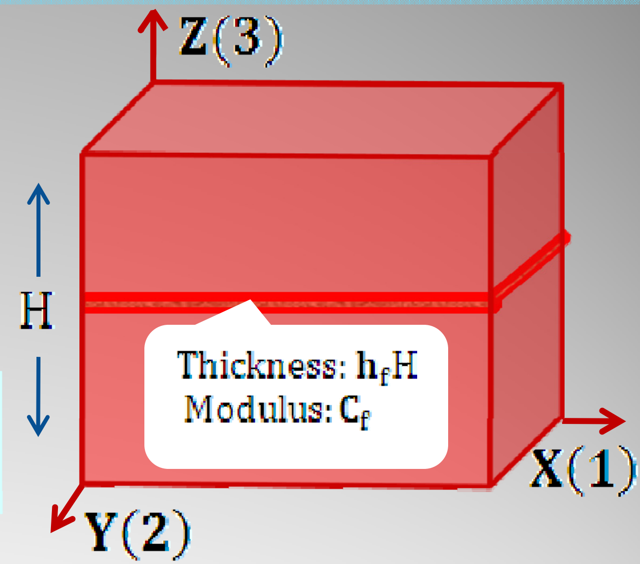
Group theory

Fractured media decomposed into fracture medium and background medium

Fractured medium moduli transform

to a commutative group

$$[H, H\rho, g(3), g(4), g(5),]$$



$$g(3) = H\begin{bmatrix} 1/C33 & 0 & 0 \\ 0 & 1/C44 & 0 \\ 0 & 0 & 1/C44 \end{bmatrix} \quad g(4) = H\frac{C13}{C33}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g(5) = H\left\{\begin{bmatrix} C11 & C11-2C66 & 0 \\ C11-2C66 & C11 & 0 \\ 0 & 0 & C66 \end{bmatrix}\right\} - \frac{C13}{C33}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Subtracting group element of fracture $g_f(3)$, from the fractured medium group

$$g_f(3) = \begin{bmatrix} Z_N & 0 & 0 \\ 0 & Z_T & 0 \\ 0 & 0 & Z_T \end{bmatrix}$$

$$[H, H\rho, g(3) - g_f(3), g(4), g(5),]$$

Group theory

Fractured media decomposed into fracture medium and background medium

The background medium stiffness moduli 6x6 matrix

$$\begin{bmatrix}
C11+\frac{C13^2 Z_N}{1-C33Z_N} & C11+2C66\frac{C13^2 Z_N}{1-C33Z_N} & \frac{C13}{1-C33Z_N} & 0 & 0 & 0 \\
C11+2C66\frac{C13^2 Z_N}{1-C33Z_N} & C11+\frac{C13^2 Z_N}{1-C33Z_N} & \frac{C13}{1-C33Z_N} & 0 & 0 & 0 \\
\frac{C13}{1-C33Z_N} & \frac{C13}{1-C33Z_N} & \frac{C13}{1-C33Z_N} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C44}{1-C44Z_T} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{C44}{1-C44Z_T} & 0 \\
0 & 0 & 0 & 0 & C65 & C66
\end{bmatrix}$$

$$\text{Fractured media} \leftarrow \text{Fracture} + \text{background}$$

$$\text{Fractured media} - \text{Fracture} \rightarrow \text{background}$$

Group theory

Fractured media decomposed into fracture and background medium

Fracture compliances with a isotropic background medium:

$$Z_T = \frac{1}{C44} - \frac{1}{C66} \qquad Z_N = \frac{1}{C33}\left[1 - \frac{C33 - C13}{2C66}\right]$$

$$E_T = \mu Z_T = \frac{C66}{C44} - 1 \qquad E_N = (2\mu + \lambda) Z_N = \frac{2C66}{C33 - C13} - 1$$

$$S_T = \frac{E_T H}{\mu} \qquad S_N = \frac{E_N H}{2\mu + \lambda}$$

Conclusion

- The linear slip interface (non-welded contact) exhibits displacement discontinuity and stress continuity inherent in the boundary conditions.
- In the long wavelength assumption, the linear slip interface can simulate the fracture and moduli can be derived so that fractured medium can be described with four parameters
  μ, λ, E_N and E_T .
- Group theory can decompose fractured media into a fracture medium and a background medium.

Future work

- Model horizontal and vertical fractures (linear slip interface) embedded in isotropic or anisotropic background media.

- Study a medium moduli of the non-linear slip interface by considering the viscosity parameter.

- Study a case of fractured media-wormholes caused by cold heavy oil production.

References

- Aki, K., and Richards, P. G., 1980, Quantitative seismology, theory and methods, volume 1: W H Freeman and Co, Cambridge Press, 144 – 154.
- Crampin, S. 1985, Evidence for aligned cracks in the Earth's crust:First Break, 3, no. 3, 12-15.
- Pyrak-Nolte., L.J., L.R. Myer. And N.G.W. Cook (1990b). Transmission os seismic wave across single natural fracture: J. Geophys. Res. 95. 8617-8638.
- Coates, R.T.; and Schoenberg, M.; 1995, "Finite-difference modeling of faults and fractures", *Geophysics*, 60, 1514-1526.
- Hood and Schoenberg, 1989; Estimation of vertical fracturing from measured elastic moduli: *Journal of Geophysical Research*, 94, 15,611–15,618.
- Hsu, C.-J., and Schoenberg, M., 1993, Elastic waves through a simulated fractured medium: *Geophysics*, 58, 964–977.

- Pyrak-Nolte., L.J., L.R. Myer. And N.G.W. Cook (1990b). Transmission os seismic wave across single natural fracture: J. Geophys. Res. 95. 8617-8638.

- Raphael A. Slawinski and Edward S. Krebes,2000, Finite-difference modeling of SH-wave propagation in nonwelded contact media: *Geophysics*, vol 67, No5.

References

- Ruger, Andreas. 1998, Variation of P-wave reflectivity with offset and azimuth in anisotropic media: Geophysics 63, 935.
- Schoenberg, M.; 1980, Elastic wave behavior across linear slip interfaces, *J. Acoust. Soc. Am.*, 68, 1516-1521.
- Schoenberg, M.; and Douma, J.; 1988, Elastic wave propagation in media with parallel fractures and aligned cracks, *Geophysical Prospecting*, 36, 571-590.
- Schoenberg, M.; and Muir, F.; 1989, A calculus for finely layered anisotropic media, *Geophysics*, 54, 581–589.
- Schoenberg, M.; and Sayers, C.M.; 1995, Seismic anisotropy of fractured rock, *Geophysics*, 60, 204-211.

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Thank you!