Velocity-Stress Finite-Difference Modeling of Poroelastic Wave Propagation

Shahin Moradi

and

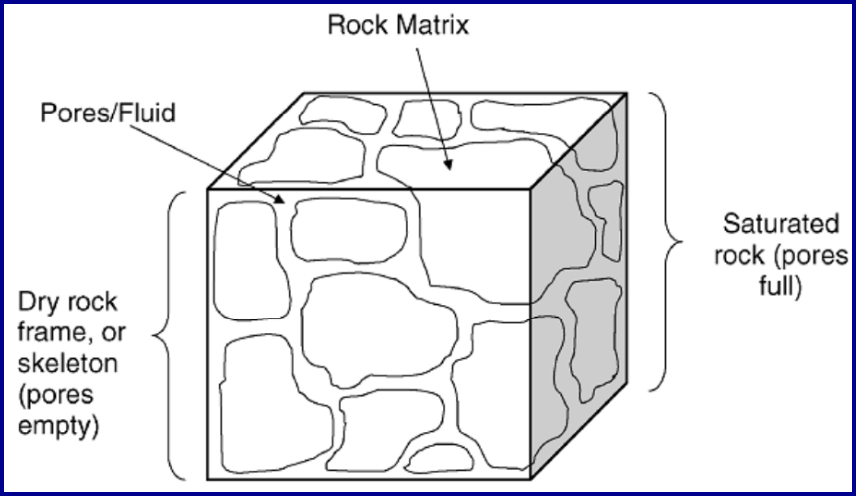
Don Lawton

Outline

- Introduction
- Biot's Theory
- Staggered-Grid Finite Difference
- Numerical Examples
- Conclusion
- Acknowledgement

Introduction

- Poroelastic Medium



(Russell et al., 2003)

- Biot (1962): anelastic effects from the relative movement of the fluid.
- Biot's theory: Important in oil and gas exploration, CO2 storage monitoring and hydrogeology.
- The Theory predicts two compressional waves and one shear wave.

Biot's Theory(1962)

Assumptions :

- Elastic rock frame
- Connected pores
- Seismic wavelength $\gg$ average pore size
- Small deformations
- Statistically isotropic medium

- Stress-Strain Relation For Porous Media (Biot, 1962)

Solid Stress $$\tau_{ij} = 2\mu e_{ij} + (\lambda_c e_{kk} + \alpha M \varepsilon_{kk})\delta_{ij}$$

Fluid Pressure $$P = -\alpha M e_{kk} - M\varepsilon_{kk}$$

$$\alpha = 1 - \frac{K_{Dry}}{K_{Solid}}$$

$$e_{ij} = \nabla . u = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$$

$$\varepsilon_{ij} = \nabla .(u - U)$$

$$M = [\frac{\phi}{K_{Fluid}} + \frac{(\alpha - \phi)}{K_{Solid}}]$$

Coupling Modulus

u :Solid Particle Displacement

U : Fluid Particle Displacement

λ & μ : Lame Parameters of the Saturated Rock.

- Equations of motion for a statistically isotropic porous media saturated with viscous fluid:

$$(m\rho - \rho_f^2)\frac{\partial^2 u_i}{\partial t^2} = m\frac{\partial \tau_{ij}}{\partial x_j} + \rho_f b\frac{\partial w_i}{\partial t} + \rho_f\frac{\partial P}{\partial x_i}$$

$$(m\rho - \rho_f^2)\frac{\partial^2 w_i}{\partial t^2} = -\rho_f\frac{\partial \tau_{ij}}{\partial x_j} - \rho b\frac{\partial w_i}{\partial t} - \rho\frac{\partial P}{\partial x_i}$$

Effective Fluid Density $m = T\frac{\rho_f}{\phi}$

ρ_f : Fluid Density

ρ : Density of Saturated Rock

Fluid Displacement Relative to the Solid $w = u - U$

Mobility $b = \eta/\kappa$

$\eta : Viscosity$

$\kappa : Permeability$

Substituting $\boxed{V = \frac{\partial u}{\partial t}}$ and $\boxed{W = \frac{\partial w}{\partial t}}$ in the equations of motion and taking derivatives with respect to time from both sides of the stress-strain relationship we have:

$$(m\rho - \rho_f^2)\frac{\partial V_i}{\partial t} = m\frac{\partial \tau_{ij}}{\partial x_j} + \rho_f bW + \rho_f \frac{\partial P}{\partial x_i}$$
$$(m\rho - \rho_f^2)\frac{\partial W_i}{\partial t} = -\rho_f \frac{\partial \tau_{ij}}{\partial x_j} - \rho bW - \rho \frac{\partial P}{\partial x_i}$$

and

$$\frac{\partial \tau_{ij}}{\partial t} = 2\mu \frac{\partial e_{ij}}{\partial t} + (\lambda_c \frac{\partial e_{kk}}{\partial t} + \alpha M \frac{\partial \varepsilon_{kk}}{\partial t})\delta_{ij}$$

$$\frac{\partial P}{\partial t} = -\alpha M \frac{\partial e_{kk}}{\partial t} - M \frac{\partial \varepsilon_{kk}}{\partial t}$$

- 2D case:

$$\frac{\partial \tau_{xx}}{\partial t} = (\lambda_c + 2\mu)\frac{\partial V_x}{\partial x} + \lambda_c(\frac{\partial V_z}{\partial z}) + \alpha M(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z}) \quad (1)$$

$$\frac{\partial \tau_{zz}}{\partial t} = (\lambda_c + 2\mu)\frac{\partial V_z}{\partial z} + \lambda_c(\frac{\partial V_x}{\partial x}) + \alpha M(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z}) \quad (2)$$

$$\frac{\partial \tau_{xz}}{\partial t} = \mu(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z}) \quad (3)$$

$$\frac{\partial P}{\partial t} = -\alpha M(\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z}) - M(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z}) \quad (4)$$

$$\frac{\partial V_x}{\partial t} = A(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}) + BW_x + C\frac{\partial P}{\partial x} \quad (5)$$

$$\frac{\partial V_z}{\partial t} = A(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z}) + BW_z + C\frac{\partial P}{\partial z} \quad (6)$$

$$\frac{\partial W_x}{\partial t} = D(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}) + EW_x + F\frac{\partial P}{\partial x} \quad (7)$$

$$\frac{\partial W_z}{\partial t} = D(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z}) + EW_z + F\frac{\partial P}{\partial z} \quad (8)$$

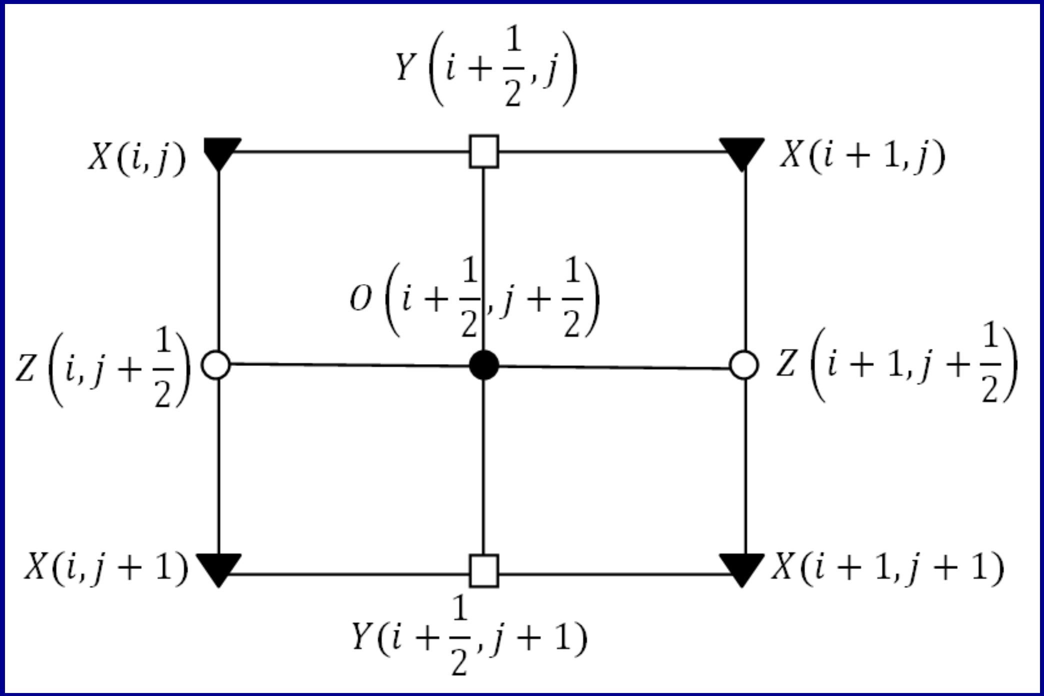
Staggered-Grid Finite Difference(Levander, 1988)

X : τ_{xx}, τ_{zz} and P

Y : V_x and W_x

Z : V_z and W_z

O: τ_{xz}

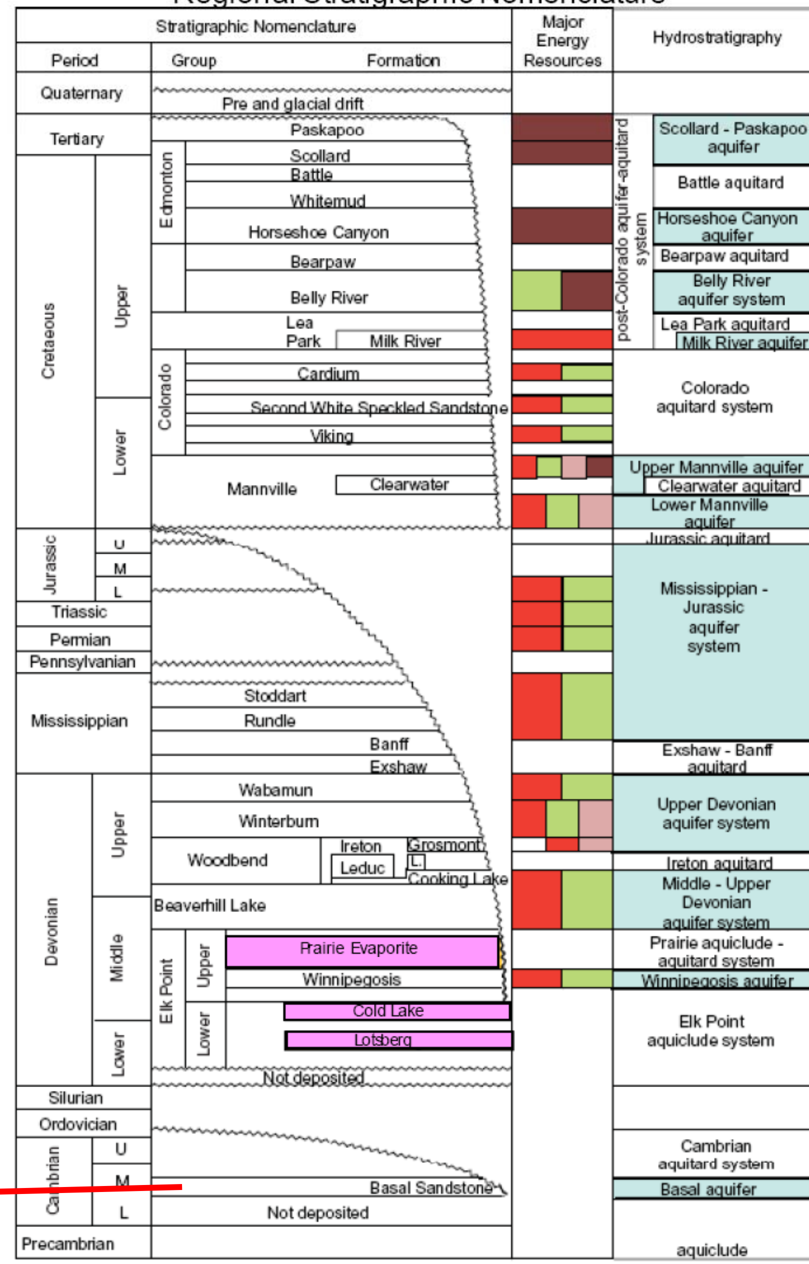


Numerical Examples

Single layer model based on QUEST Project

- CO_2 storage in Basal Cambrian Sands or BCS, which is a saline aquifer within Western Canadian Sedimentary Basin (WCSB)
- Data from well SCL-8-19-59-20W4

Regional Stratigraphic Nomenclature

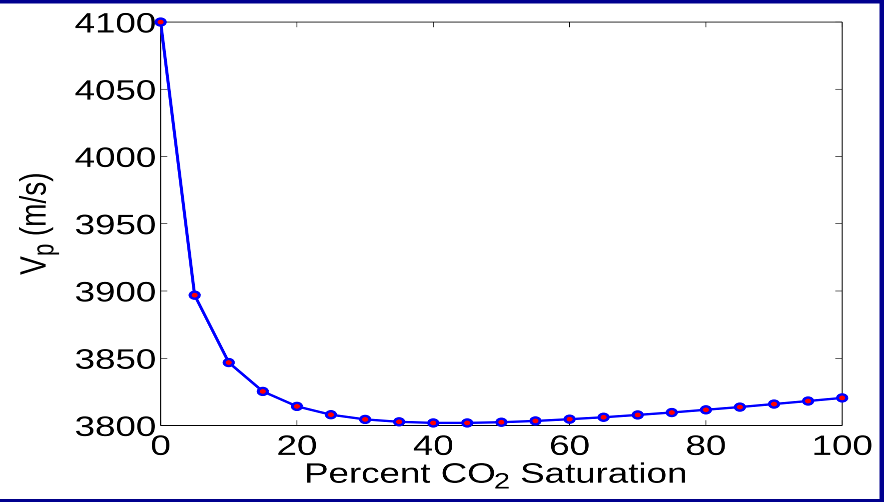


Quest

Modified after Bachu et al. 2000.

Single Layer Model

Gassmann Fluid Substitution



ρ_f	$937\ (kg/m^3)$
ρ	$2370\ (kg/m^3)$
V_p	$3800\ (m/s)$
V_s	$2400\ (m/s)$
ϕ	16%
κ	$1 (mD)$

BCS: 40% CO_2

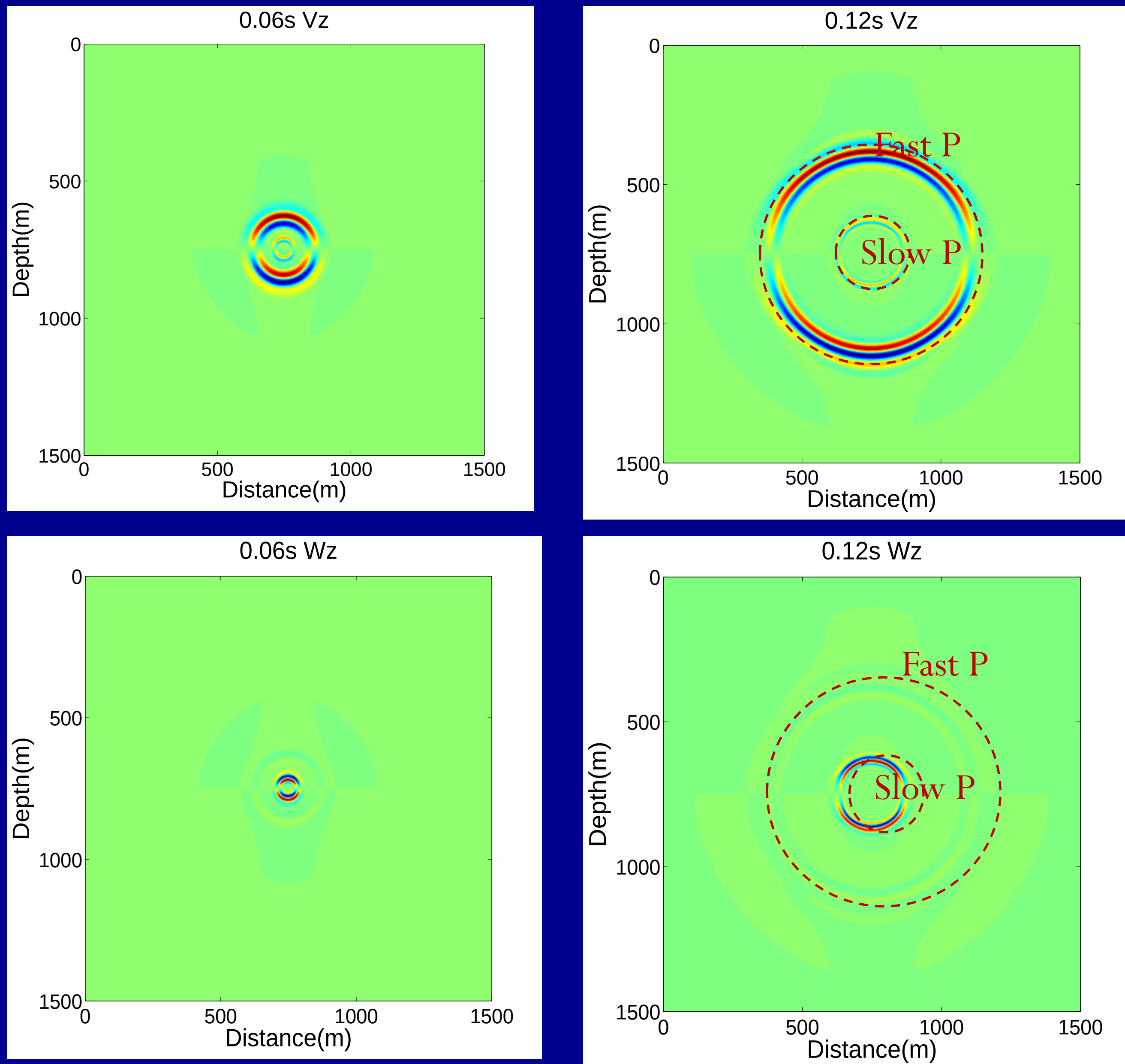
- Fourth order in space and second order in time.
- The stability condition is the same as the one in the elastic case (Zhu:1991)

$$\Delta t \leq \frac{h}{(V_p^2 - V_s^2)^{1/2}}$$

$$h = 3m \qquad dt = 0.2\ ms$$

- The size of the model was 1500 m by 1500 m
- Explosive source: Ricker wavelet with dominant frequency 50 Hz
- Source location : $(x, z) = (750, 750)m$

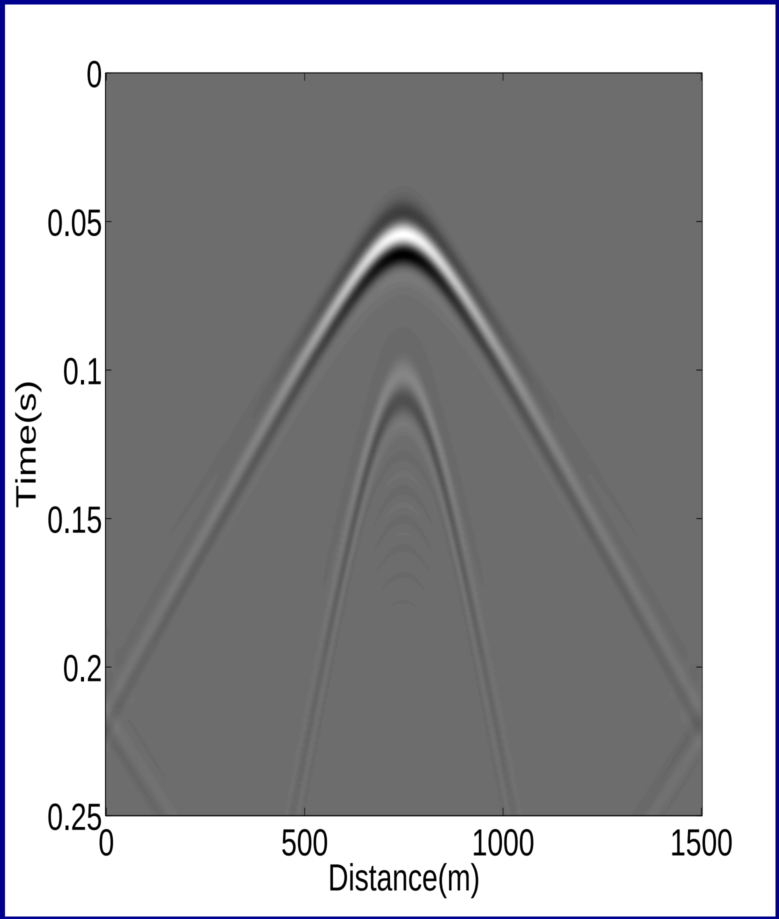
- Vertical Particle Velocity Snapshots:

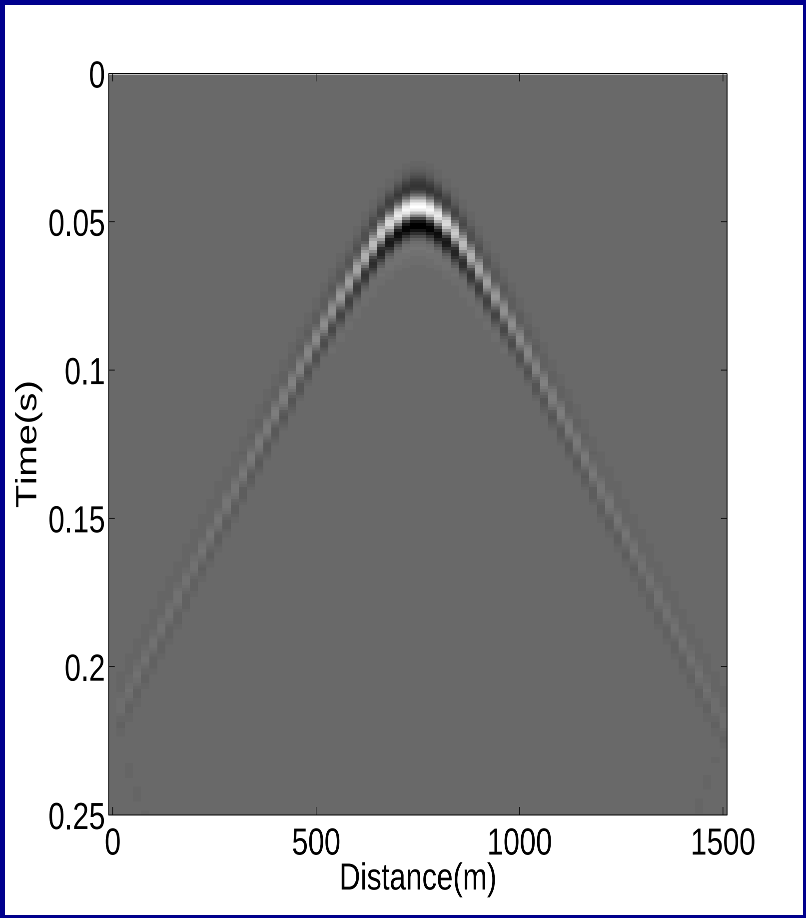


Solid

Fluid

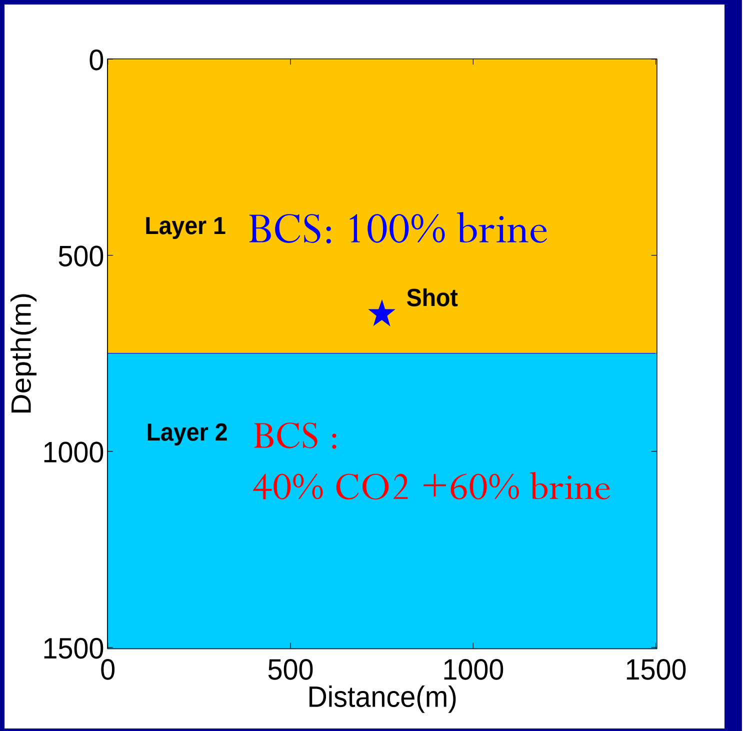
- Comparison with elastic algorithm

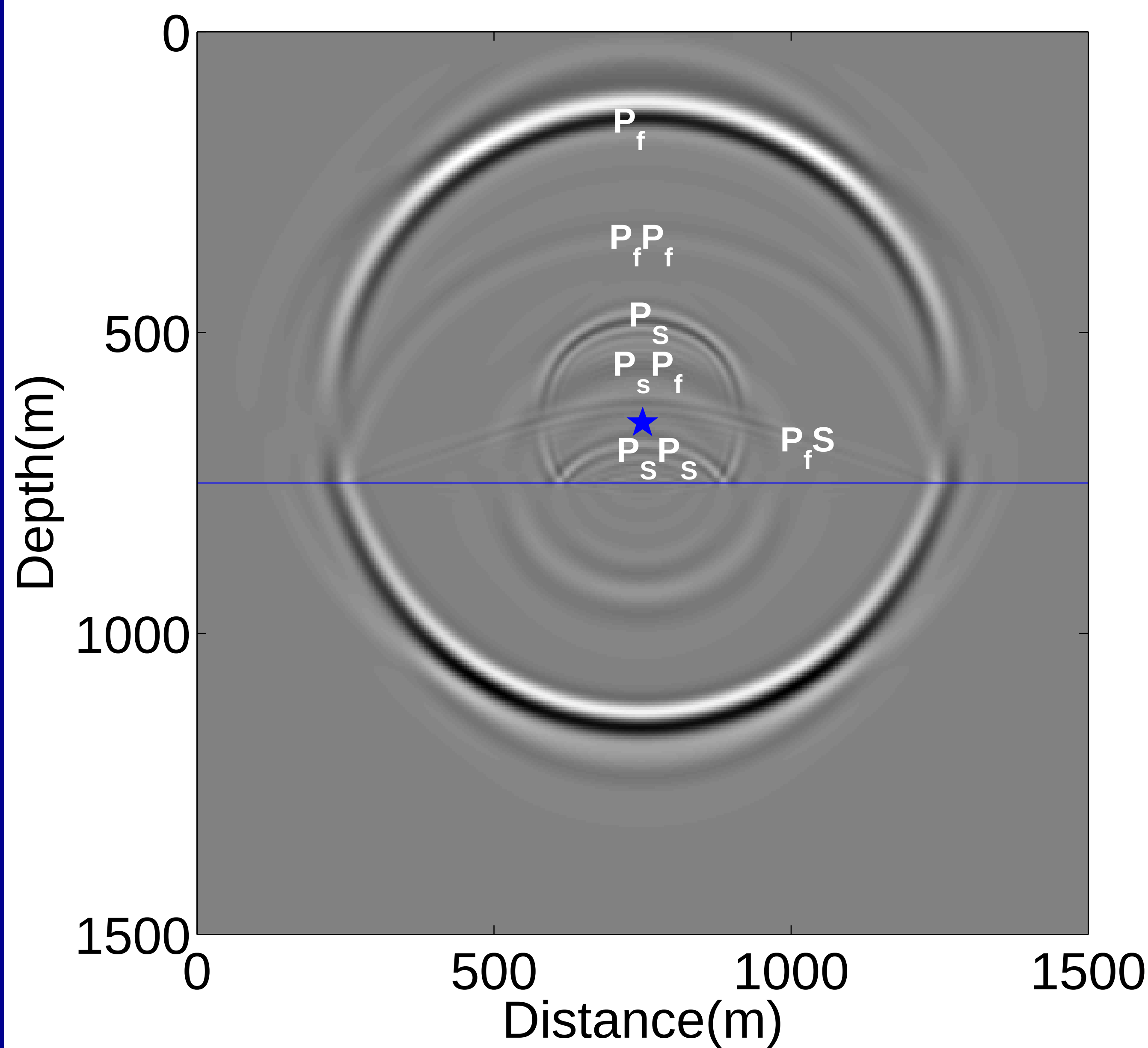




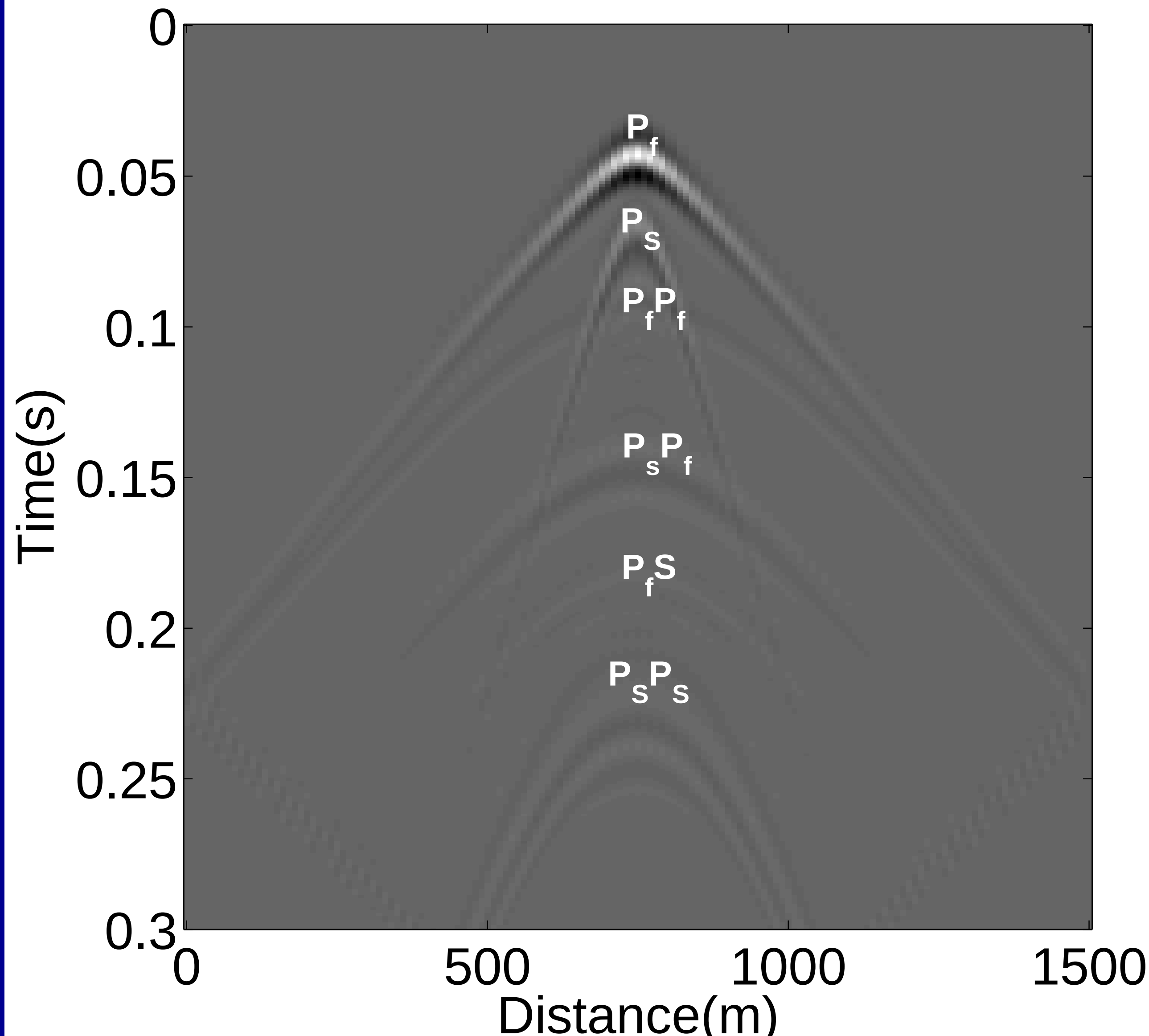
Two-Layered Model

	Top Layer	Bottom Layer
ρ_f	$1070\ (kg/m^3)$	$937\ (kg/m^3)$
ρ	$2400\ (kg/m^3)$	$2370\ (kg/m^3)$
V_p	$4100 (m/s)$	$3800\ (m/s)$
V_s	$2390 (m/s)$	$2400\ (m/s)$
ϕ	16%	16%
κ	$1(mD)$	$1(mD)$





0
500
1000
1500
Depth(m)
0
500
1000
1500
Distance(m)
P f
P f P f
P S
P s P f
P S P S
P f S


0
0.05
0.1
0.15
0.2
0.25
0.3
Time(s)
0
500
1000
1500
Distance(m)
P f
P s
P f P f
P s P f
P f S
P s P s

Conclusion and Future Goals

- The Poroelastic algorithm Generates slow compressional wave as predicted by Biot's theory.
- At a poroelastic boundary the slow P-wave is converted to a fast P-wave.
- The algorithm handles layered models and should be examined for more complex models.
- The algorithm could be used for inversion to obtain porous media properties that are ignored in elastic algorithms.

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