

Velocity-Stress Finite-Difference Modeling of Poroelastic Wave Propagation

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Carbon
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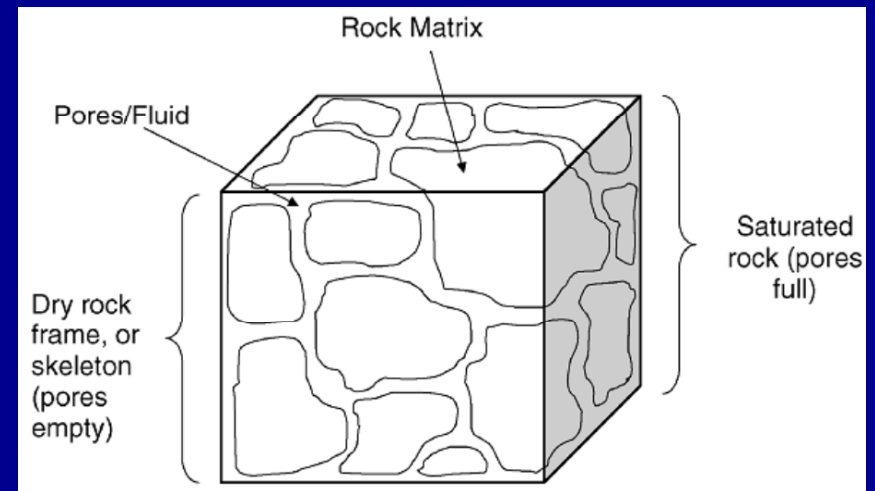
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Outline

- Introduction
- Biot's Theory
- Staggered-Grid Finite Difference
- Numerical Examples
- Conclusion
- Acknowledgement

Introduction

- Poroelastic Medium



(Russell et al., 2003)

- Biot (1962): anelastic effects from the relative movement of the fluid.
- Biot's theory: Important in oil and gas exploration, CO₂ storage monitoring and hydrogeology.
- The Theory predicts two compressional waves and one shear wave.

Biot's Theory(1962)

Assumptions :

- Elastic rock frame
- Connected pores
- Seismic wavelength \gg average pore size
- Small deformations
- Statistically isotropic medium

- Stress-Strain Relation For Porous Media (Biot, 1962)

Solid Stress $\tau_{ij} = 2\mu e_{ij} + (\lambda_c e_{kk} + \alpha M \varepsilon_{kk}) \delta_{ij}$

Fluid Pressure $P = -\alpha M e_{kk} - M \varepsilon_{kk}$

$$e_{ij} = \nabla \cdot u = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\varepsilon_{ij} = \nabla \cdot (u - U)$$

$$\alpha = 1 - \frac{K_{Dry}}{K_{Solid}}$$

$$M = \left[\frac{\phi}{K_{Fluid}} + \frac{(\alpha - \phi)}{K_{Solid}} \right]$$

Coupling Modulus

u : Solid Particle Displacement
 U : Fluid Particle Displacement

λ & μ : Lamé Parameters
of the Saturated
Rock.

- Equations of motion for a statistically isotropic porous media saturated with viscous fluid:

$$(m\rho - \rho_f^2) \frac{\partial^2 u_i}{\partial t^2} = m \frac{\partial \tau_{ij}}{\partial x_j} + \rho_f b \frac{\partial w_i}{\partial t} + \rho_f \frac{\partial P}{\partial x_i}$$

$$(m\rho - \rho_f^2) \frac{\partial^2 w_i}{\partial t^2} = -\rho_f \frac{\partial \tau_{ij}}{\partial x_j} - \rho b \frac{\partial w_i}{\partial t} - \rho \frac{\partial P}{\partial x_i}$$

Effective
Fluid Density

$$m = T \frac{\rho_f}{\phi}$$

ρ_f : Fluid Density

ρ : Density of Saturated
Rock

Fluid Displacement
Relative to the Solid

$$w = u - U$$

Mobility
 $b = \eta / \kappa$

η : *Viscosity*

κ : *Permeability*

Substituting $V = \frac{\partial u}{\partial t}$ and $W = \frac{\partial w}{\partial t}$ in the equations of motion and taking derivatives with respect to time from both sides of the stress-strain relationship we have:

$$\begin{aligned} (m\rho - \rho_f^2) \frac{\partial V_i}{\partial t} &= m \frac{\partial \tau_{ij}}{\partial x_j} + \rho_f b W + \rho_f \frac{\partial P}{\partial x_i} \\ (m\rho - \rho_f^2) \frac{\partial W_i}{\partial t} &= -\rho_f \frac{\partial \tau_{ij}}{\partial x_j} - \rho b W - \rho \frac{\partial P}{\partial x_i} \end{aligned}$$

and

$$\frac{\partial \tau_{ij}}{\partial t} = 2\mu \frac{\partial e_{ij}}{\partial t} + \left(\lambda_c \frac{\partial e_{kk}}{\partial t} + \alpha M \frac{\partial \varepsilon_{kk}}{\partial t} \right) \delta_{ij}$$

$$\frac{\partial P}{\partial t} = -\alpha M \frac{\partial e_{kk}}{\partial t} - M \frac{\partial \varepsilon_{kk}}{\partial t}$$

- 2D case:

$$\frac{\partial \tau_{xx}}{\partial t} = (\lambda_c + 2\mu) \frac{\partial V_x}{\partial x} + \lambda_c \left(\frac{\partial V_z}{\partial z} \right) + \alpha M \left(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \quad (1)$$

$$\frac{\partial \tau_{zz}}{\partial t} = (\lambda_c + 2\mu) \frac{\partial V_z}{\partial z} + \lambda_c \left(\frac{\partial V_x}{\partial x} \right) + \alpha M \left(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \quad (2)$$

$$\frac{\partial \tau_{xz}}{\partial t} = \mu \left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z} \right) \quad (3)$$

$$\frac{\partial P}{\partial t} = -\alpha M \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} \right) - M \left(\frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \quad (4)$$

$$\frac{\partial V_x}{\partial t} = A \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) + BW_x + C \frac{\partial P}{\partial x} \quad (5)$$

$$\frac{\partial V_z}{\partial t} = A \left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) + BW_z + C \frac{\partial P}{\partial z} \quad (6)$$

$$\frac{\partial W_x}{\partial t} = D \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) + EW_x + F \frac{\partial P}{\partial x} \quad (7)$$

$$\frac{\partial W_z}{\partial t} = D \left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) + EW_z + F \frac{\partial P}{\partial z} \quad (8)$$

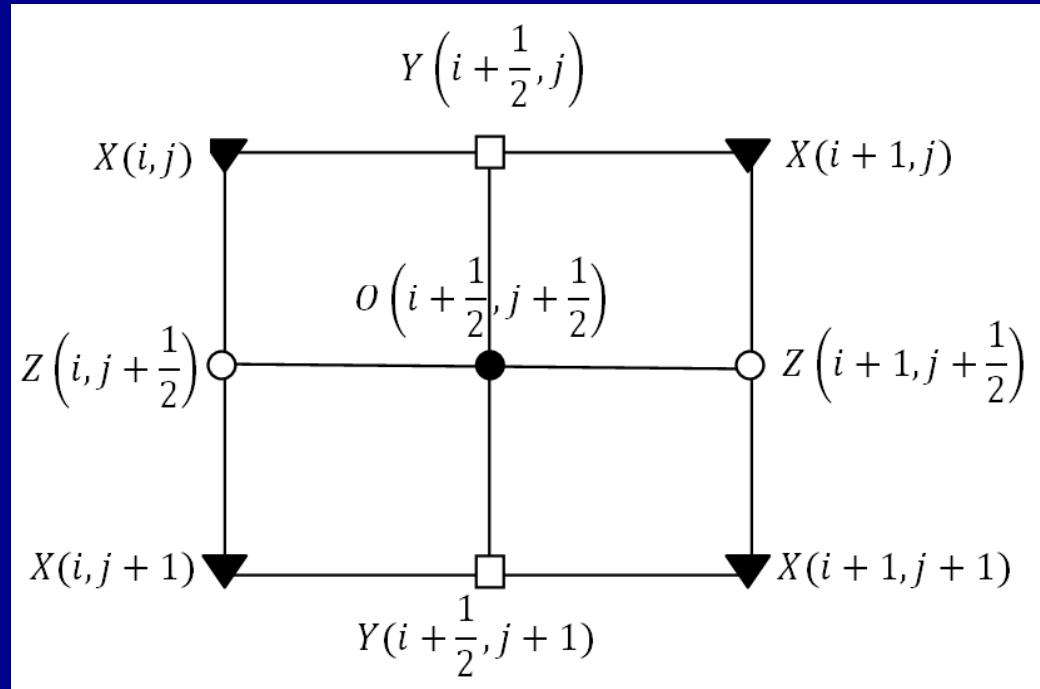
Staggered-Grid Finite Difference(Levander, 1988)

$X : \tau_{xx}, \tau_{zz}$ and P

$Y : V_x$ and W_x

$Z : V_z$ and W_z

$O : \tau_{xz}$



Numerical Examples

Single layer model based on QUEST Project

- CO₂ storage in Basal Cambrian Sands or BCS, which is a saline aquifer within Western Canadian Sedimentary Basin (WCSB)
- Data from well SCL-8-19-59-20W4

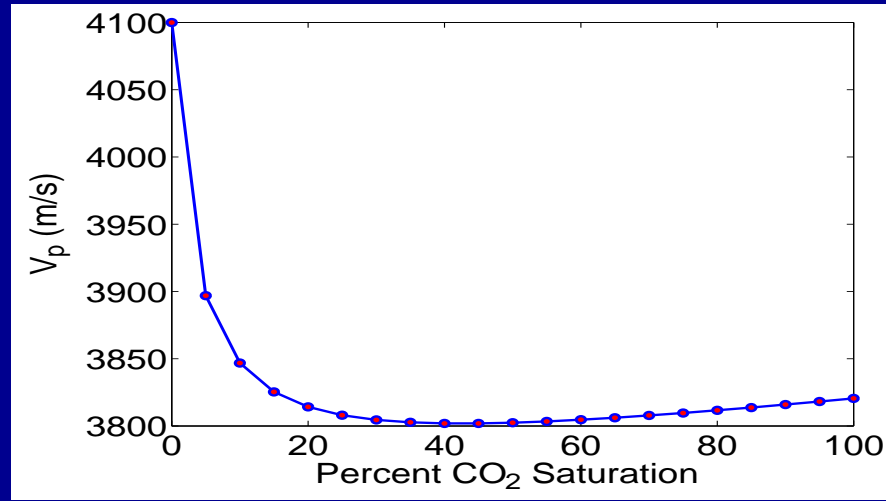
Quest

Stratigraphic Nomenclature			Major Energy Resources	Hydrostratigraphy	
Period	Group	Formation			
Quaternary	Pre and glacial drift				
Tertiary	Cretaceous	Upper	Paskapoo	post-Colorado aquifer-aquitard system	Scollard - Paskapoo aquifer
			Scollard		Battle aquitard
			Battle		Horseshoe Canyon aquifer
			Whitemud		Bearpaw aquitard
			Horseshoe Canyon		Belly River aquifer system
			Bearpaw		Lea Park aquitard
			Belly River		Milk River aquifer
			Lea Park		
			Milk River		
			Colorado		
	Lower	Cardium	Colorado aquitard system		
		Second White Speckled Sandstone			
		Viking			
		Mannville			
		Clearwater			
Jurassic	U	Jurassic aquitard			
	M				
	L				
Triassic		Mississippian - Jurassic aquifer system			
Permian					
Pennsylvanian					
Mississippian		Stoddart	Exshaw - Banff aquitard		
		Rundle			
		Banff			
		Exshaw			
Devonian	Upper	Wabamun	Upper Devonian aquifer system		
		Winterburn			
		Woodbend			
		Ireton			
		Grosmont			
	Middle	Leduc	Ireton aquitard		
		Cooking Lake			
		Beaverhill Lake			
		Prairie Evaporite			
		Winnipegosis			
Lower	Elk Point	Cold Lake	Middle - Upper Devonian aquifer system		
		Lotsberg			
		Not deposited			
Silurian					
Ordovician					
Cambrian	U	Basal Sandstone	Elk Point aquiclude system		
	M				
	L				
Precambrian		Not deposited			
			Cambrian aquitard system		
			Basal aquifer		
			aquiclude		

Modified after Bachu et al. 2000.

Single Layer Model

Gassmann Fluid Substitution



ρ_f	937 (kg/m ³)
ρ	2370 (kg/m ³)
V_p	3800 (m/s)
V_s	2400 (m/s)
ϕ	16%
κ	1(mD)

BCS: 40% CO₂

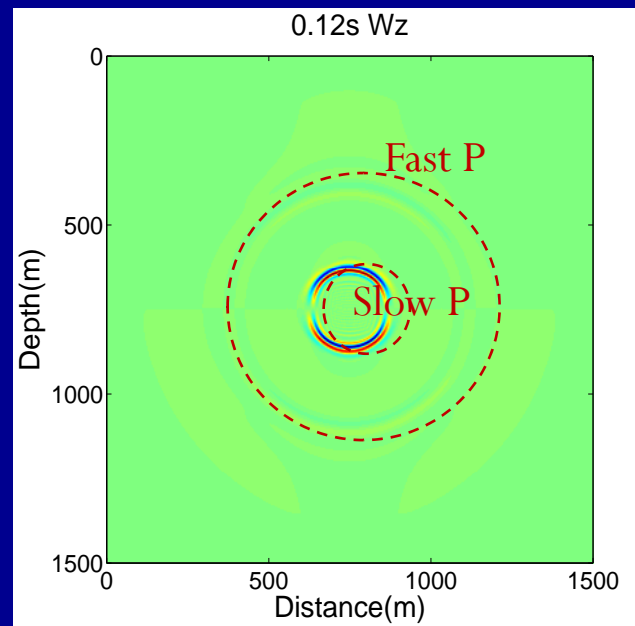
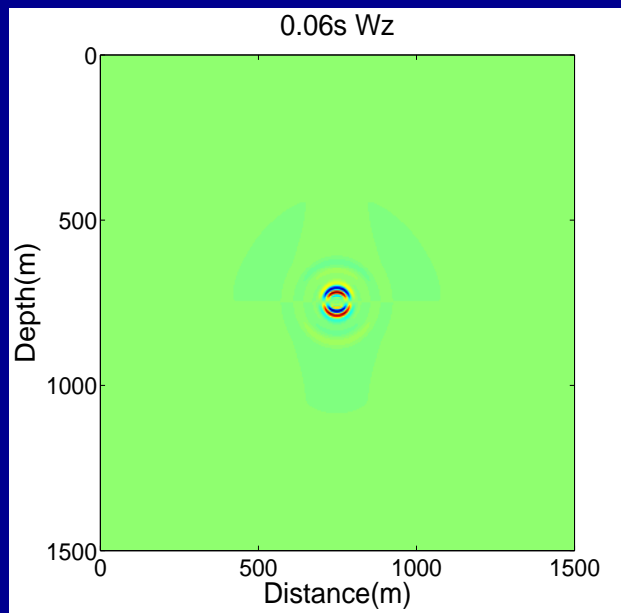
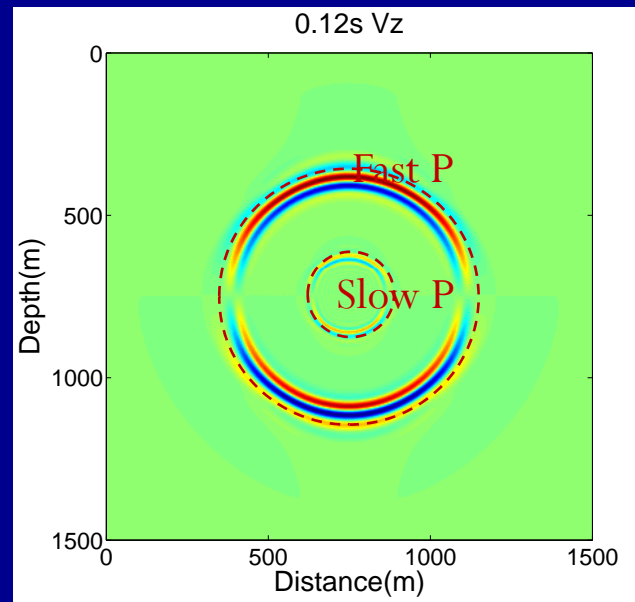
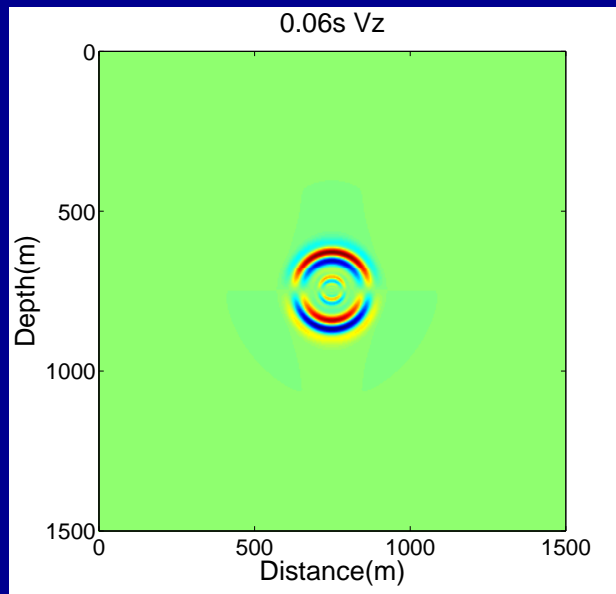
- Fourth order in space and second order in time.
- The stability condition is the same as the one in the elastic case (Zhu:1991)

$$\Delta t \leq \frac{h}{(V_p^2 - V_s^2)^{1/2}}$$

$$h = 3m \quad dt = 0.2 \text{ ms}$$

- The size of the model was 1500 m by 1500 m
- Explosive source: Ricker wavelet with dominant frequency 50 Hz
- Source location : $(x, z) = (750, 750)m$

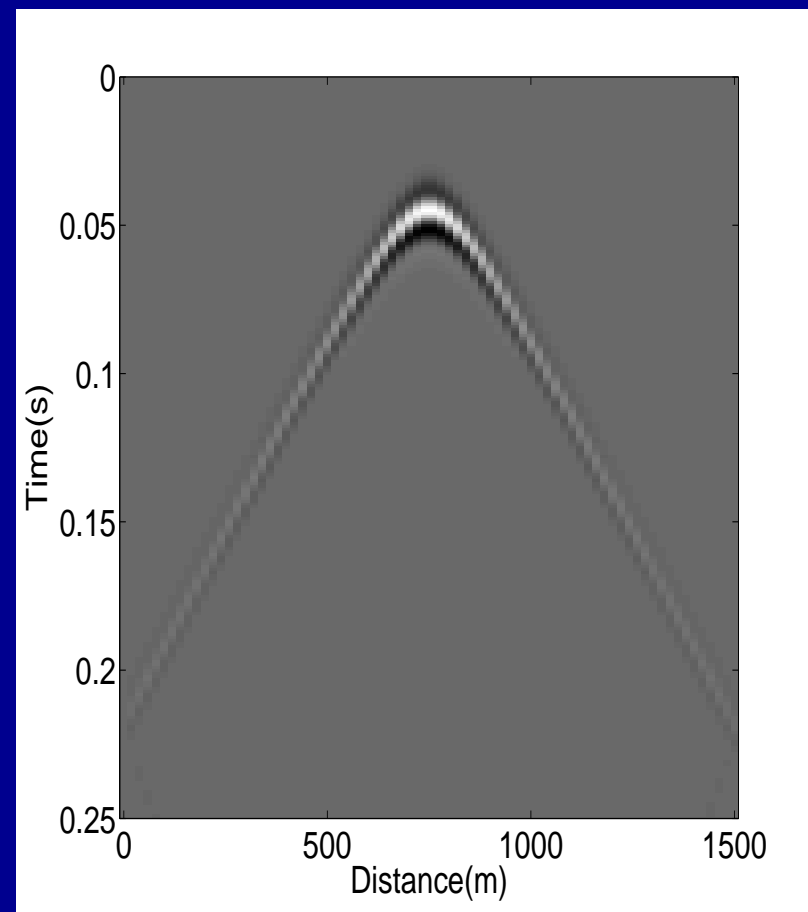
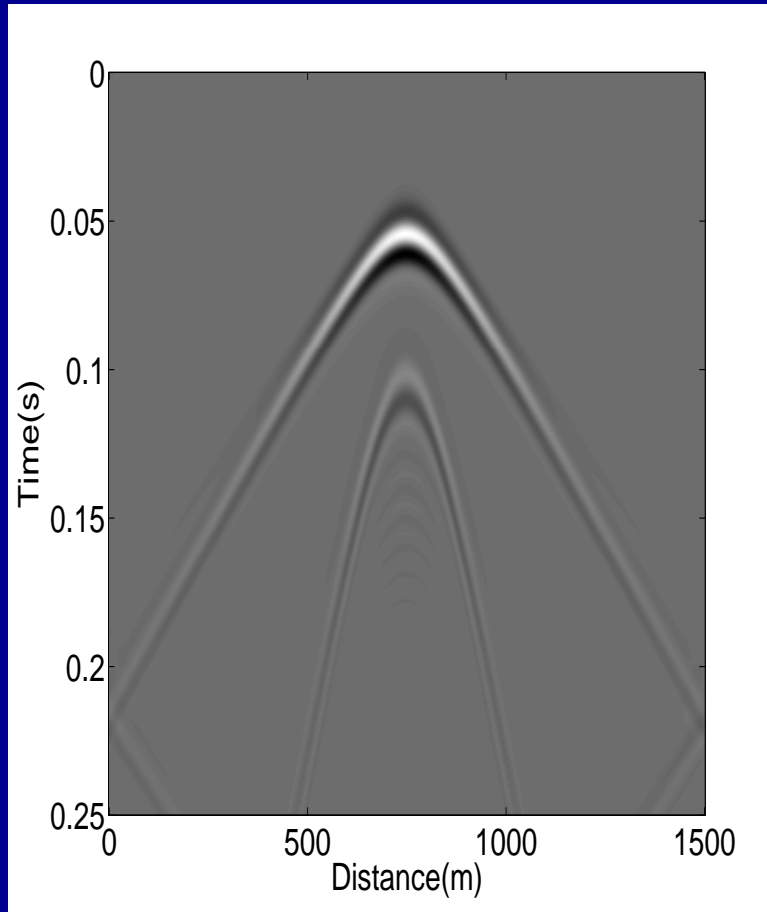
- Vertical Particle Velocity Snapshots:



Solid

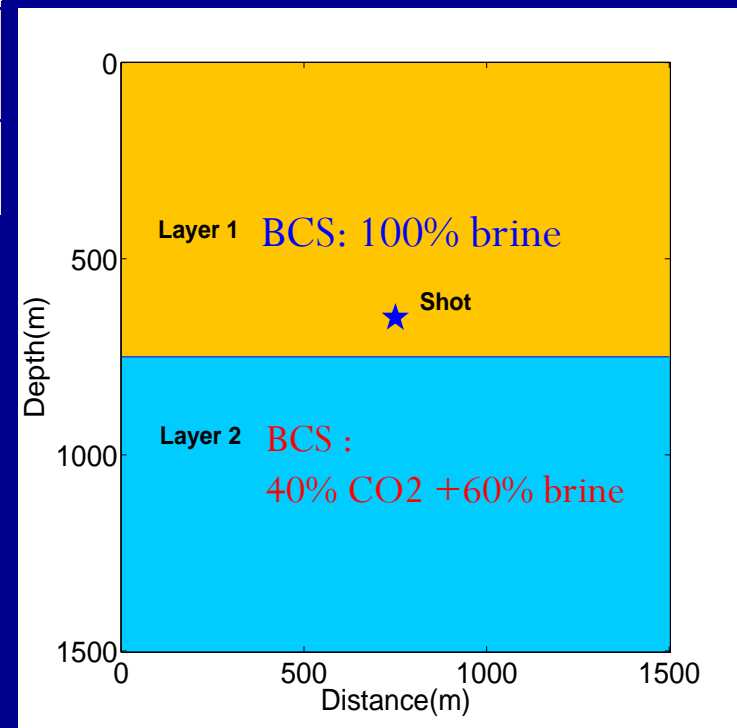
Fluid

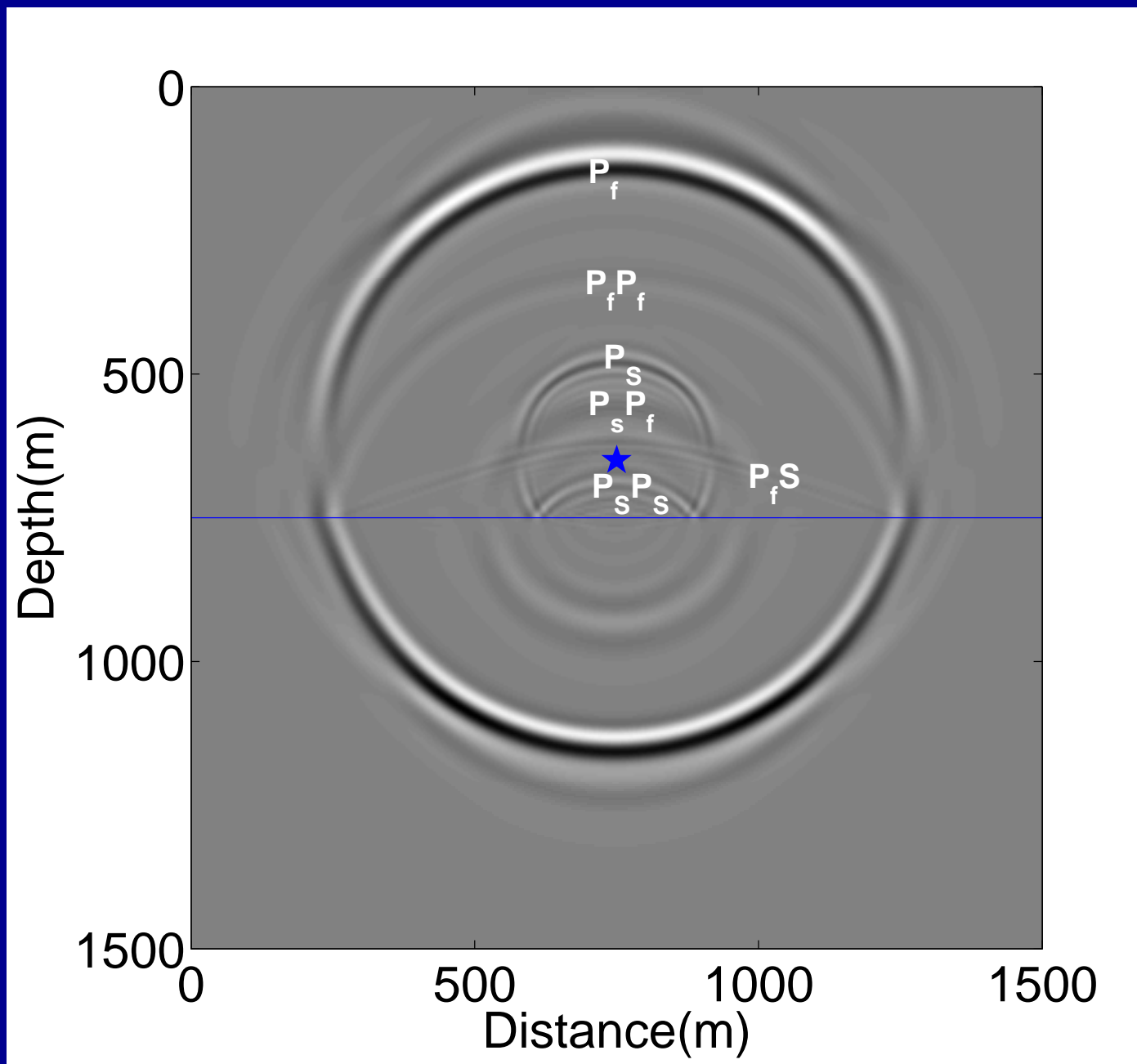
- Comparison with elastic algorithm

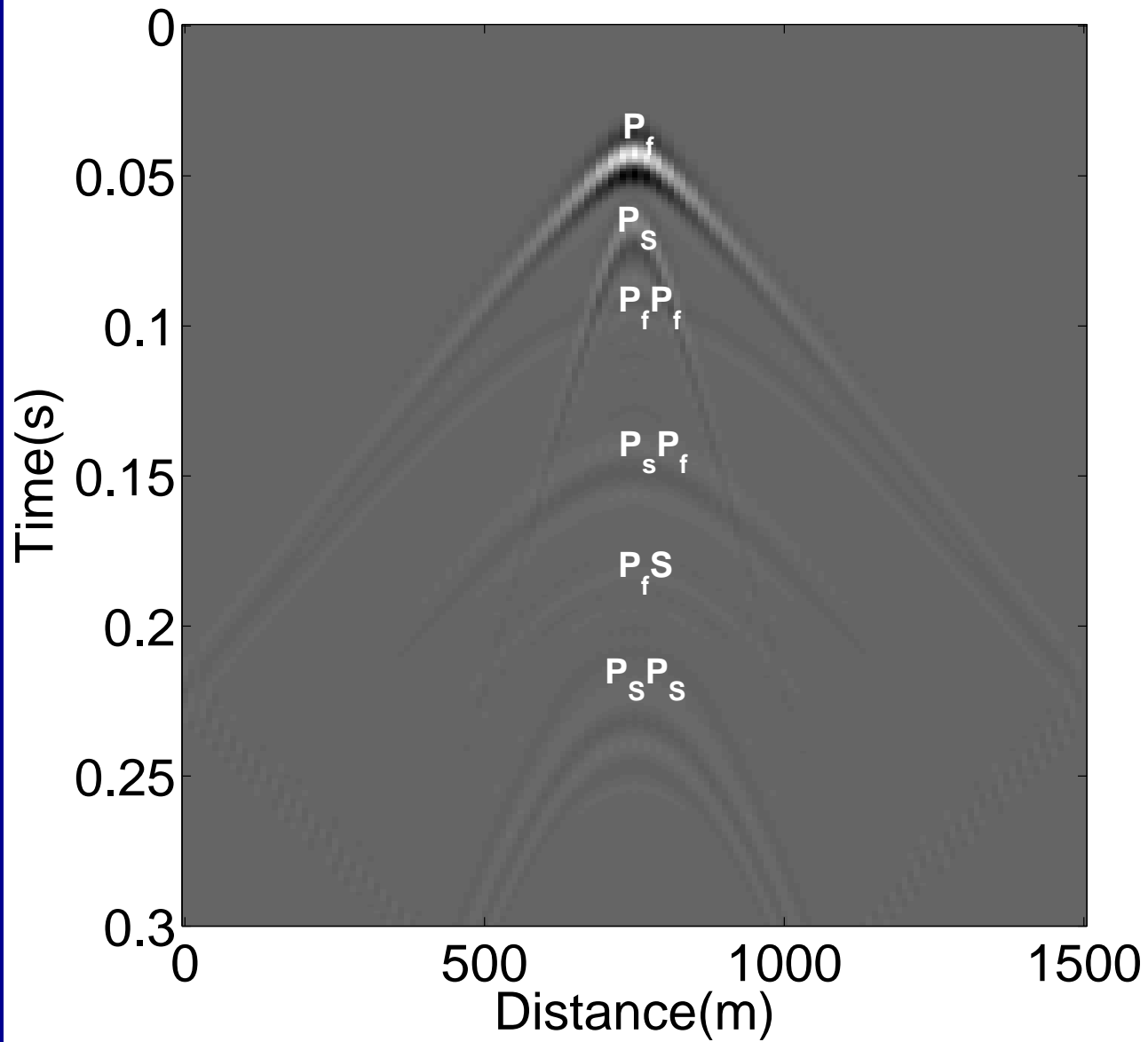


Two-Layered Model

	Top Layer	Bottom Layer
ρ_f	1070 (kg/m^3)	937 (kg/m^3)
ρ	2400 (kg/m^3)	2370 (kg/m^3)
V_p	4100(m/s)	3800 (m/s)
V_s	2390(m/s)	2400 (m/s)
ϕ	16%	16%
κ	1(mD)	1(mD)







Conclusion and Future Goals

- The Poroelastic algorithm Generates slow compressional wave as predicted by Biot's theory.
- At a poroelastic boundary the slow P-wave is converted to a fast P-wave.
- The algorithm handles layered models and should be examined for more complex models.
- The algorithm could be used for inversion to obtain porous media properties that are ignored in elastic algorithms.

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