5D Interpolation by Arbitrarily Sampled Fourier Transform

John Zheng Guo, University of Calgary

Supervisor: Prof. Wenyuan Liao
Mentor: Dr. Ye Zheng at Geo-X Exploration Services

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Introduction: What Is 5D Interpolation and Why It's Important?

- Seismic data processing methods such as migration often work better if input traces are from an equally spaced ("equispaced") or regular midpoint-offset grid.

- However, this ideal condition is often difficult to achieve due to several reasons, including equipment failure and malfunction, locations unable to reach, and economics. Also, due to change of grid specification.

- Therefore, in practice, "data interpolation", which interpolates or extrapolates irregular input seismic data to a desired regular grid or any point in general, becomes an important processing step.

- Since the data has one dimension in time/sample (temporal), and four dimensions in midpoint-offset (spatial), so the method is called "5D interpolation."

Introduction: Categories of Seismic Data Interpolation Methods

- Binning
  - Primitive
- f-x Methods
  - Results not satisfactory
- Radon, Curvelet Based
  - So far mainly used in 3D interpolation
- **Fourier-Based**
  - Best results in 5D interpolation
  - Computationally expensive

Fourier-Based 5D Interpolation Methods

- Existing popular methods
  - Projection Onto a Convex Set (POCS)
  - Minimum Weighted Norm Interpolation (MWNI)
  - Anti-Leakage Fourier Transform (ALFT).

- Based on the same principle
  - Computing “spatial frequency contents” in the f-K domain

- Differences in the actual implementations.
  - True or binned input trace positions
  - Iterative or not
  - These differences have an impact on effectiveness, speed and quality
  - By tuning various parameters, those methods can convergence

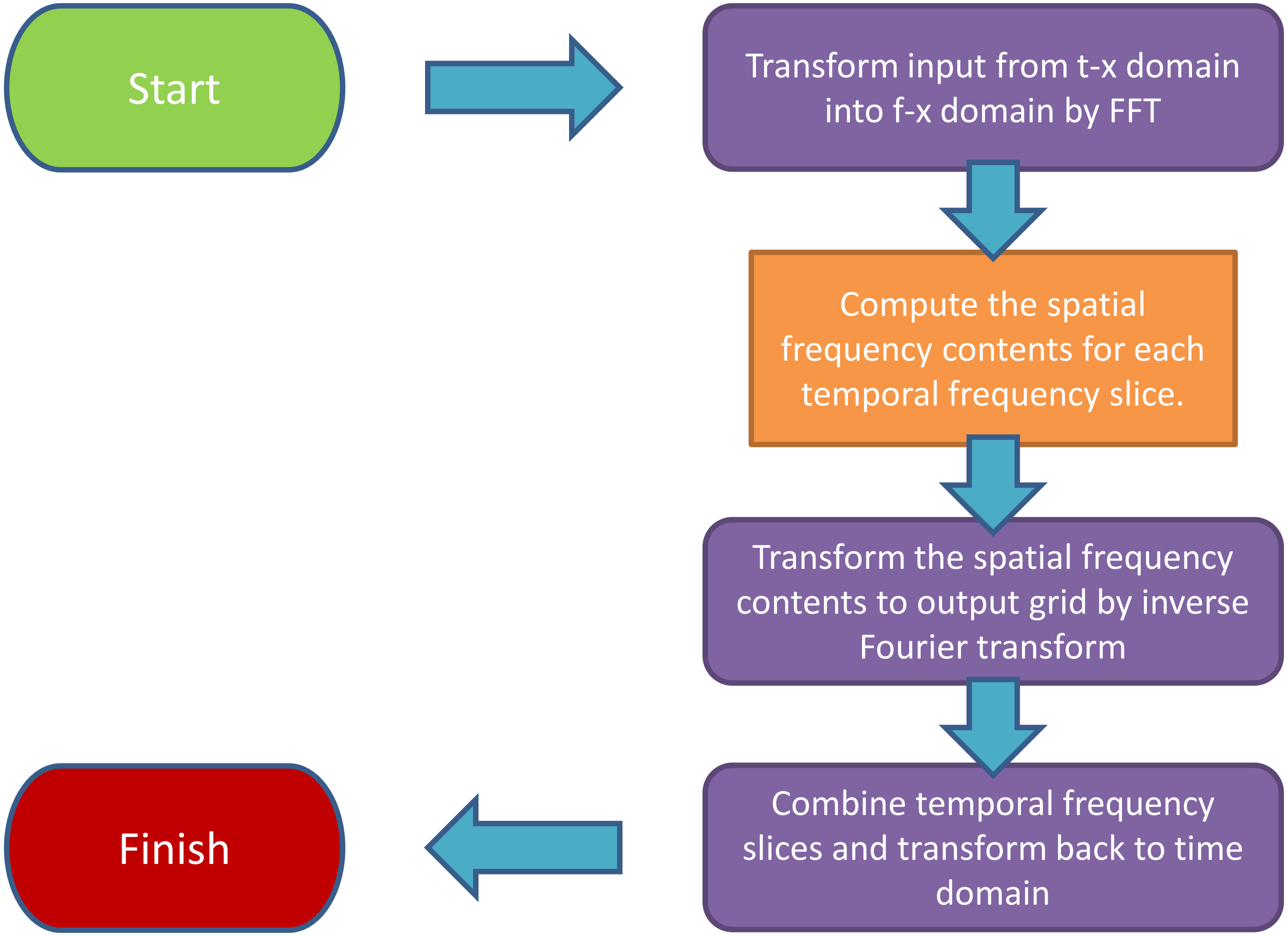
Fourier-Based Methods: Key Points

- Operate in the frequency-wavenumber (f-K) domain.
  - f is the "temporal frequency" transformed by FFT from the 1D time/sample dimension.
  - K is the wavenumber corresponding to the Fourier coefficients in the 4D midpoint-offset domain, it's also called "spatial frequency".

- Each temporal frequency is treated independently.
  - "Temporal frequency slice"

- For each temporal frequency slice
  - Spatial frequency contents computed
  - Then transformed back onto the output grid by inverse Fourier transform.

Fourier-Based Methods: Flow Chart



Fourier-Based Methods: Spatial Frequency Contents

- We define that "spatial frequency contents" consist of many Fourier coefficients.

- Each Fourier coefficient is a complex value for its energy and associated with a 4-dimentional vector for its location in wavenumber domain.

- All solved Fourier coefficients together form the estimated spatial frequency contents, and interpolated traces at regularized locations are computed by inverse Fourier transform.

- Mathematically, let $c = (c_K, c_E)$ be a Fourier coefficient, c_K be its wavenumber and c_E be the associated energy, the inverse Fourier transform to location x is, with appropriate normalization,

  $$f^{-1}(c, x) = c_E e^{2\pi i < c_K, x >}$$

  where < > is the inner product operator.

- The final interpolated result at x is obtained by summing up the inverse transform from the entire spatial frequency contents.

Fourier-Based Methods: POCS

- Iterative
- Uses binned input traces
- Perform FFT on them
- Apply a threshold
- Only keep the Fourier coefficients larger than the threshold
- Fast but suboptimal interpolation accuracy
- Many variations and improvements since the original version (2006)

Fourier-Based Methods: MWNI

- Non-iterative
- Uses binned input traces
- Tries to solve an optimization problem
- Cost function is the norm of “residual data”
- Spatial frequency contents are computed by the cost function
- Many variations by tuning the cost function

Fourier-Based Methods: ALFT

- ALFT computes the spatial frequency contents in an iterative manner: in each round, it first calculates the estimated energy associated with each of the Fourier coefficients from a pre-selected set of wavenumbers (typically a regular grid of points) in the f-K domain by a weighted DFT method.

- Let T be input data in the f-x domain indexed by l, and each trace t_l be associated with location x_l, energy e_l, and an appropriately defined weight w_l. For a Fourier coefficient c with wavenumber c_K, its energy is estimated as

$$c_E \approx \sum_{t_l \in T} w_l e_l e^{-i2\pi < c_K, x_l >}$$

- Then, it selects c with the largest c_E and adds it to the spatial frequency contents, subtract its inverse from current input data to get “residual” data, and proceeds into the next round with residual data as new input until a threshold is reached.

- According to tests by Geo-X, ALFT is superior to POCS and MWNI but is much slower.

The Arbitrarily Sampled Transform (ASFT) Method for 5D Interpolation

- ASFT is an iterative method, similar to POCS and ALFT.
- In each round, it adaptively computes one or several Fourier coefficients by an optimization scheme.
- This is done in two stages: selection, and optimization.
- Residual data becomes input data for next round.
- Adaptive threshold is used to determine termination of computation.
- The Fourier coefficients are collected and inversely transformed to the desired output grid.

ASFT: Fourier Coefficient Selection

- Role similar to the weighted DFT used in ALFT, but better accuracy with optimization.
- Let T be input data in the f-x domain indexed by l, and each trace t_l be associated with location x_l and energy e_l.
- Pre-select c_K from a grid. The cost for the energy c_E is:

$$\mathcal{J}(c_E) := \sum_{t_l \in T} \|e_l - f^{-1}(c, x_l)\|^2$$

  where $c = (c_K, c_E)$.
- Minimize c_E
- Done analytically, as efficient as the weighted DFT used in ALFT
- Then pick the Fourier coefficient $c = (c_K, c_E)$ with the largest c_E.

ASFT: Fourier Coefficient Optimization

- Let T be input data in the f-x domain indexed by l, and each trace data t_l be associated with location x_l and energy e_l. The cost for Fourier coefficient c is the same as before:

$$\mathcal{J}(c) := \sum_{t_l \in T} \| e_l - f^{-1}(c, x_l) \|^2$$

- However, at the optimization stage, we would take the previously selected c as input and use the cost function to optimize both c_E and c_K for c.

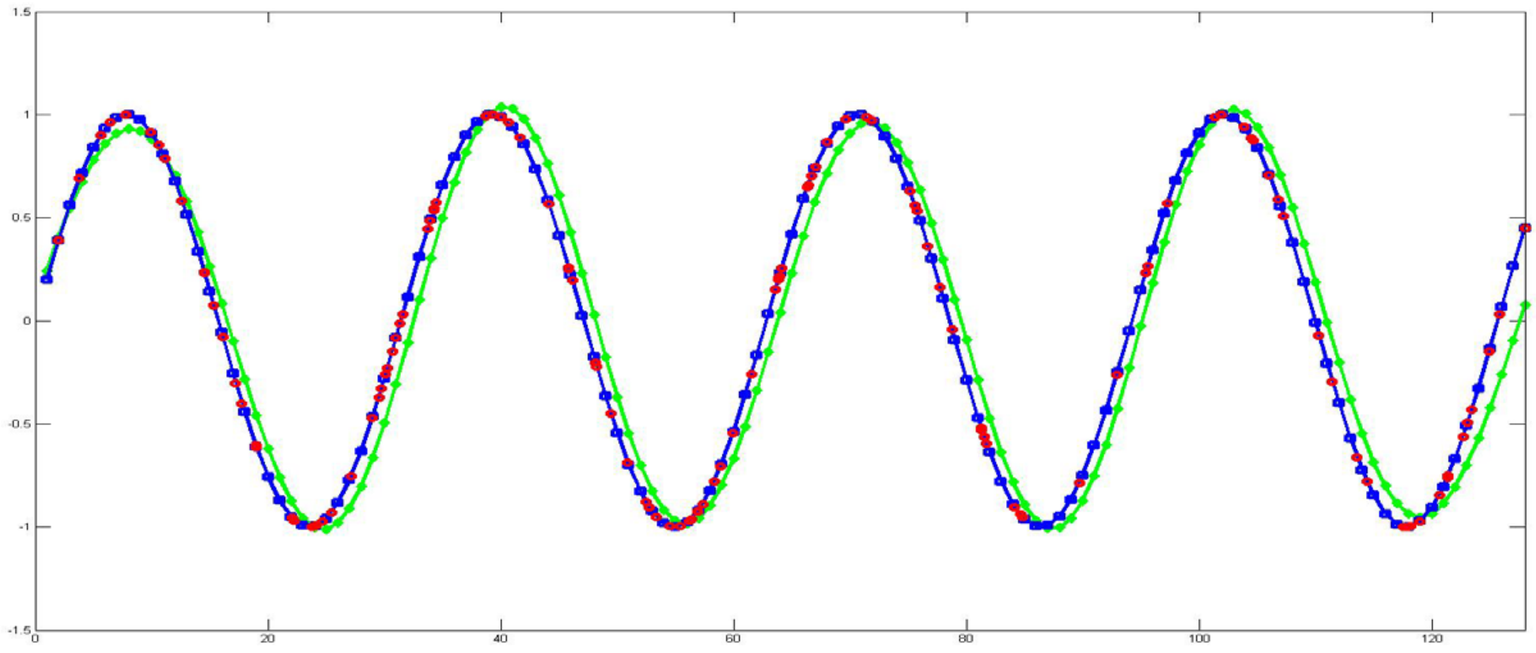
- The Fourier coefficient is optimized by a gradient-based optimization algorithm such as CG or BFGS.

- As a result, for ASFT, true positions of the input traces are used for computation and the spatial frequency contents are allowed to be at an arbitrary point in the f-K domain.

ASFT: Advantages

- Uses true input trace position
- Iterative, ALFT-like
  - Reduces Leakage
- Optimizes Fourier coefficient by a cost function
  - The best idea from MWNI
- Able to compute several Fourier coefficients in one round
  - Best combination of speed and quality
- **Unifying and improving upon POCS, MWNI and ALFT**

ASFT: 1D Demo



Function: y = sin(x/5), x = 1…128

Red	Input
Green	Reconstruction by ALFT
Blue	Reconstruction by ASFT

WSCB Real Data Test (1)

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Iline stacks after migration without (left) and with(right) ASFT interpolation

WSCB Real Data Test (2)

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xline stacks after migration without (left) and with(right) ASFT interpolation

WSCB Real Data Test (3)

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Shallow time slices after migration without (left) and with(right) ASFT interpolation

WSCB Real Data Test (4)

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Deep time slices after migration without (left) and with(right) ASFT interpolation

Conclusions

- 5D interpolation is a useful technique to improve migration quality.
- Fourier-based methods provide the best interpolation results.
- ASFT improves upon existing popular methods such as POCS, MWNI, and ALFT.
- ASFT produced excellent results on Western Canadian Sedimentary Basin (WCSB) data.

Thank You!

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