Amplitude analysis of the inverse scattering series for internal multiple prediction

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Overview

- **Multiple**: Seismic energy that has been reflected more than once (SEG wiki)
  - **long-path multiple**: arrives as a distinct event
  - **short-path multiple**: arrives so soon after the primary that it merely adds tail to the primary (i.e., changes the waveshape).

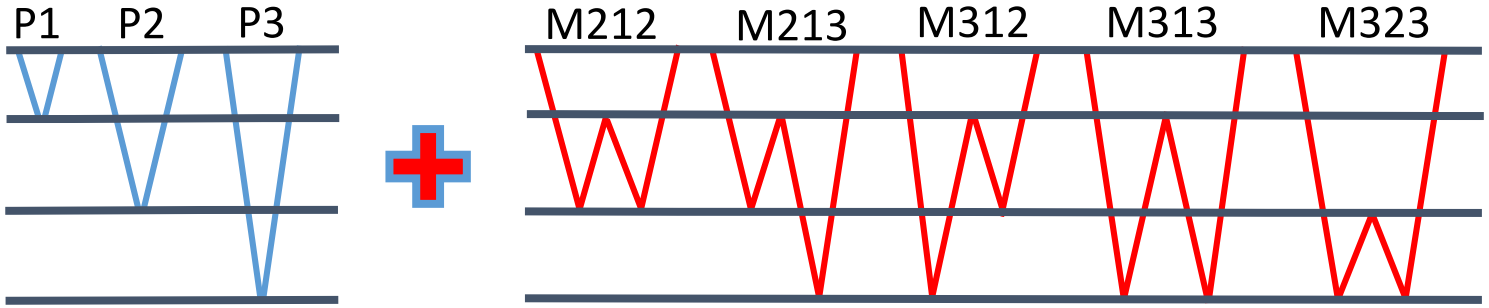
- For this project the focus is internal long-path multiple attenuation using the inverse scattering series

Objectives

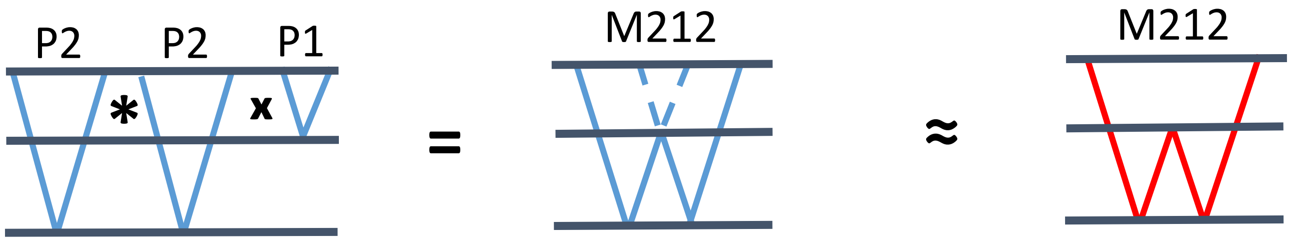
- **Goal of internal multiple prediction**:
  - Correctly predict the amplitudes of all internal multiples without predicting primaries
- **In practice**:
  - Optimal approximation to amplitudes and minimize artifacts of prediction
  - Prediction then input into adaptive subtraction

Background Information and Theory

Primaries **Internal Multiples (first order)**



$$b_3(\omega) = \int_{-\infty}^{\infty} dz_1 e^{-i2\frac{\omega}{c_0}z_1} b_1(z_1) \left[\int_{z_1+\varepsilon}^{\infty} dz_2 e^{i2\frac{\omega}{c_0}z_2} b_1(z_2) \right]^2$$



Amp = $T_{01}R_2T_{10}\ T_{01}R_2T_{10}R_1$

Amp = $T_{01}R_2R_1R_2T_{10}$

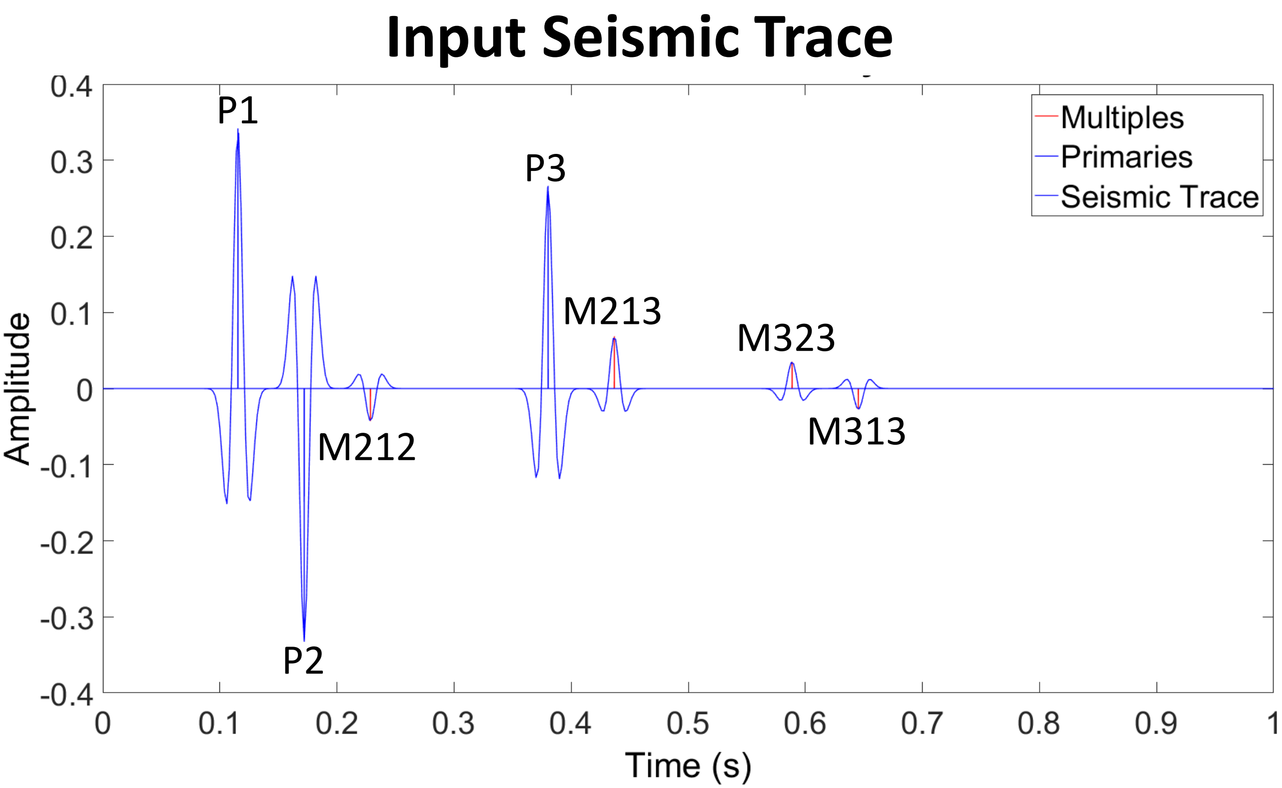
Error = $T_{01}T_{10}$

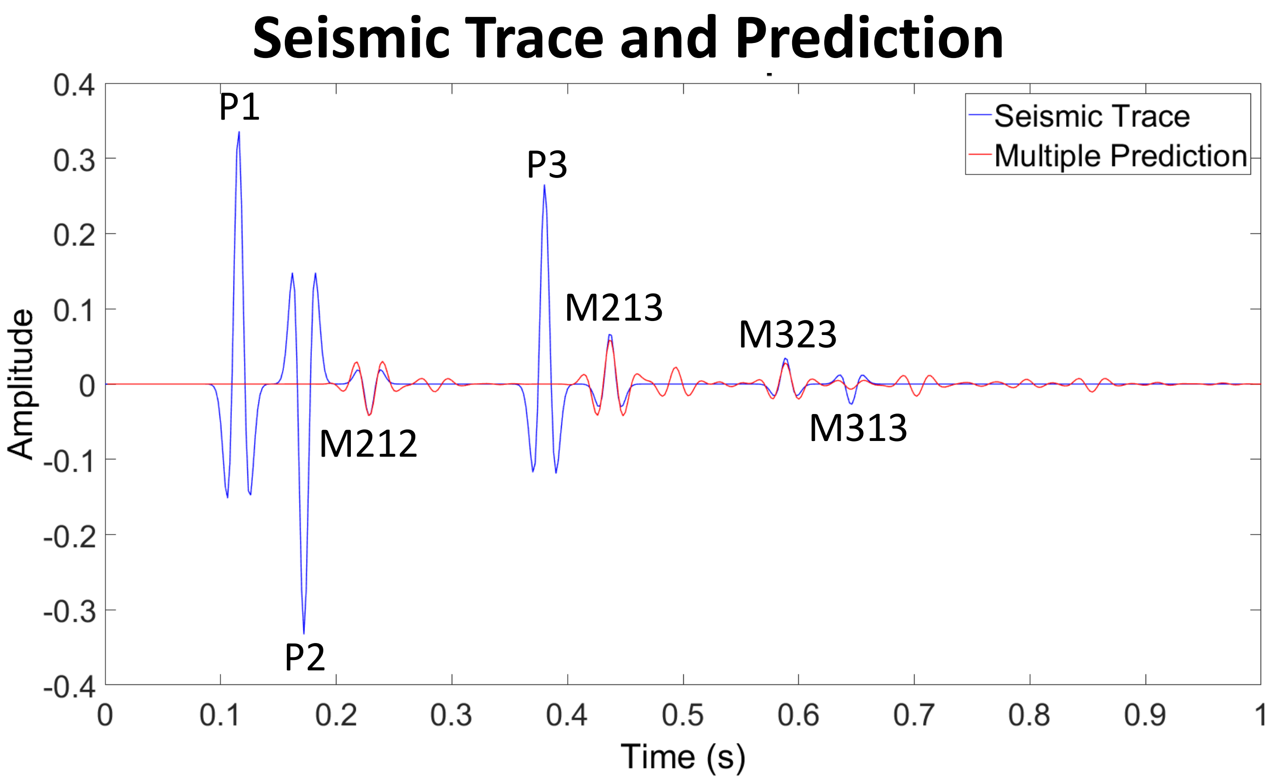
b_1 = Input Data

ε = Search limiting parameter

- Using ISS
  - Only input data and epsilon required
  - Predict multiples from sub events in the data that obey lower-higher-lower relationship
  - Prediction will be in error due to transmission losses across generator

1D Internal Multiple Prediction





- ISS has successfully predicted multiples from input data
- Has also predicted higher order multiples not originally modeled
- Small amplitude differences between the trace and prediction

Order of Operations

Original Method

```
for ik = kz1:kzmax {

   Ipos = exp(i*kzPos(ik)*z)
   Ineg = exp(-i*kzPos(ik)*z)
   intPos = b1z*Ipos
   intNeg = b1z*Ineg

   for iz = z1:zmax {

      inner = sum(intPos(iz+ε:zmax))
      pred(ik) = pred(ik) + intNeg(iz)*inner*inner
   }
}
```

Method with Scalar

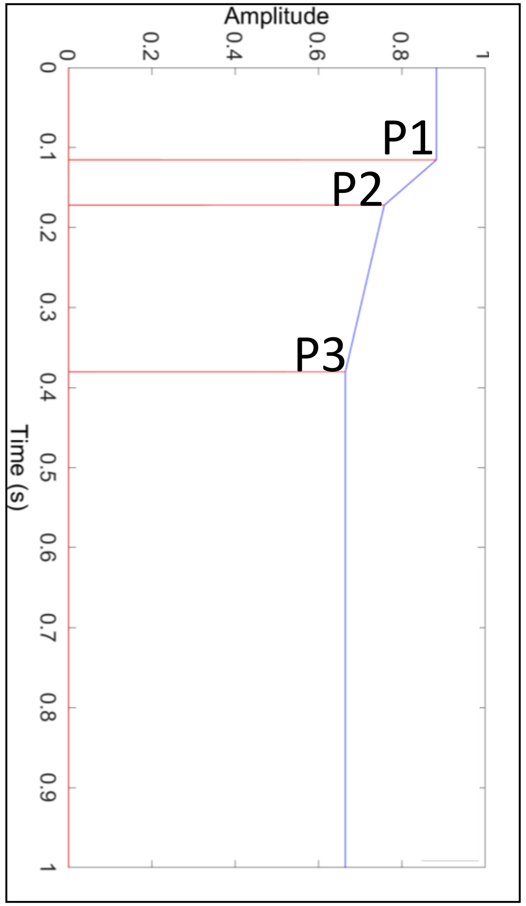
```
for iz = z1:zmax {

   for ik = kz1:kzmax {

      Ipos = exp(i*kzPos(ik)*z)
      Ineg = exp(-i*kzPos(ik)*z)
      intPos = b1z*Ipos
      intNeg = b1z*Ineg

      inner = sum(intPos(iz+ε:zmax))
      pred(iz, ik) = intNeg(iz)*inner*inner
   }
   pred(iz, : ) = pred(iz, : )*φ(iz)
}
spred(ik) = sum(pred(iz, ik))
```

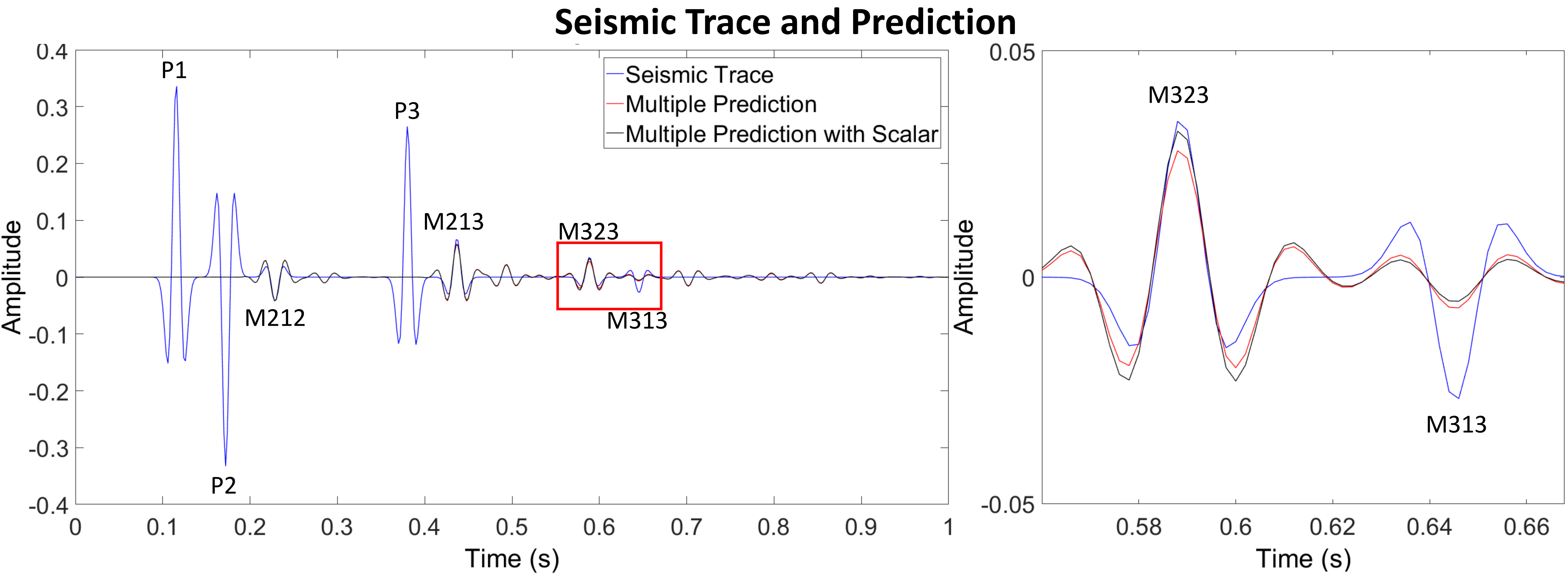
Transmission Loss



$$b_3(\omega) = \int_{-\infty}^{\infty} dz_1 e^{-i2\frac{\omega}{c_0}z_1} b_1(z_1) \left[\int_{z_1+\varepsilon}^{\infty} dz_2 e^{i2\frac{\omega}{c_0}z_2} b_1(z_2) \right]^2$$

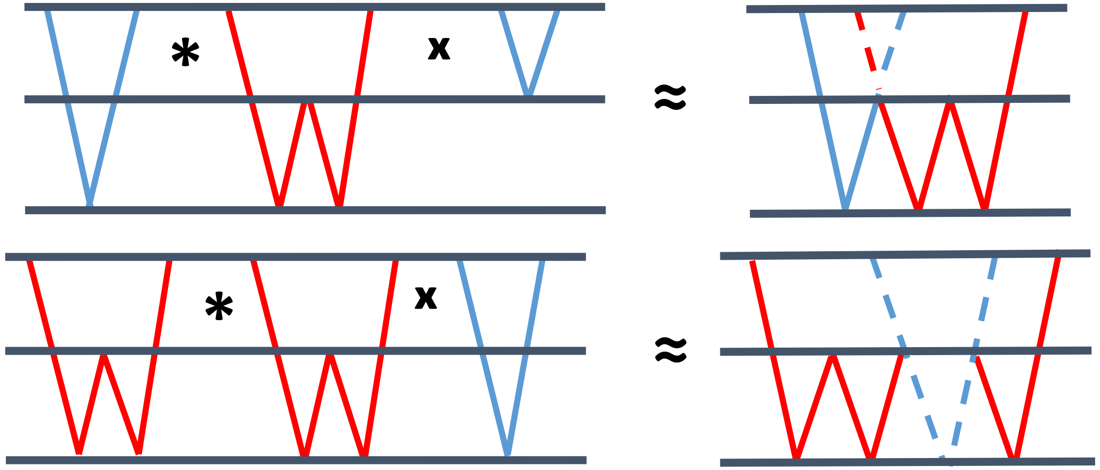
$$\varphi = \frac{1}{\text{Transmission Loss}}$$

1D Multiple Prediction with Scalar



- Prediction with scalar has been beneficial to multiple M323
- Scalar has been detrimental to M313

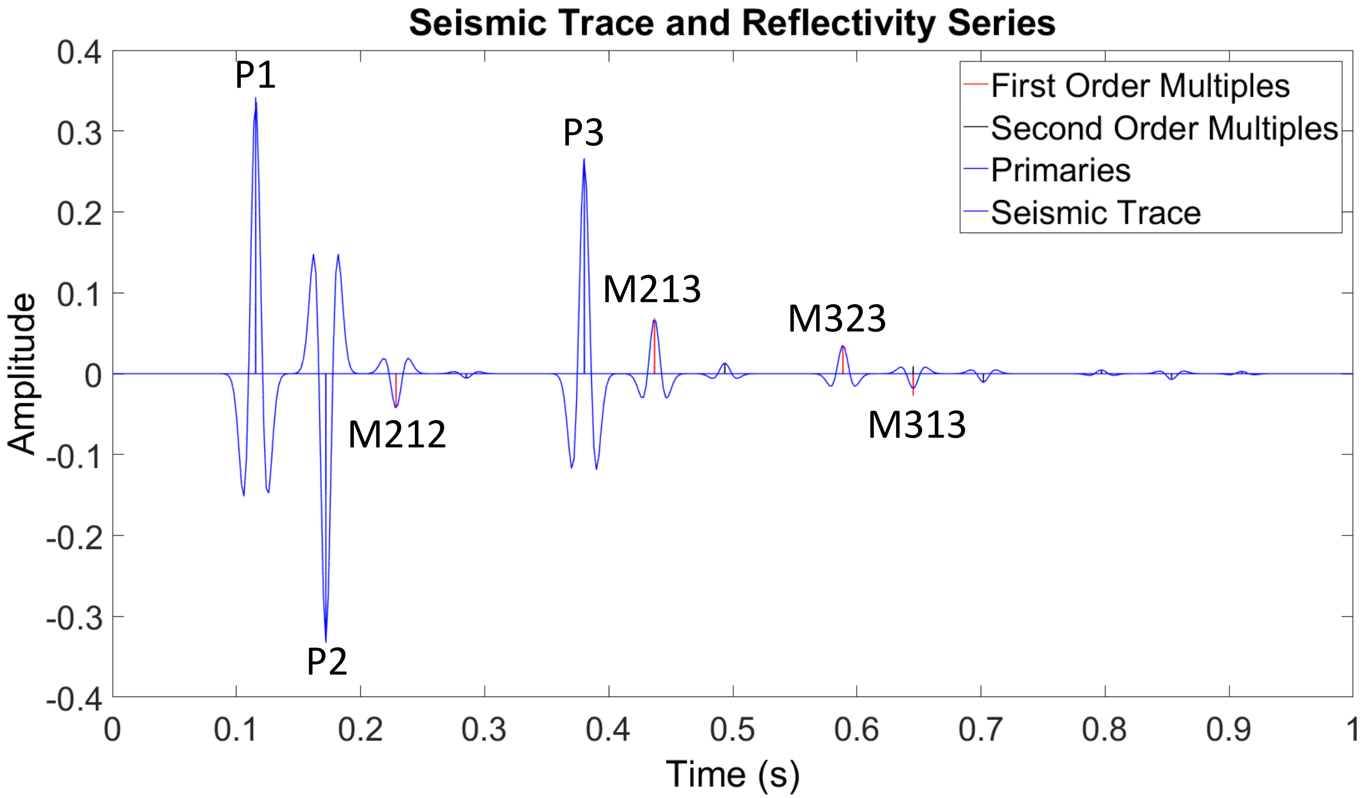
2^{nd} Order Multiples



Error = $(2T_{01}T_{10})$

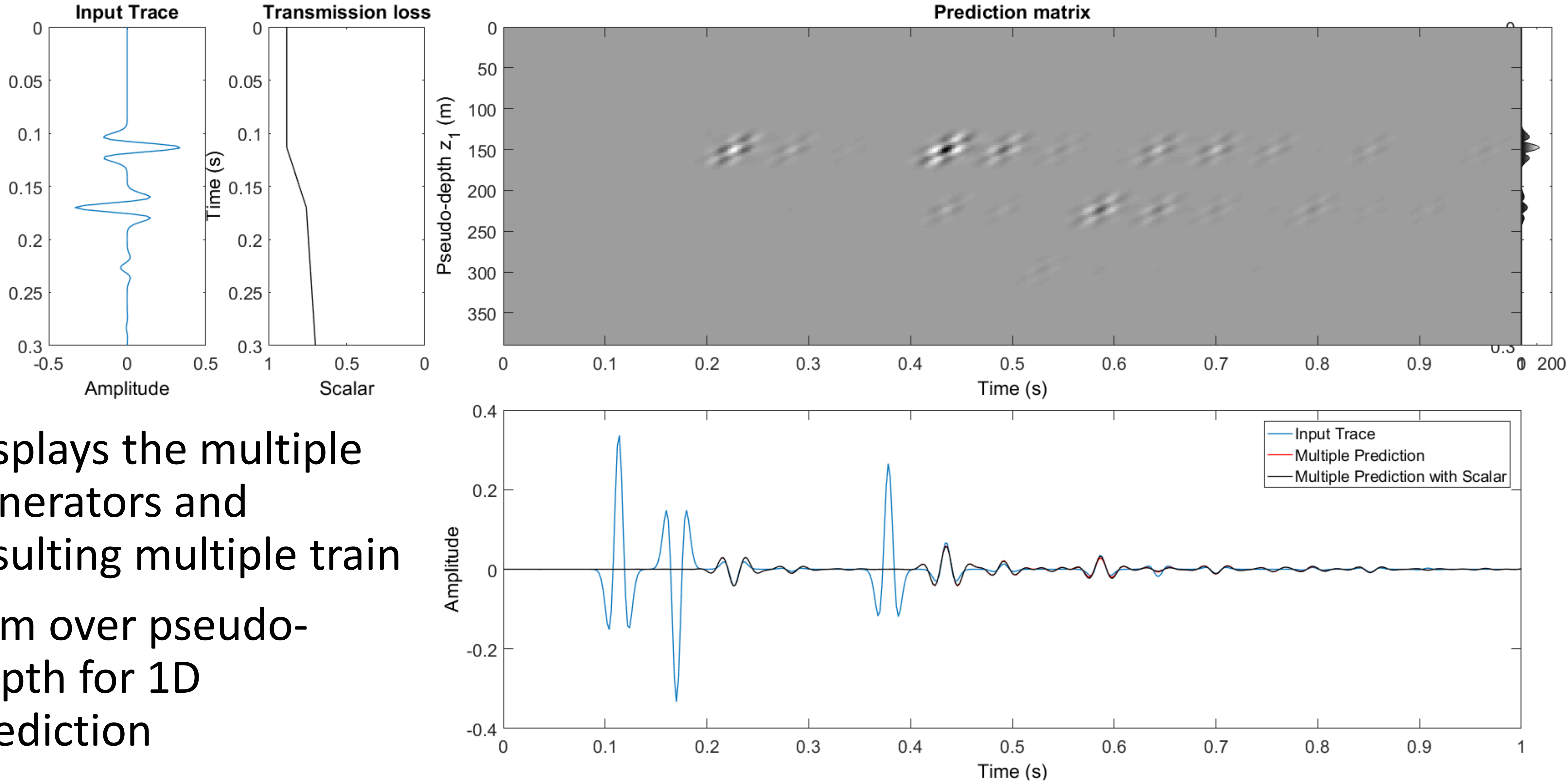
Error = $(T_{01}R_2T_{10})^2$

Combined Error = $[(2T_{01}T_{10})- (T_{01}R_2T_{10})^2]$ which should be a factor between 1 and 2 for $1-R_1^2 < 1$ (Zhang and Shaw 2010)



- Second order multiples are generally overpredicted where first order multiples are under predicted

1D Prediction in Pseudo-Depth Time Plot

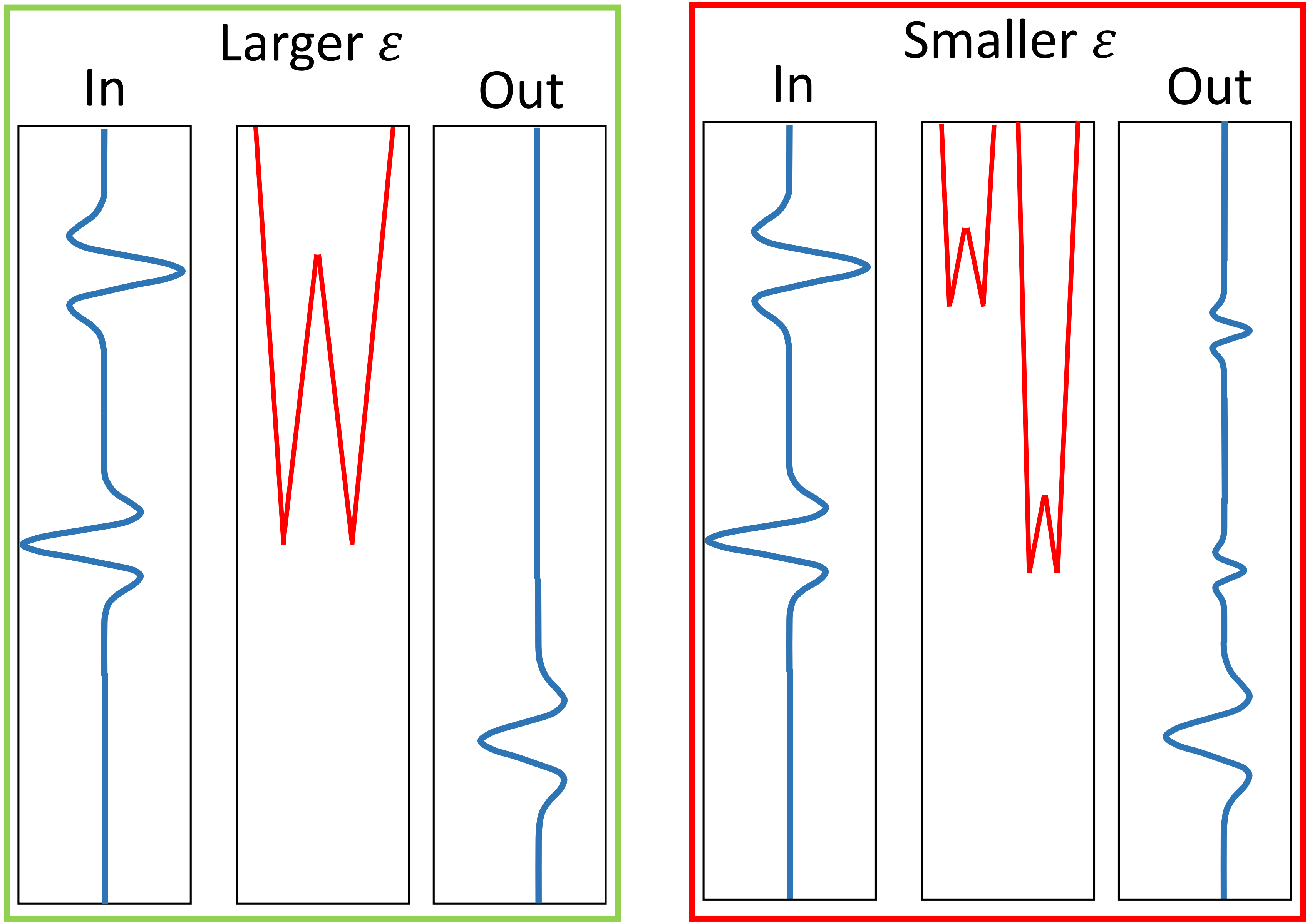


- Displays the multiple generators and resulting multiple train
- Sum over pseudo-depth for 1D prediction

Epsilon (ε)

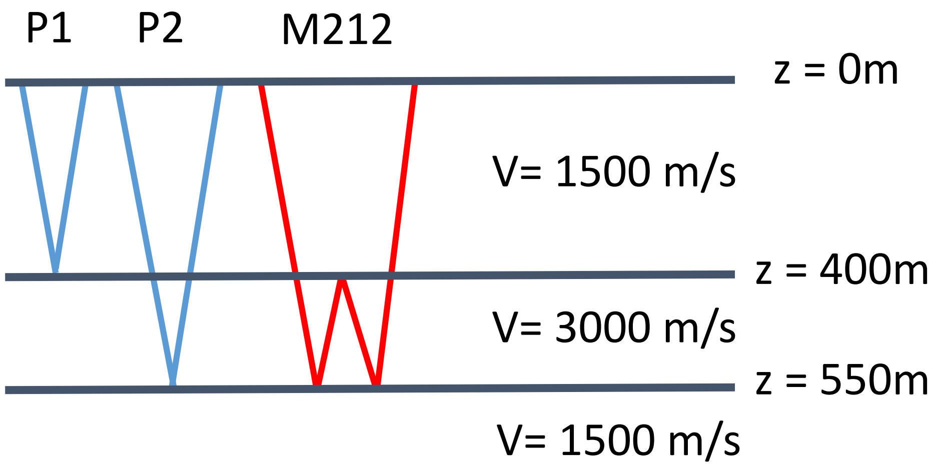
ε = Search limiting parameter

- If output domain varies from input
  - Difficult to vary epsilon
  - eg. Previous example (ω)
- If output domain is the same as input
  - Can use nonstationary epsilon
  - Next example (t, x)

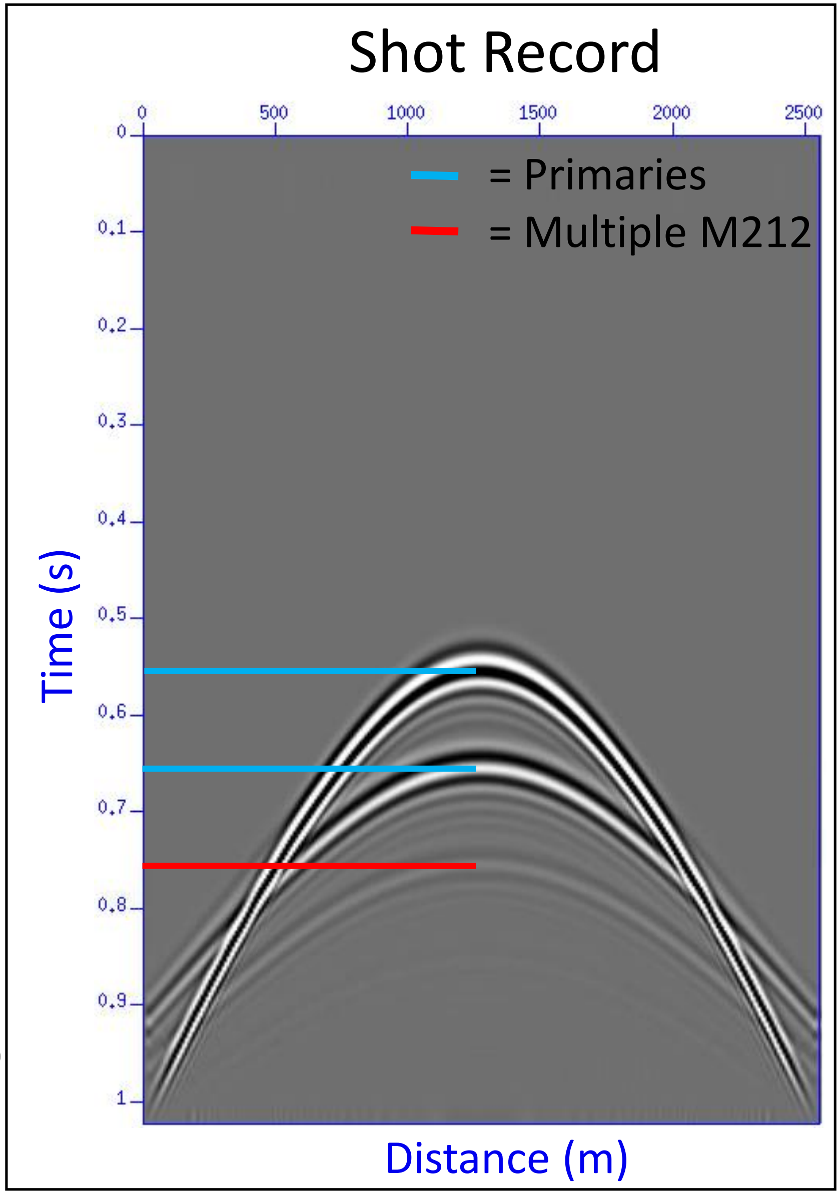


1.5D Time offset Domain Prediction

- Shot record created using finite difference modeling in MATLAB



- Note due to the time offset equation epsilon can be nonstationary

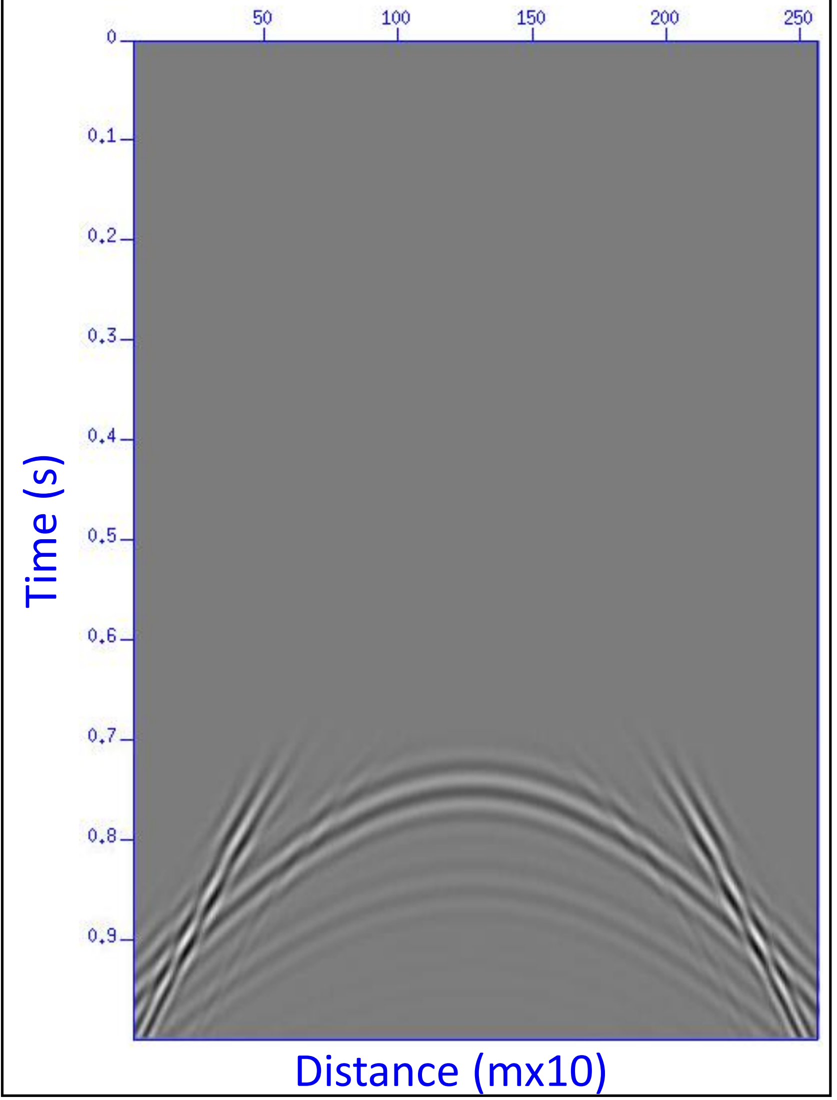


Convolution Definition $(f * g)(\mathrm{t}) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$

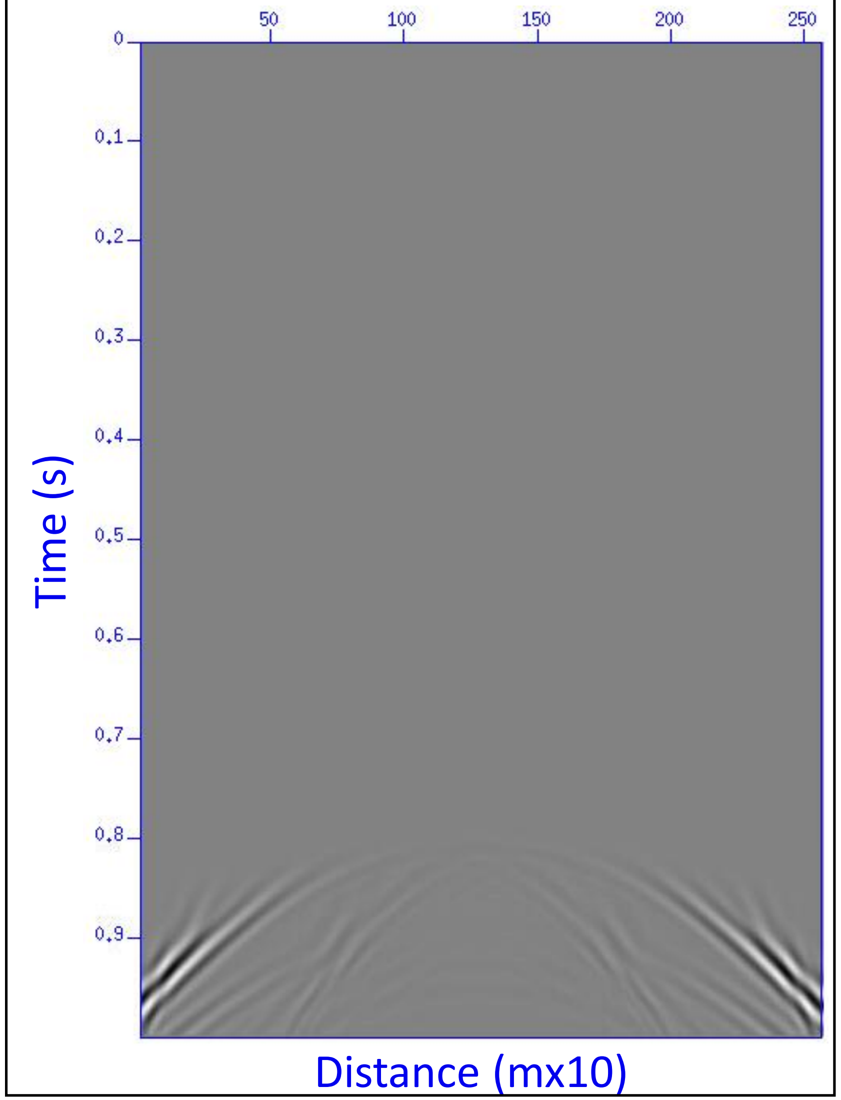
$$b_3(x,t) = \int dx' \int dt' s(x-x', t'-t) \int dx'' \int_{t'-(t-\varepsilon)}^{t-\varepsilon} dt'' s(x'-x'', t'-t'') s(x'', t'')$$

Stationary epsilon

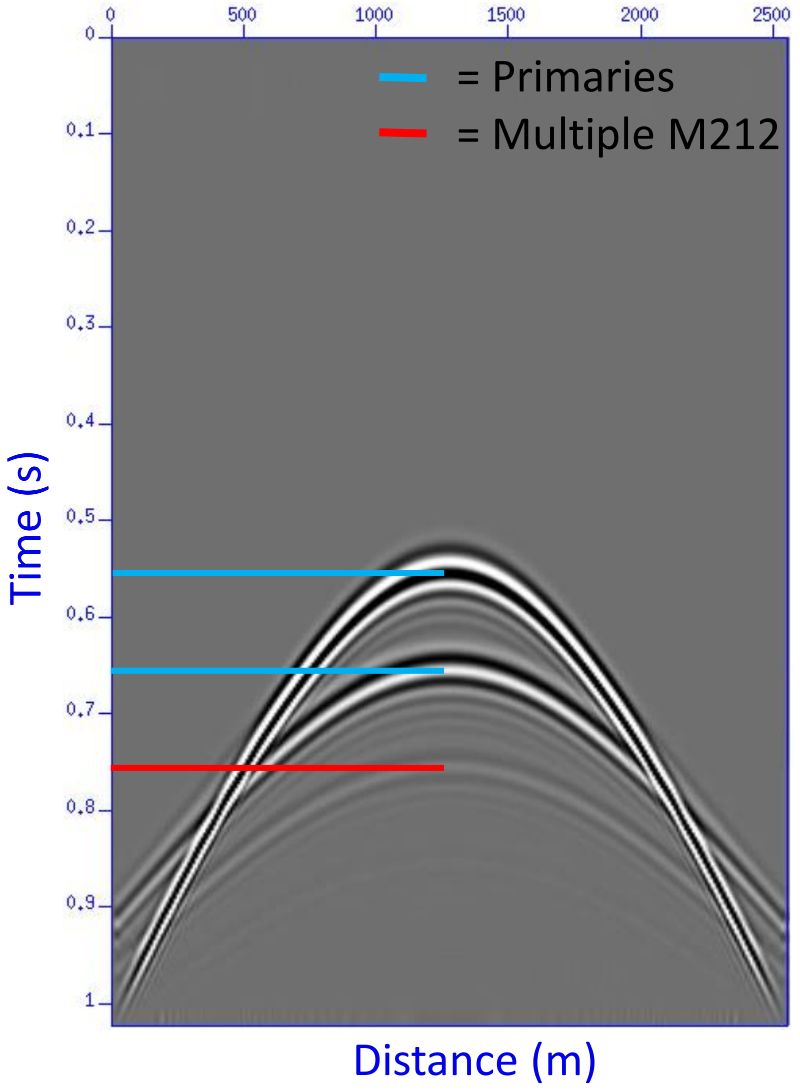
Epsilon = 30



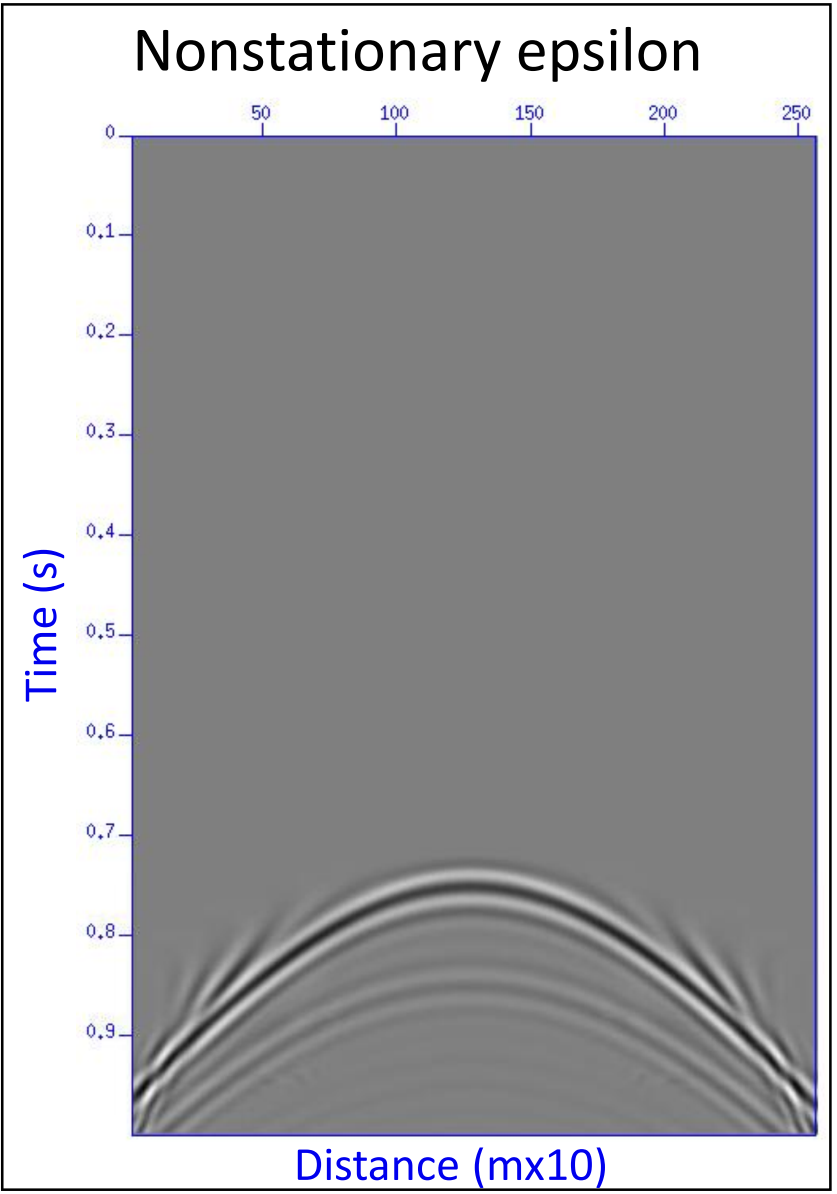
Epsilon = 70

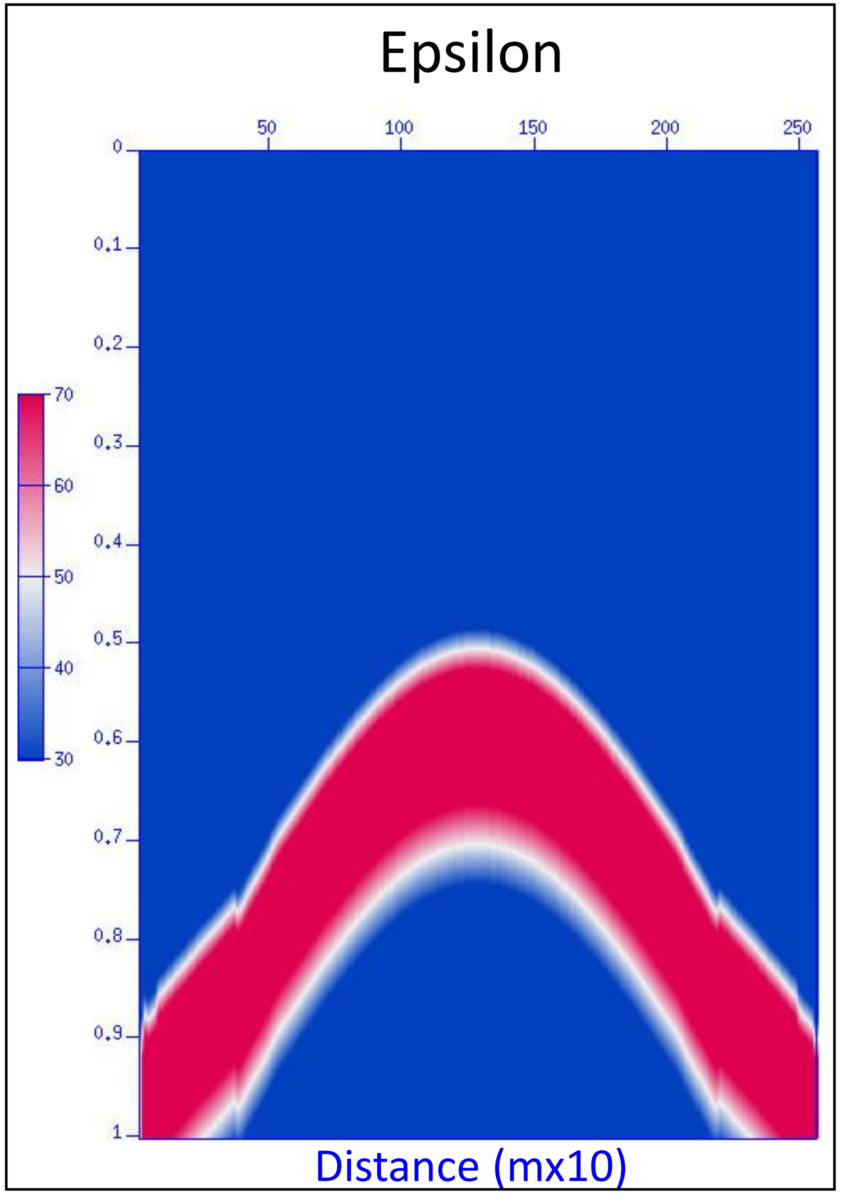


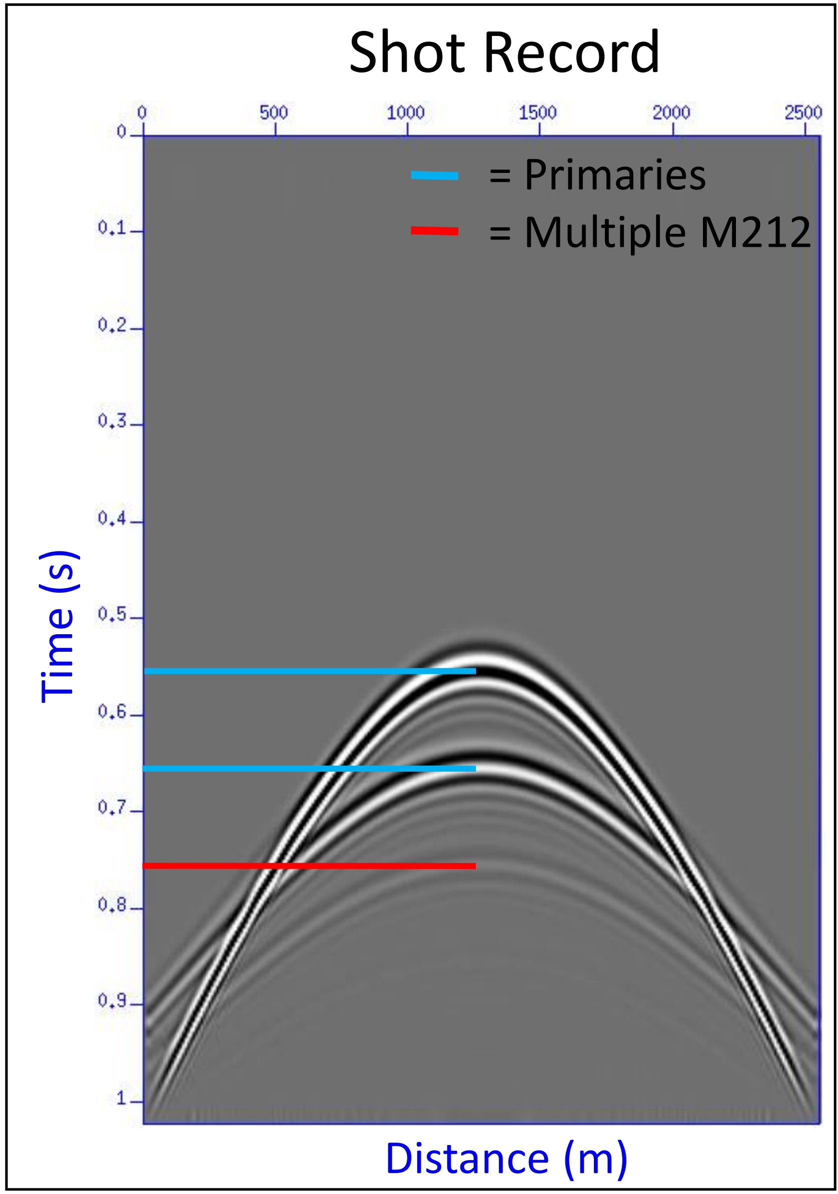
Shot Record



Nonstationary epsilon







Conclusions and future work

Conclusions:

- In 1D using scalar assisted in correcting transmission losses
- In 1.5D time space domain was able to reduce artifacts through nonstationary epsilon
- Combined these assist in the goal of an accurate amplitude prediction

Future Work:

- Reduce computational expense
- Goal of project is to implement ISS on land seismic data
  - How to calculate epsilon?
  - How to manage irregular spatial sampling?
  - What stage of seismic processing workflow to apply multiple attenuation?
    - Amplitude recovery/gain, statics, deconvolution, …

Acknowledgments

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Questions?