Amplitude Migration in v(z) media

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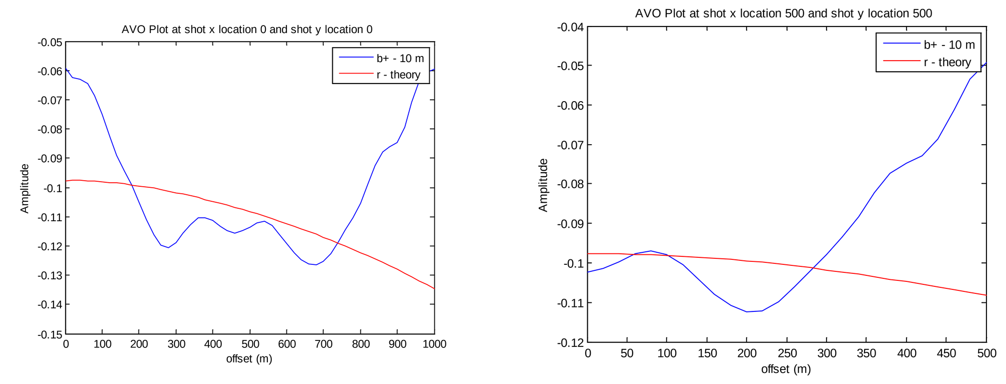
Objective

- **Purpose**: To describe and implement a Kirchhoff type algorithm that correctly preserves seismic amplitudes in a v(z) medium.

Background

- Lahr and Margrave (2015) found following results for shot record migration



- Looking for AVO 3 event reflector, indicated by red line
- Ended up with 'shifted' migrated amplitudes (blue)
- Migrated shot on right at top left corner , on right at middle, of 1000x1000 m survey

Conclusion: Repeat experiment with simpler data and migration to account for v(z) media.

Theory

- General migration algorithm:

$$\beta(\mathbf{y}) = \frac{1}{8\pi^3}\int d^2\xi \frac{|h(\mathbf{y},\boldsymbol{\xi})|}{a(\mathbf{y},\boldsymbol{\xi})|\nabla_y \emptyset(\mathbf{y},\boldsymbol{\xi})|}\cdot\int i\omega d\omega e^{-i\omega\emptyset(\mathbf{y},\boldsymbol{\xi})} u_{S(\mathbf{x}_g,\mathbf{x}_s,\omega)}$$

- where the amplitude weights are given as:

$$w(\mathbf{y},\boldsymbol{\xi}) = \frac{|h(\mathbf{y},\boldsymbol{\xi})|}{a(\mathbf{y},\boldsymbol{\xi})|\nabla(\tau_s+\tau_g)|^2}$$

- For v(z) medium have

$$w(\mathbf{y},\boldsymbol{\xi}) = \frac{\sqrt{cos(\alpha_{s0})}\sqrt{cos(\alpha_{g0})}}{v_0}\sqrt{\frac{\psi_s}{\psi_r}}\sqrt{\frac{\sigma_s}{\sigma_r}}$$

- defining in-plane spreading and

$$\psi_s = cos(\alpha_s)\int_0^z \frac{v(\zeta)}{cos^3\alpha_s(\zeta)}d\zeta = cos(\alpha_s)\frac{\partial\rho_s}{\partial p_s}$$

- out-of-plane spreading

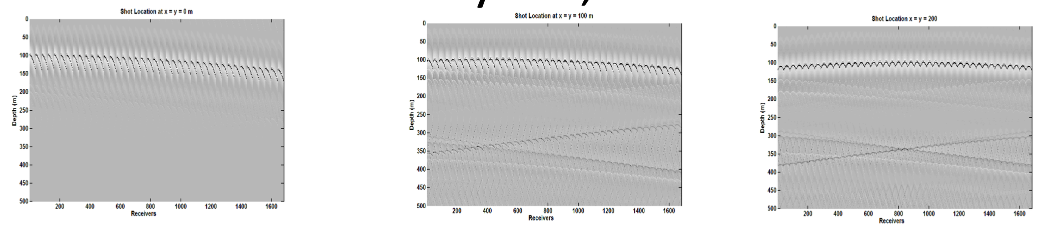
$$\sigma_s = \int_0^z \frac{v(\zeta)}{cos\ \alpha_s(\zeta)}d\zeta = \frac{\rho_s}{p_s}$$

- For constant media, migration algorithm reduces to

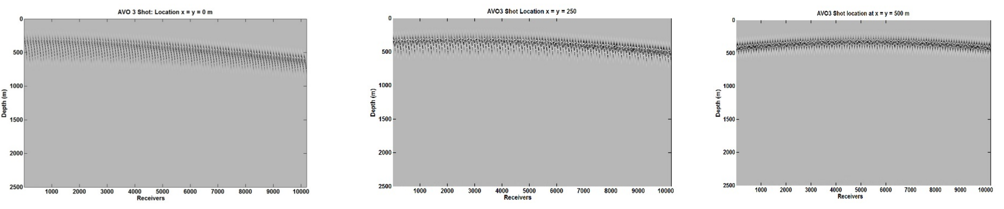
$$\beta(\mathbf{y}) = \frac{2y_3}{\pi c^2}\int d^2\xi \frac{r_s}{r_g^2}cos\theta\cdot\int i\omega d\omega e^{-i\omega[r_s+r_g]/c} u_{S(\mathbf{x}_g,\mathbf{x}_s,\omega)}$$

Input Data

- Flat reflector
  - 400x400 m survey, 500 m depth
  - v(z) = 2000 + 0.3z
  - Reflector at 100 m
  - Shots at x = y = 0, 100 and 200 m

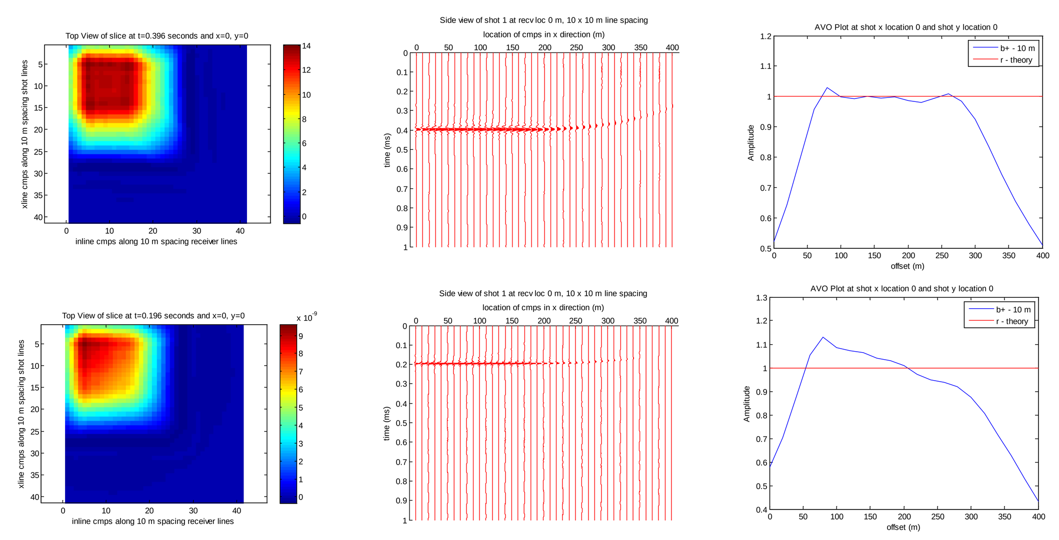


- Simple AVO 3 reflector
  - 1000x1000 m survey, 2500 m depth
  - Two layers, background and AVO 3
  - Reflector at 300 m

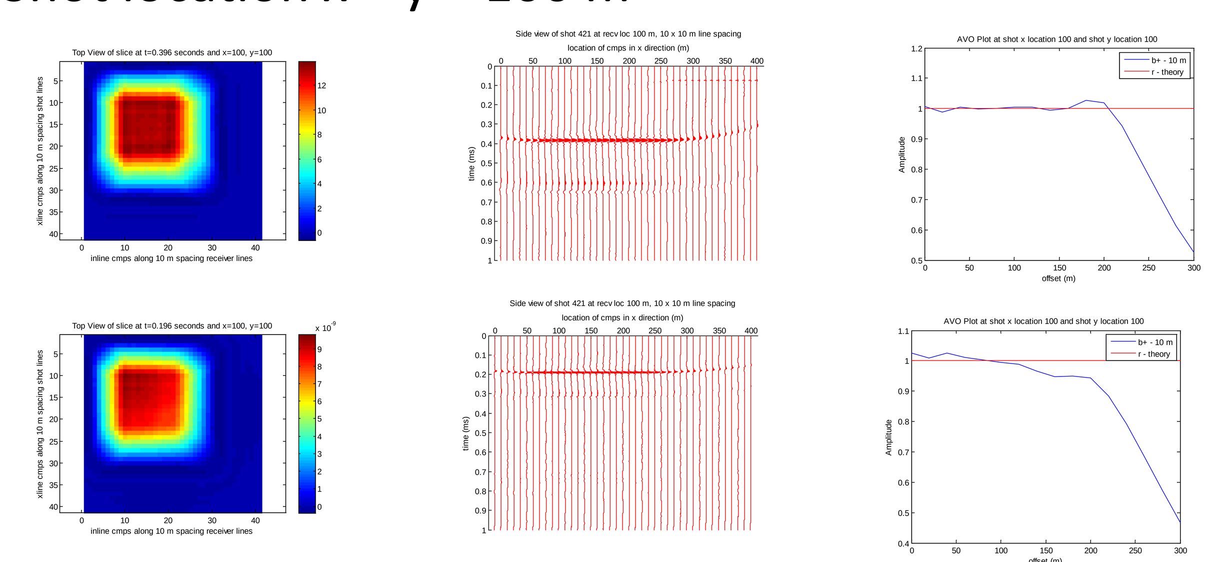


Results

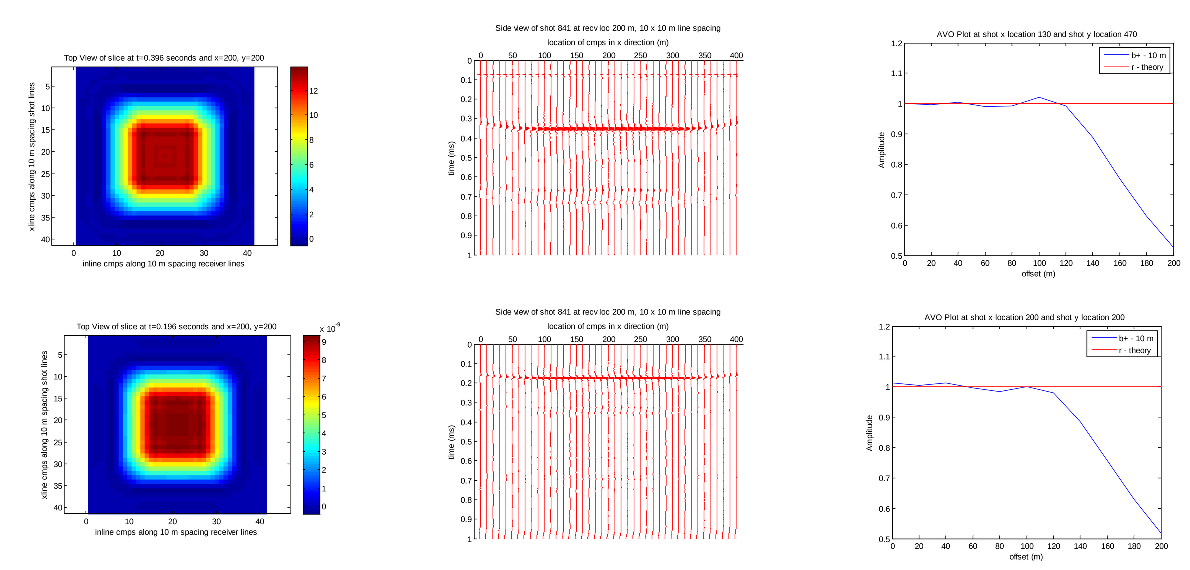
- Flat Reflector – top v(z) vs bottom v(c)
- ❖ Shot location x = y = 0 m



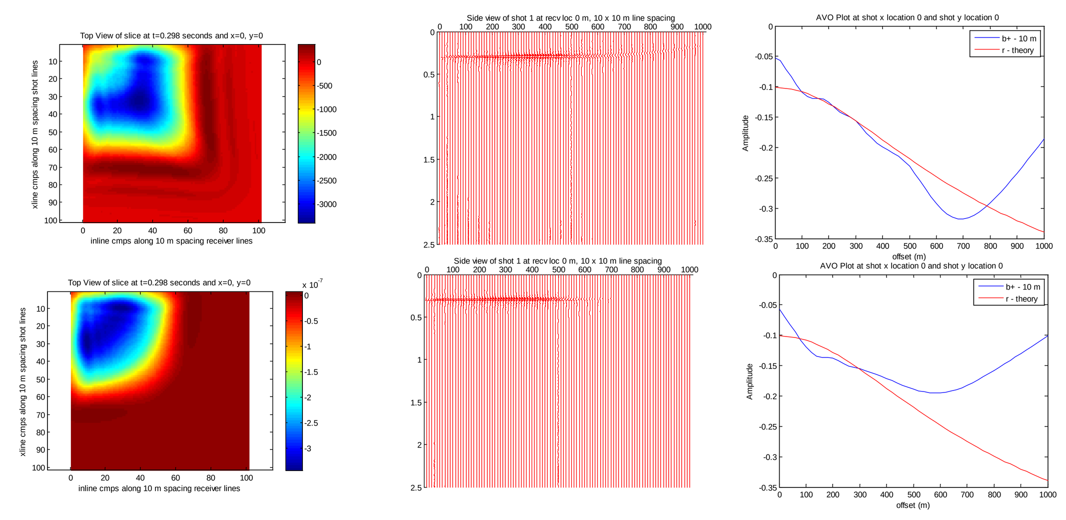
- ❖ Shot location x = y = 100 m



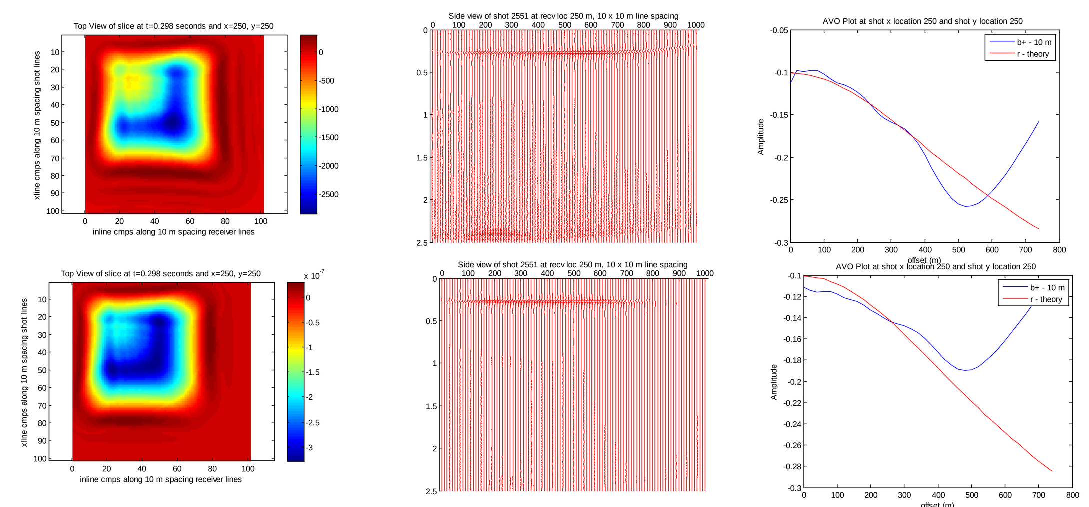
- ❖ Shot location x = y = 200 m



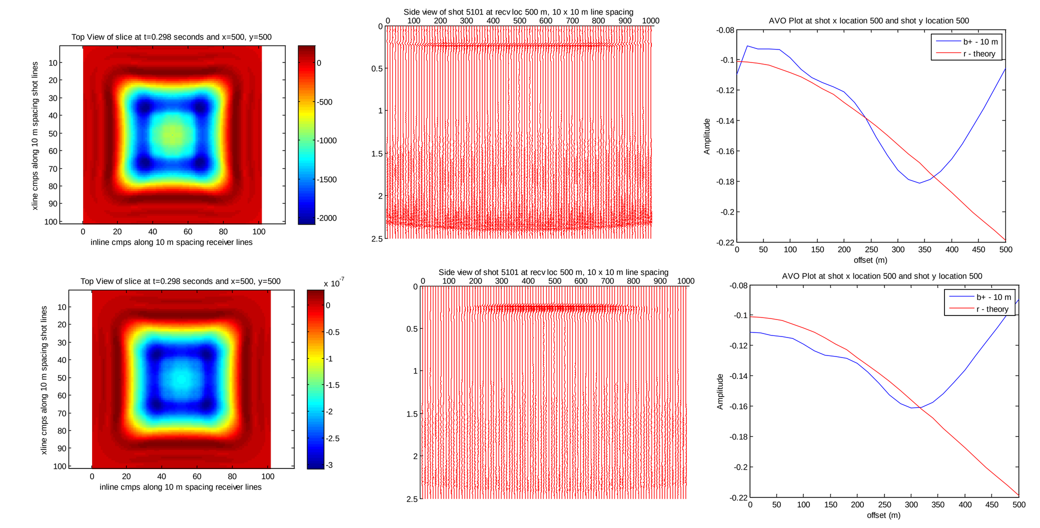
- Simple AVO 3 reflector: v(z) top, v(c) bottom
- ❖ Shot Location: x = y = 0 m



- ❖ Shot Location: x = y = 250 m



- ❖ Shot Location: x = y = 500 m



Conclusions

- Definite improvement of amplitudes when using v(z) algorithm
- See edge effects for all shots in corner(at x = y = 0 m)
- Adding layers and events leads to more 'volatile' responses – need to investigate more

References

Bleistein. et. al., 2001, Mathematics of Multidimensional Seismic Imaging, Migration, and Inversion: Springer.

Lahr and Margrave, 2015, Preservation of AVO after Migration, CREWES Research Report, Volume 27.