Removal of water layer multiples and peg-legs by wave-equation approach

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ABSTRACT

Removal of water-layer multiples and peg-legs is still one of the major processing problems in offshore exploration. The requirements on the optimal wave-equation approach for the suppression of such multiples are the following: 1) Without knowledge of the subsurface structure (except the approximate geometry of the water-bottom) it should correctly predict the kinematics of multiples. 2) The adaptive subtraction of the predicted multiples should be consistent with the data model (correct multiple suppression operator) and should involve as few parameters as possible. A long filter or time-varying filter used in the adaptive subtraction step changes the form of the predicted multiples, so that they fit not just the recorded multiples, but the sum of primaries and multiples. In our new wave-equation approach we try to fulfill the two essential requirements above. The approach is an extension of our data-consistent deconvolution *Remul* (Lokshtanov, 1999-A) for structures with strong inline lateral variations. If structural variations in the crossline direction are not severe and the main free-surface multiples are water-layer multiples and peg-legs, the new approach performs very well and is computationally efficient.

INTRODUCTION

The main differences between our scheme and other wave-equation methods (see references) are the following: 1) By applying adaptive subtraction of the predicted multiples in the tau-p domain, we take into account the angle-dependency of the reflection coefficients from the water-bottom; 2) We do not split the subtraction procedure into two steps – first subtraction of the 'pure' multiples and receiver-side peg-legs, then subtraction of source-side peg-legs. Due to overlapping of the receiver-side and source-side peg-legs, such splitting is possible only when we know exactly the reflection coefficients from the water-bottom, but not when we estimate them as parameters of the multiple suppression operator. In this respect our approach is similar to the work of Levin (1987); 3) The conventional approach for prediction of multiples (wavefield extrapolation through a water-layer with an irregular sea-floor) is based on the Kirchhoff integral (Berryhill & Kim, 1986). In the conventional approach the extrapolation from the receiver-side is applied to the common-shot gathers, while the extrapolation from the source-side is applied to the common-receiver gathers. In our scheme the prediction of multiples is performed in the same domain as used for multiple subtraction. The prediction procedure starts from the Radon transformed input CMP gathers and results in the Radon transformed CMP gathers of the predicted multiples. Therefore no additional sorting or additional Radon transformations are required.

The comparison above of our scheme with other schemes does not include the SRME approach (Berkhout et al., 1997; Verschuur et al., 1997; Weglein, 1999) which at least theoretically is the most general wave-equation method for suppression of free-surface multiples (water-layer multiples and peg-legs included). The main objection of this author to the SRME approach is the following. The SRME iterations do indeed converge to the correct result (response without free-surface multiples) when the correct weights are used at each iteration, but not when they are defined from the minimum energy criterion. According to the data model, the second iteration of SRME should attack multiples of second and higher orders, while the multiples of the first-order should already have been removed by the first iteration. But they are not removed when the amplitude weights at the first iteration are derived from the minimum energy criterion. So the second iteration as well as the first will do something very different from what is dictated by 'ideal' theory. For a hard water-bottom the problem increases with decreasing water depth – more iterations are needed and the multiples of different orders interfere while requiring different amplitude corrections. This fundamental practical limitation of the SRME (even for 1D structures) can be partly overcome by combining the first iteration for multiple prediction and a pattern recognition technique for multiple suppression (Spitz, 2000). At the same time, such a combination fails in the simplest situation of 'locally' 1D structure when the multiples have the same pattern as primaries.

Note that in our approach we also use the minimum energy criterion to estimate the sea-floor reflection coefficients entering the multiple suppression operator, but in contrast to the SRME scheme, for any water depth we need to estimate at most three reflection filters (see formula 3). These filters are estimated simultaneously by suppressing all water-layer multiples and peg-legs in one step. Nevertheless the procedure can fail if the primaries are strongly correlated at the period of multiples.

SUPPRESSION OF WATER-LAYER MULTIPLES AND PEG-LEGS BY WAVE-EQUATION APPROACH

The rigorous derivation of our wave-equation multiple suppression operator is given in Lokshtanov (1999-B). Here we will follow a more intuitive approach. Denote by D the pre-stack data (in whatever domain) along the profile. Denote by P_g the operator for receiver-side extrapolation of the input data through the water-layer. P_g includes the propagation of the recorded wavefield down to the sea-floor, reflection from the sea-floor (multiplied by the reflection coefficient of the free-surface) and propagation up to the free-surface. As a result of such extrapolation the primary water-bottom reflection is 'transferred' to the first-order multiple; the first-order multiple is 'transferred' to the second-order multiple; each primary reflection from below the water-bottom is 'transferred' to the first-order receiver-side peg-leg; each first-order receiver-side peg-leg is 'transferred' to the second-order peg-leg and so on. Therefore the operator $(I + P_g)$ applied to the data D removes all 'pure' water-layer multiples and water-layer receiver-side peg-legs: the result $F_1 = (I + P_g)D$ is free from all 'pure' water-layer multiples and receiver-side peg-legs. Suppose now that we exchange the positions of sources and receivers using the reciprocity principle (neglecting the difference in source and receiver directivity patterns). The source-side peg-legs for the original geometry become the receiver-side peg-legs for the new geometry. Similarly to the previous step, these peg-legs can be removed from data F_1 by the operator $(I+P_s)$ where P_s is the source-side extrapolation operator. Note that the result F_1 contains the primary reflection from the water-bottom. Consequently the operator P_s applied to F_1 creates the first-order 'pure' water-layer multiple which is already removed from F_1. Therefore the data F without all 'pure' water-layer multiples and peg-legs can be obtained as follows:

$$F = \left(I + P_s\right)\left(I + P_g\right)D - P_s D_w, \tag{1}$$

where D_w is the primary reflection from the water-bottom. This reflection is easily separated from the rest of the data in the tau-p domain. The formula (1) can be rewritten as:

$$F = D + D_g + D_s + D_{sg} \ , \tag{2}$$

where $D_g = P_g D$, $D_s = P_s(D - D_w)$, $D_{sg} = P_s P_g D$. So far we have assumed that the extrapolation operators include the reflection coefficients of the water-bottom. In practice, we do not know these coefficients. Therefore we calculate the results D_g, D_s, D_{sg} (see next section) assuming that the reflection coefficients are equal to one for all angles of incidence and all reflection points along the sea-floor. Such extrapolation predicts correctly the kinematics of multiples, but not their amplitudes. Consequently, all extrapolation results in (2) should be properly scaled. The 'scaled version' of (2) is applied CMP by CMP, trace by trace. For each p-trace the operator has the form:

$$f(\tau) = d(\tau) + r_g(\tau) * d_g(\tau) + r_s(\tau) * d_s(\tau) + r_{sg}(\tau) * d_{sg}(\tau), \tag{3}$$

where $d(\tau), d_g(\tau), d_s(\tau), d_{sg}(\tau)$ are p-traces for the input data and the results of extrapolation from the receiver-side, source-side (of muted input data) and source-side after receiver-side respectively. The filters $r_g(\tau), r_s(\tau), r_{sg}(\tau)$ account for angle-dependent reflection coefficients from the water-bottom and small phase-shifts due to imperfect knowledge of the water-bottom geometry. The filters are estimated from the criterion of minimum energy of f. Note that for 'locally' 1D structure the operator (3) is transformed into our single-channel data-consistent deconvolution operator *Remul* (Lokshtanov, 1999-A).

EXTRAPOLATION OF THE RADON TRANSFORMED CMP GATHERS

The multiple suppression operator (3) requires the Radon transformed CMP gathers of input data and of extrapolation results. The general case of irregular sea-floor is considered in Lokshtanov (1999-B). Here we describe an efficient procedure for calculation of the required gathers assuming 'locally' horizontal water-bottom and arbitrary 2D structure below it. Note that in contrast to the conventional phase shift extrapolation our procedure does not require separate FK or Radon transforms (of common shots and common receiver gathers) for source-side and receiver-side extrapolations. Assume that CMP coordinate y increases in the shot direction. Then the input Radon transformed CMP gathers $D(p,y)$ can be represented as follows (Lokshtanov, 1999-B):

$$D(p,y) = \frac{\omega}{2\pi}\int R(p,p_d)\exp\{i\omega p_d y\}dp_d, \tag{4}$$

where $R(p,p_d)$ is the complex (frequency dependent) amplitude of the reflected plane wave with slowness p_g due to the incident plane wave with slowness p_s; $p_g = p - p_d/2$, $p_s = p + p_d/2$. With these notations the results of receiver- side extrapolation $D_g(p,y)$ can be obtained as:

$$D_g(p,y) = \frac{\omega}{2\pi}\int R(p,p_d)\exp\{i\omega(p_d y + 2q_g h)\}dp_d =$$
$$\frac{\omega}{2\pi}\int D(p,x)\left\{\int \exp\{i\omega[p_d(y-x) + 2q_g h]\}dp_d\right\}dx, \tag{5}$$

where h is the 'local' water-bottom depth, while $q_g = (1/c^2 - p_g^2)^{1/2}$ is the vertical slowness; c is water velocity. The integral in the curly brackets can be calculated by the stationary phase approximation. For each pair x, y the stationary point p_d^{st} corresponds to a simple relation (figure 1): $x - y = h\ \mathrm{tg}\alpha_g$ where $p_g = c\sin\alpha_g$ and $p_g = p - p_d^{st}/2$. The extrapolation from the source-side is performed in a similar way.

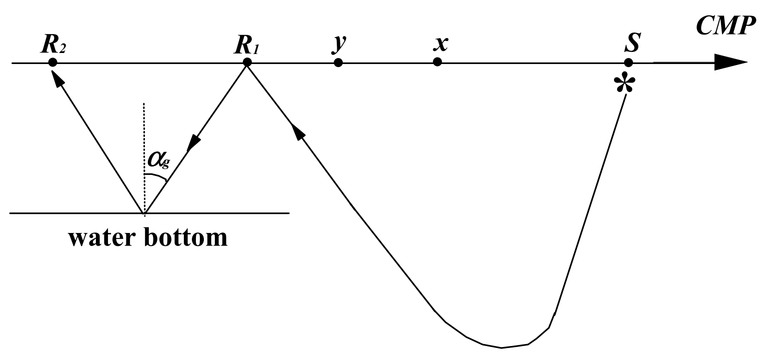

FIG. 1. Illustration of the stationary phase result: x and y are CMP positions of the primary and multiple events respectively. They are related by: $x - y = 0.5(R_1 - R_2) = h\ \mathrm{tg}\ \alpha_g$.

EXAMPLES

We generated synthetic finite difference data with all multiples for a model with an irregular water-bottom and a single complex interface below the water-bottom. The data were CMP sorted and Radon transformed. Figure 2 shows constant P sections (the same *p*-trace for all CMPs) for the input data (left), for data after our new wave-equation (WE) multiple suppression (centre) and after *Remul* (right). The result after WE is almost perfect, while after *Remul* we still have residuals of water-layer peg-legs from the second reflector. Note the difference in the positioning of the source-side and the receiver-side peg-legs in the input data. The kinematics of these peg-legs is correctly predicted by the WE approach, but not by *Remul*. The difference in the results of these two approaches increases with increasing slowness *p* (or increasing offsets or incident angle).

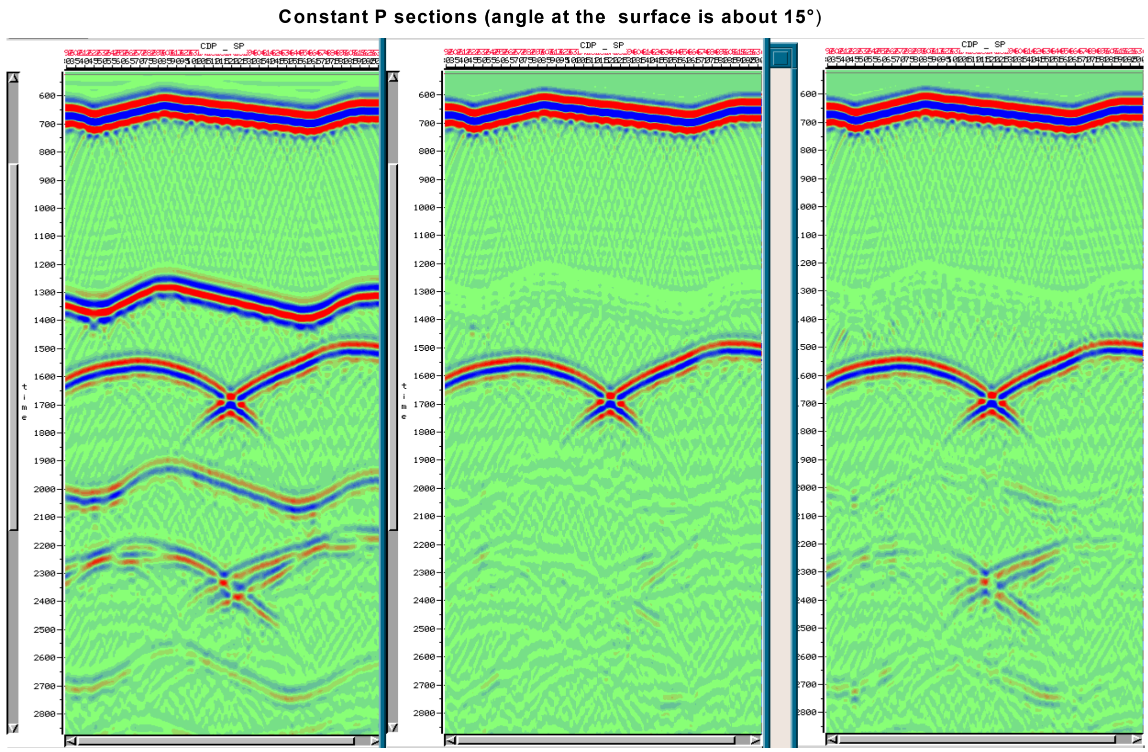

FIG. 2. Constant *P* –section before multiple suppression (left), after multiple suppression by the new wave-equation (WE) approach (centre) and after *Remul* (right).

Figure 3 shows real data stacks before and after WE multiple suppression. Both stacks were created with the same velocity and mute libraries. The results on pre-stack level are given in Figure 4. It shows constant *P* sections (the same *p-trace* for all CMPs) before multiple suppression, after WE multiple suppression and the difference. Note how the weak dipping primaries are extracted from below strong water-layer peg-legs from Top and Base Cretaceous. Other synthetic and real data examples can be seen in (Lokshtanov, 2000).

FURTHER DEVELOPMENTS

An interesting extension of the operator (3) was suggested by Hugonnet (2002). His work is a trade off between the wave-equation and SRME approaches. He calculates the terms required by (3) (extrapolation from the receiver-side, source-side and source-side

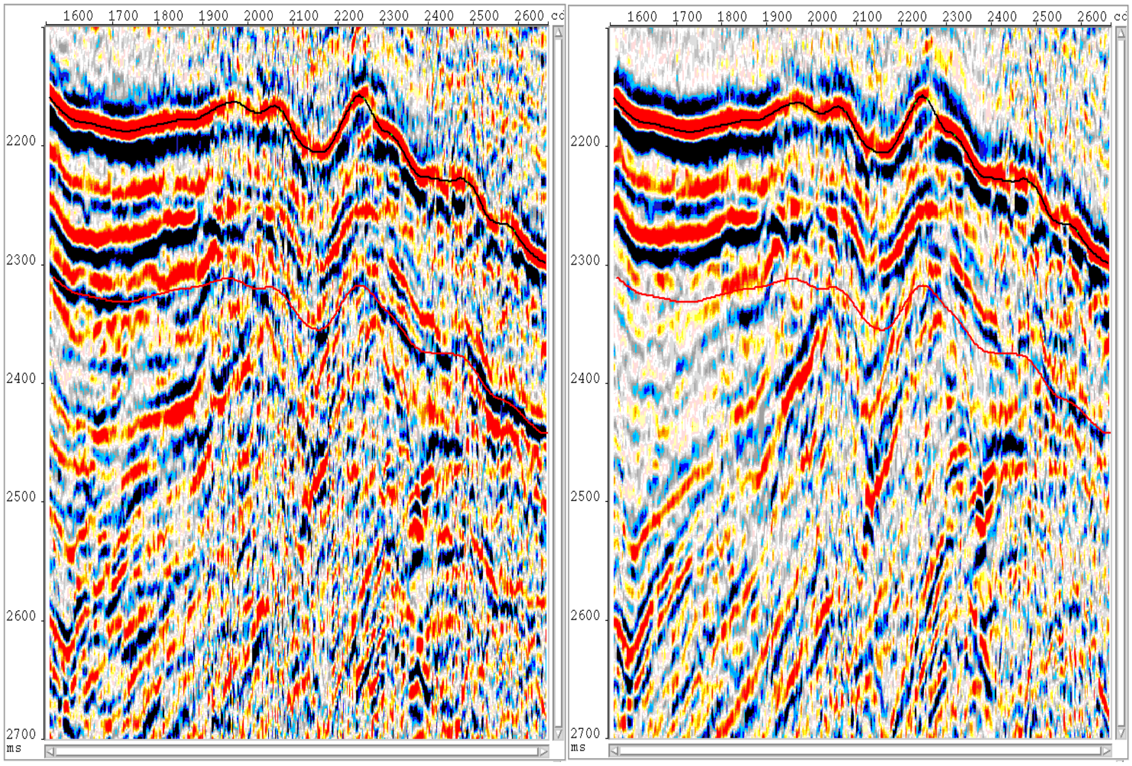

FIG. 3. Stack before multiple suppression (left) and after WE multiple suppression (right). The pink line shows the expected position of the first-order water-layer peg-leg (expected blue event) from the Top Cretaceous (black line). The multiple period is about 140 msec.

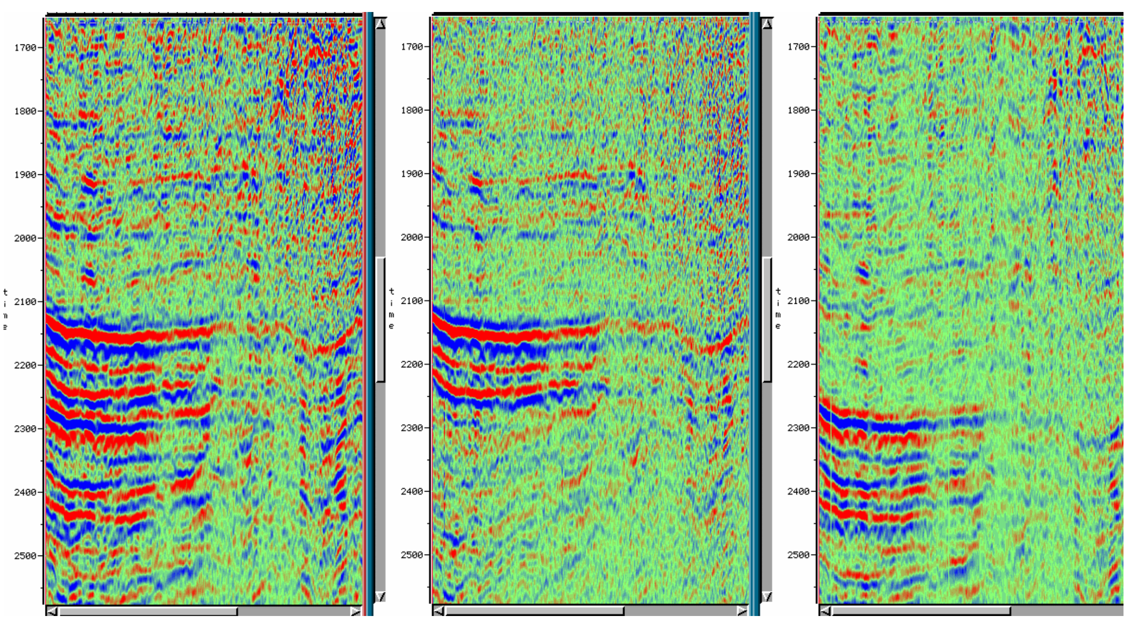

FIG. 4. Constant *P* sections for input data (left), after WE multiple suppression (centre) and the difference (right). The corresponding vertical angle of wave propagation at the surface is about 8°. Note how the weak dipping primaries are extracted from below strong multiples.

after receiver-side) by convolving the input data with the primary reflection response from a 'shallow' part of the structure. Therefore the predicted multiples are not just water-layer multiples and peg-legs, but all free-surface multiples generated by a shallow part of the structure. He uses (3) for adaptive subtraction of the predicted multiples in one step, thereby avoiding SRME iterations. The filters in (3) are no longer reflection coefficients from the sea-floor, but designature operators.

CONCLUSIONS

If structural variations in the crossline direction are not severe and the main free-surface multiples are water-layer multiples and peg-legs, our wave-equation (WE) approach performs very well and is computationally efficient. In the WE approach the kinematics of multiples are predicted from the given water-bottom geometry. Therefore the adaptive subtraction procedure requires fewer unknown parameters than the multichannel version of *Remul* with a large number of channels. As a consequence, in the WE approach the primaries are better preserved while the multiples are better suppressed.

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