Seismic modelling with the reflectivity method

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ABSTRACT

Numerical seismic modelling is a powerful tool in seismic imaging, interpretation and inversion. Wave propagation in complicated earth models can be simulated, thanks to sophisticated modelling algorithms and high performance computing technology. Although wave equation based modelling methods and other well developed modelling techniques, such as finite difference method and ray tracing methods (Carcione, Herman, and Kroode, 2002), are playing more and more important roles in numerical modelling, reflectivity modelling is still widely used due to its unique properties. This method can model almost all kinds of waves propagating in elastic or anelastic media with high numerical stability and accuracy but relatively less computation cost. Reflectivity modelling can also simulate wave propagation in fine layered earth models. The main purpose of this paper is to give an introduction on the reflectivity modelling method, demonstrate the modelling results using the reflectivity modelling method and release our modelling software to our sponsors.

INTRODUCTION

The reflectivity modelling method was at first proposed by Thomson (1950). Haskell (1953) modified the method to simulate surface wave propagation. This modelling method represents wave propagation in the frequency-wavenumber domain, and it mainly deals with coefficient (or propagator) matrix computation in the frequency-wavenumber domain (Kennett, 1975; Kind, 1976; Kennett, 1983; Müller, 1985). Finally, by use of Fourier transforms, the seismic modelling results can be transformed back into the time-space domain. Excellent work was done by Fuchs and Müller (1971) in the early 1970s. Their work was followed by Kennett (1975, 1979, 1983), Kind (1976), Kennett and Kerry (1979), Kennett and Illingworth (1981), Fryer (1981), and Kennett and Clarke (1983).The theories of reflectivity modelling are fully presented in Kennett (1983). A good tutorial on the reflectivity modelling method may be found in Müller (1985). Reflectivity modelling is always carried out in a cylindrical coordinate system, through which one can conveniently reduce wave equations into 1D. The modelling theory describes wave behaviour in stratified earth models in a convenient way, where all wave types can be decomposed into upgoing and downgoing waves; and waves can be decoupled into P-SV and SH wave types (Kennett, 1983). Reflections, transmissions, conversions of all wave modes, and the corresponding multiples inside sandwiched (thin) layers inserted between two half spaces or a free surface and a half space can be fully modelled. Moreover, modelling in the frequency-wavenumber domain makes it easy to handle absorptions in anelastic media (Temme and Müller, 1982).

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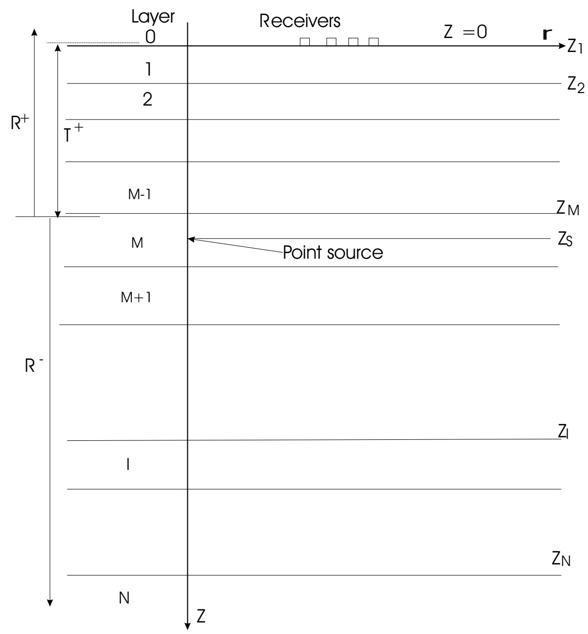


FIG. 1. A layered earth model (after Müller (1985)).

THEORY

Starting from the equation of motion and a constitutive relation, the wave equation used in the reflectivity modelling method can finally be written as

$$\frac{\partial \mathbf{b}}{\partial z} = \omega \mathbf{A}\mathbf{b} + \mathbf{F}, \tag{1}$$

(Kennett, 1983), where $\mathbf{b}$ is a vector composed of displacements and traction, $\mathbf{A}$ is a matrix reflecting the properties of a medium through which waves propagate, and $\mathbf{F}$ is driving force.

There are two main methods to calculate responses on the surface. One is called the *“propagator matrix”* (Aki and Richards, 1980; Kennett, 1983); the other is called the *“reflectivity method”* (Müller, 1985). Our codes follow the algorithms from Müller (1985).

The boundary condition for wave propagation in such a stacking-layer model is: the displacement and traction (vector $\mathbf{b}$) are continuous when waves cross interfaces. In the propagator matrix method, vector $\mathbf{b}$ is transformed into a wave vector related to reflection and transmission coefficient matrices at interfaces (Kennett, 1983).

In the stratified system shown in Figure 1, Müller (1985) gave the following formulas to calculate far field P-SV waves at the surface ($z = 0$).

$$\begin{pmatrix} u_r \\ u_z \end{pmatrix} = \frac{\omega}{4\pi\rho_m} \sum_{i=1}^{2} \epsilon_i \int_0^\infty \mathbf{J_i U T^+ [I - R^- R^+]^{-1} (S_i^u + R^- S_i^d)} du, \tag{2}$$

where u_r and u_z are the radial and vertical components of displacement, respectively; ρ_m is the density at level z_m; $\mathbf{J_i}$ denotes matrices related to Bessel functions of the first kind, i.e., J_0 and J_1; the arguments of Bessel functions are $u\omega r$, which are not written out; $\mathbf{R}^+$ is the total reflection coefficient matrix between z_m and $z_1(=0)$; $\mathbf{R}^-$ is the total reflection matrix between the lower half space and z_m; T^+ is the total transmission matrix between z_m and z_1; $\mathbf{S_i^u}$ and $\mathbf{S_i^d}$ are source radiation terms in the upgoing and downgoing directions, respectively; and ϵ_i is related to the magnitudes of a point source buried at depth z_s (see Figure 1). For explicit formulas regarding the terms in equation (2), see Müller (1985, Eqns. (46), (47), (66), (67), (71), (72), (81) and (82)).

We give a brief summary of the calculation of those coefficient matrices in (2). From the bottom of the model (the lower homogeneous half space) waves cross the first interface and pass through the first homogeneous layer above the half space. These two procedures (crossing interface and passing through a layer) will be repeated as waves transverse more layers in order to reach the free surface or enter the upper half space. Coefficient matrices at an interface corresponding to upgoing and downgoing waves can be calculated with densities and slowness on both sides of an interface (Müller, 1985, Table 1 and Table 2). When waves travel within a homogeneous layer, they experience phase shift. In computation, we can use a *recursive algorithm* (Müller, 1985, Eqns. (22) and (31)) to obtain total coefficient matrices.

$$\mathbf{MT_i} = \mathbf{E_i MB_i E_i} \tag{3}$$

and

$$\mathbf{MB_i} = \mathbf{R^d_{i+1}} + \mathbf{T^u_{i+1}}[\mathbf{I} - \mathbf{MT_{i+1} R^u_{i+1}}]^{-1} \mathbf{MT_{i+1} T^d_{i+1}}, \tag{4}$$

(Müller, 1985) where i and $i+1$ denote layer numbers; super script u denotes upgoing; super script d denotes downgoing; $\mathbf{MT_i}$ or $\mathbf{MB_i}$ is the reflectivity matrix on top or at the bottom of layer i, respectively; $\mathbf{T_{i+1}}$ is the transmission matrix in layer $i+1$; $\mathbf{E_i}$ is a phase shift term in layer i.

MODELLING EXAMPLES

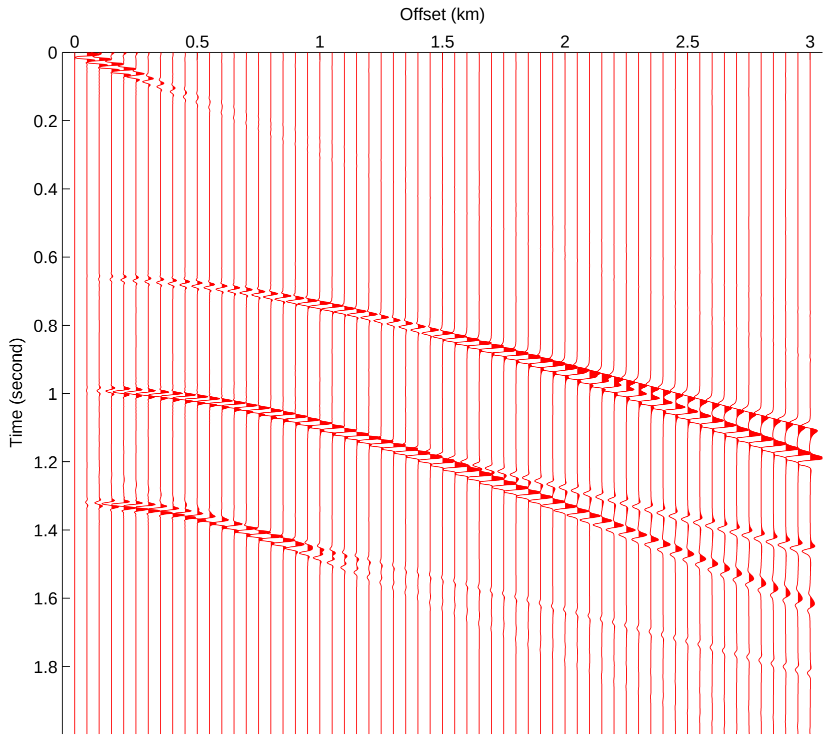
Modelling primary waves and their conversions

The modelling results will illustrate that our modelling codes can simulate partial responses as required. The physical parameters of a layered earth model are shown in Table 1.

Table 1. A two-layered earth model

Layer	thickness (km)	α (km/sec)	β (km/sec)	Q_p	Q_s	ρ (g/cm^3)
1	1.0	3.0	1.5	1000	1000	2.0
2	2.0	5.5	3.0	1000	1000	3.5

a)



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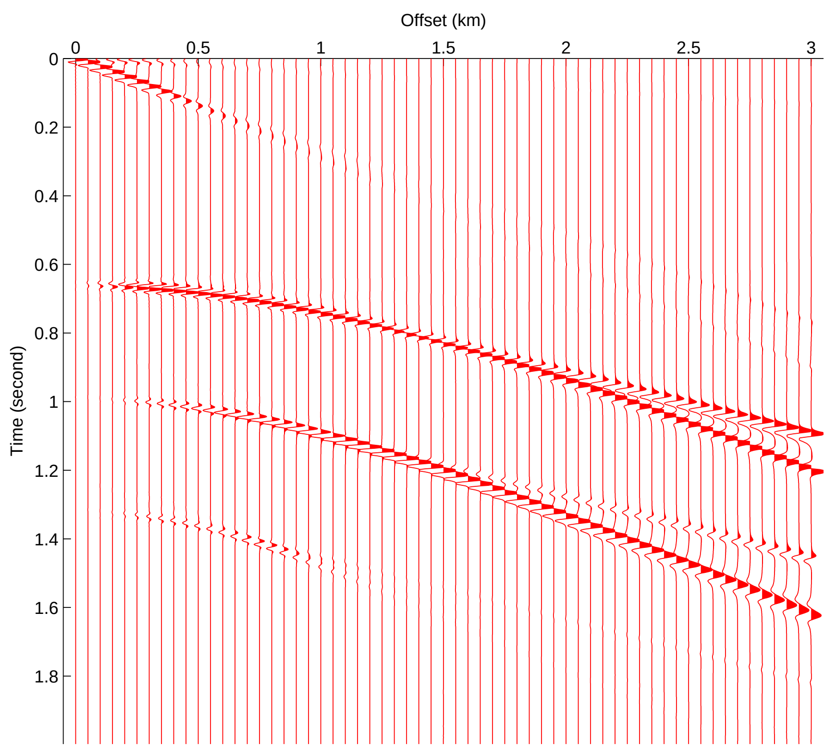
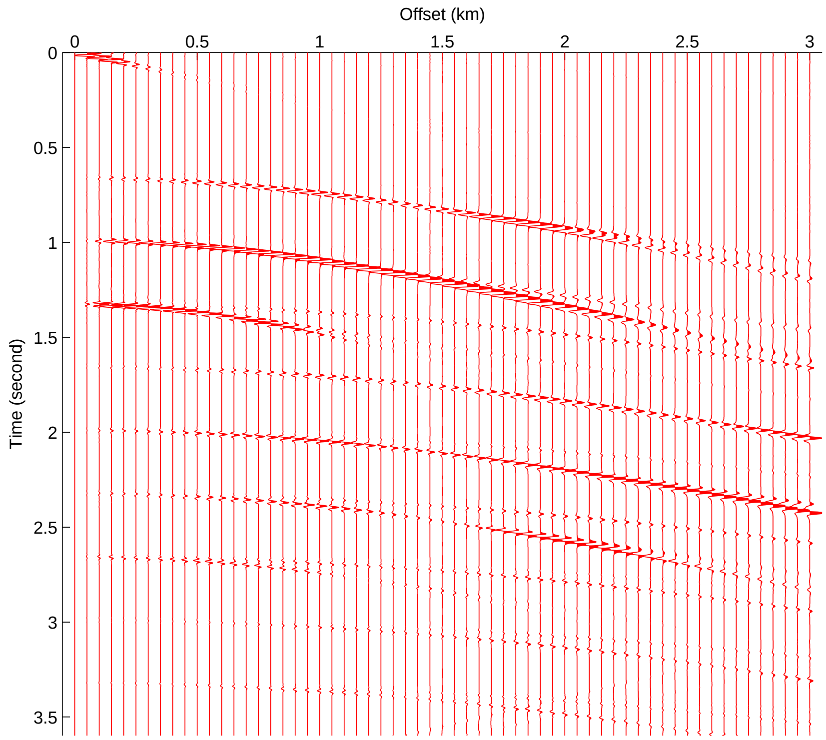


FIG. 2. Displacements of early arrivals (P-SV waves) in a two-layered earth model. a) Radial component; b) Vertical component. Plotted with trace equalization.

a)



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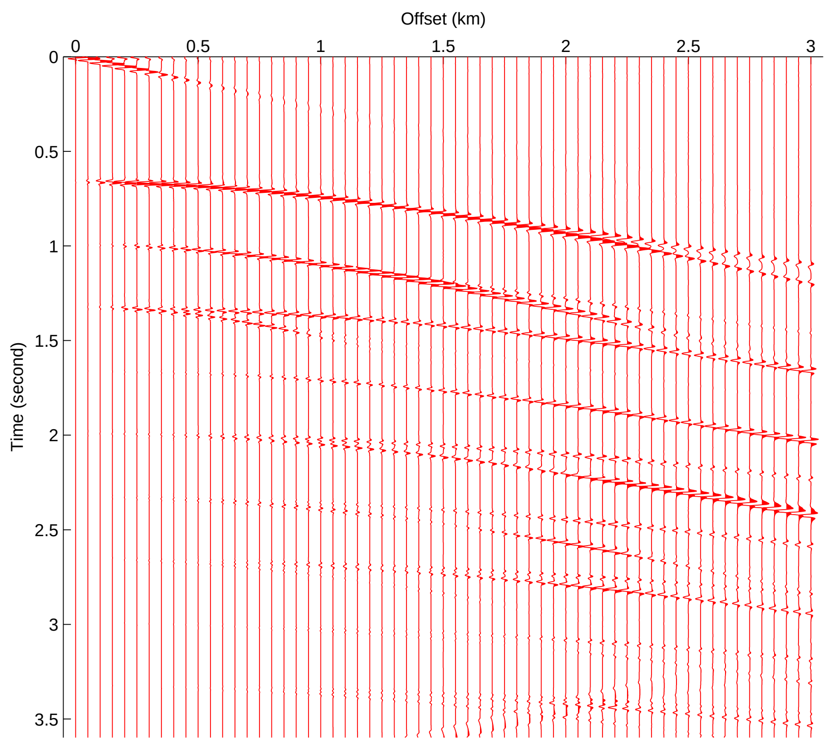

FIG. 3. Displacements of multiples and early arrivals (P-SV waves) in a two-layered earth model. a) Radial component; b) Vertical component. Plotted with trace equalization.

In Table 1, α denotes P wave velocity, β is S wave velocity, Q_p and Q_s are Q factors of P waves and S waves, respective, and ρ is the density of the medium.

In the first model, we do not calculate multiples between the free surface and the reflector. In our modelling codes, parameters can be set up to model with or without direct arrivals or multiples, which shows that the reflectivity modelling algorithms are versatile. Since the point source radiates both P (P) and S (S) waves, we can see both reflected P waves (PP) and S waves (SS) corresponding to the incident P waves and S waves among the early arrivals (Figure 2 a) and b)). In this model (parameters shown in Table 1), we can work out the first arrival of reflected P (PP) wave at zero offset will be at t = 0.67 second and that of reflected S (SS) wave will be at t = 1.33 second; we can also calculate the arrival times for conversions from P to S (PS) and S to P (SP). Their arrival times are between those of reflected P (PP) and reflected S (SS) waves. Head waves at or beyond critical angles (corresponding to different wave types: PP, SS, etc.) can also be observed in synthetic seismograms. In this simple single reflector earth model, all reflections and converted wave modes predicted in seismic theory have been presented to us, which proves that our modelling software gives correct modelling results.

Modelling multiples

The second model is designed to show multiples within the same earth model as shown in Table 1. We can see in Figure 3 how complicated those multiples are even though we only have a single reflector in the model; these modelling results show the power and beauty of the reflectivity modelling technique. Besides multiples, those early arrivals demonstrated in the first model (PP, SS, PS or SP) can also be seen in Figure 3. This example shows us a full response modelling given by the reflectivity modelling method.

CONCLUSIONS

Our modelling results confirm the conclusion that the reflectivity modelling method is a very useful and powerful modelling tool in stratified earth models. Through the modelling results we can see that all wave modes in a stratified earth system can be clearly simulated. The reflectivity modelling method can supply sufficient information in layered earth models.

However, reflectivity modelling has a weakness; that is, it is truly a 1D modelling method. When we try to model more complicated earth models, such as heterogeneous media, we have to turn to other modelling methods.

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