Estimating anisotropy parameters in layered VTI media

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ABSTRACT

Anisotropy parameters in a VTI medium are obtained by anisotropic velocity analysis performed on short-spread or long-spread P-wave reflection-seismic data, in combination with check-shot or well-log data. Analysis of four reflection-traveltime approximations to the actual reflection traveltime in weakly anisotropic media shows that each reflection-traveltime approximation has its own requirements for spread length and subsurface anisotropic parameters. The accuracy of the estimated Thomsen anisotropy parameter δ depends not only on the accuracy of the picked NMO velocity but also on the subsurface anisotropy parameters. The smaller the value of $(\varepsilon - \delta)$, the higher the accuracy of the estimated δ value. The results of the four reflection-traveltime inversions by semblance analysis for synthetic seismic examples demonstrate that in estimating δ, the nonhyperbolic and the shifted-hyperbolic estimations are better than the three-term Taylor-series method, which, in turn, is better than hyperbolic estimation. Only the nonhyperbolic approximation can be used to estimate the anisotropy parameter ε accurately.

INTRODUCTION

Alkhalifah and Larner (1994) showed that accurate 2-D imaging in transversely isotropic media requires good knowledge of the Thomsen anisotropy parameters δ and ε. There are various traveltime inversion approaches for estimating anisotropy parameters (Alkhalifah and Tsvankin, 1995; Brown et al., 2000; Elapavuluri and Bancroft, 2002; Gaiser, 1990; Isaac and Lawton, 2004; White et al., 1983) but each has its own assumptions and limitations. Thomsen (1986) derived relations between normal-moveout (NMO) velocities and anisotropy parameters in a homogeneous anisotropic layer. In combination with check-shot or well-log data, we are able to use various analytic reflection-traveltime approximations over limited spread-lengths to obtain anisotropy parameters in VTI media by NMO-velocity analysis and through a Dix-type differentiation procedure. Besides hyperbolic approximation, a popular approach for estimating anisotropy is a modified three-term Taylor series approximation to the reflection moveout curve (Tsvankin and Thomsen, 1994; Alkhalifah and Larner, 1994; Tsvankin, 1995).

If one ignores the contribution of the vertical shear-wave velocity, a modified three-term Taylor-series approximation to the reflection moveout curve can be fully determined by two parameters, V_{NMO} (NMO velocity) and η ($=(\varepsilon - \delta)/(1+2\delta)$; Alkhalifah and Tsvankin, 1995), or by V_{NMO} and V_{h} (horizontal velocity). Based on the nonhyperbolic moveout equation developed by Tsvankin and Thomsen (1994), a 2-D semblance scan can be used to estimate anisotropy parameters. For convenience, we refer to this method as nonhyperbolic reflection-traveltime inversion. Elapavuluri and Bancroft (2002) showed the shifted hyperbolic approximation can also be used to estimate anisotropy parameters from P-wave reflection data.

In this paper, we compare the traveltime approximations of four reflection-traveltime inversions (hyperbolic, modified three-term Taylor-series, shifted-hyperbolic and nonhyperbolic traveltime inversions) with the exact traveltimes in VTI media. We then carry out these four inversions on synthetic seismic data examples and try to determine the relationship between the estimated anisotropy parameters and the true anisotropy parameters. Finally, we formulate some conclusions for guiding the application of these approximations.

REFLECTION TRAVELTIME APPROXIMATION

Using an approximation of the exact eikonal equation in the quasi-compressional case for so-called weak anisotropy (Daley, 2001) and relations between phase and group velocity and between phase and group angle (Thomsen, 1986), we developed multilayer ray-tracing code for modelling real traveltime-offset curves (blue solid line in Figure 1).

The P-wave traveltime approximations for four reflection-traveltime inversion methods are given as follows.

1) The hyperbolic reflection-traveltime approximation:

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\mathrm{NMO}}^2}, \tag{1}$$

where

$$V_{\mathrm{NMO}}^2(P) = \alpha_0^2(1+2\delta), \tag{2}$$

α_0 is vertical velocity for P waves, δ is Thomsen's anisotropy parameter, V_{NMO} is NMO velocity, and t_0 and t are the two-way traveltimes for zero-offset and offset x, respectively.

2) The modified three-term Taylor-series approximations (Tsvankin and Thomsen, 1994) limited to weak anisotropy:

$$t^2(x) = t_0^2 + A_2x^2 + \frac{A_4x^4}{1+Ax^2}, \tag{3}$$

where,

$$A_2 = \frac{1}{V_{\mathrm{NMO}}^2} = \frac{1}{\alpha_0^2(1+2\delta)},\ A = 1/(\alpha_0 t_0)^2 \text{ and } A_4(P) = -\frac{2(\varepsilon-\delta)}{t_0^2\alpha_0^4}, \tag{4}$$

and the parameters A_2 and A_4 are Taylor-series coefficients.

3) The shifted-hyperbolic approximation (Castle, 1994):

$$t = \tau_s + \sqrt{\tau_x^2 + \frac{x^2}{SV_{\mathrm{NMO}}^2}}\,, \tag{5}$$

where

$$\tau_s = t_0(1-\frac{1}{S})\,,\ \tau_x = \frac{t_0}{S}\,, \tag{6}$$

$$S = 1 + \frac{8(\varepsilon-\delta)}{(1+2\delta)}, \tag{7}$$

and S is the shift parameter.

4) The nonhyperbolic approximation:

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\mathrm{NMO}}^2} - \frac{[V_{\mathrm{h}}^2 - V_{\mathrm{NMO}}^2]x^4}{V_{\mathrm{NMO}}^2[t_0^2V_{\mathrm{NMO}}^4 + V_{\mathrm{h}}^2x^2]}\,. \tag{8}$$

For P-waves, the fourth-order Taylor-series coefficients, valid for arbitrary transverse isotropy (Tsvankin and Thomsen, 1994), are:

$$A_4(P) = -\frac{2(\varepsilon-\delta)}{t_0^2\alpha_0^4}\frac{\left(1+\dfrac{2\delta}{1-\beta_0^2/\alpha_0^2}\right)}{(1+2\delta)^4}\,, \tag{9}$$

and

$$A(P) = \frac{A_4}{V_{\mathrm{h}}^{-2} - A_2}\,,\ V_{\mathrm{h}}^2 = \alpha_0^2(1+2\varepsilon)\,. \tag{10}$$

where β_0 is vertical SV-wave velocity; ε is a Thomsen's anisotropy parameter, and V_{h} is horizontal velocity for P-waves. If one ignores the contribution of the vertical shear-wave velocity, which is negligible (Tsvankin and Thomsen, 1994; Tsvankin, 1995), we obtain the nonhyperbolic reflection-traveltime approximation (equation (8)) for P-waves in VTI media by substitution of equations (9) and (10) into equation (3).

Figure 1 shows examples of all four approximations to the exact reflection traveltime in which the following can be observed:

1) When $\varepsilon - \delta = 0$ (elliptical anisotropy), the three approximations reduce to the exact traveltime (Figure 1a).

2) When $0 < \varepsilon - \delta \leq 0.2$, all four closely approximate the exact traveltime for short spreads, but the hyperbolic and the three-term Taylor-series traveltimes deviate increasingly from the actual traveltime with increasing spread length (Figure 1b).

3) When $\varepsilon - \delta > 0.2$, both the shifted hyperbolic and nonhyperbolic traveltimes approximate closely the actual reflection traveltime; the three-term Taylor-series

approximation is quite poor, and the hyperbolic approximation deviates grossly from the actual reflection traveltime, even for a short spread (Figure 1c). This demonstrates that traveltime approximations depend both on anisotropy parameters and on spread length.

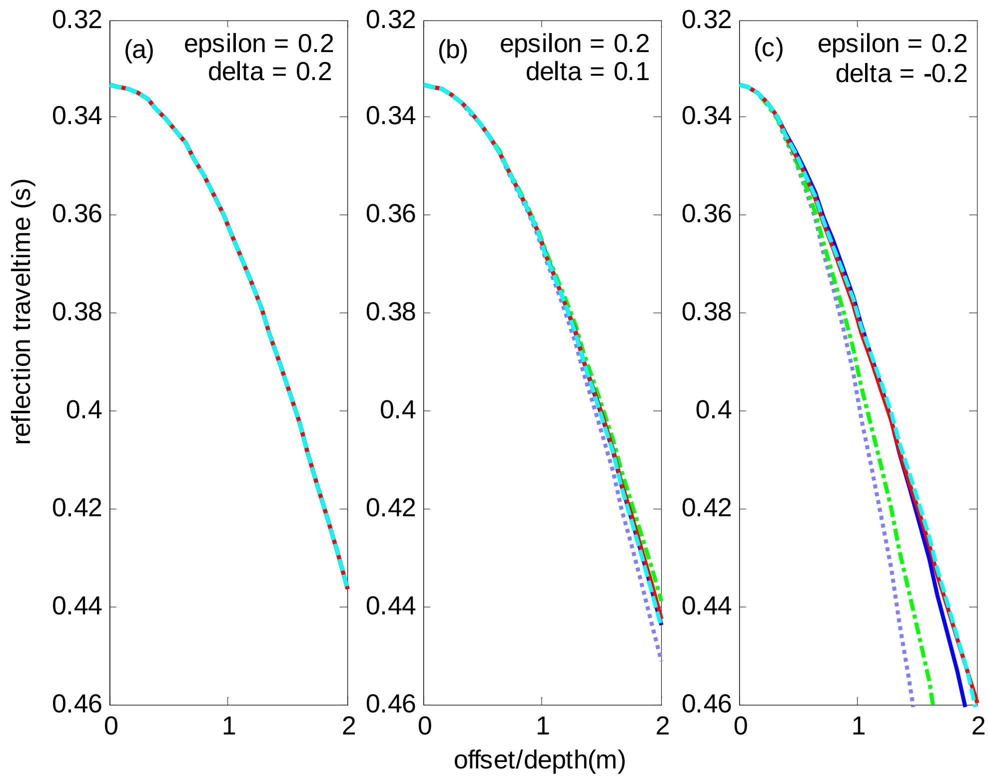
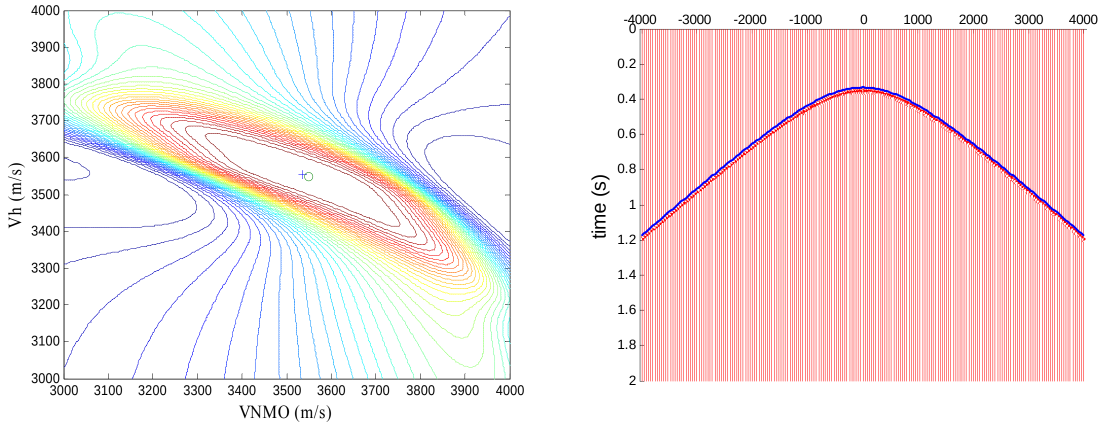


FIG. 1. Reflection-traveltime approximation to true reflection traveltime. Solid blue line: the exact traveltime; red dotted line: hyperbolic approximation; green dash-dot line: the three-term Taylor-series approximation; red solid line: the shifted-hyperbolic approximation; cyan dashed line: nonhyperbolic approximation.

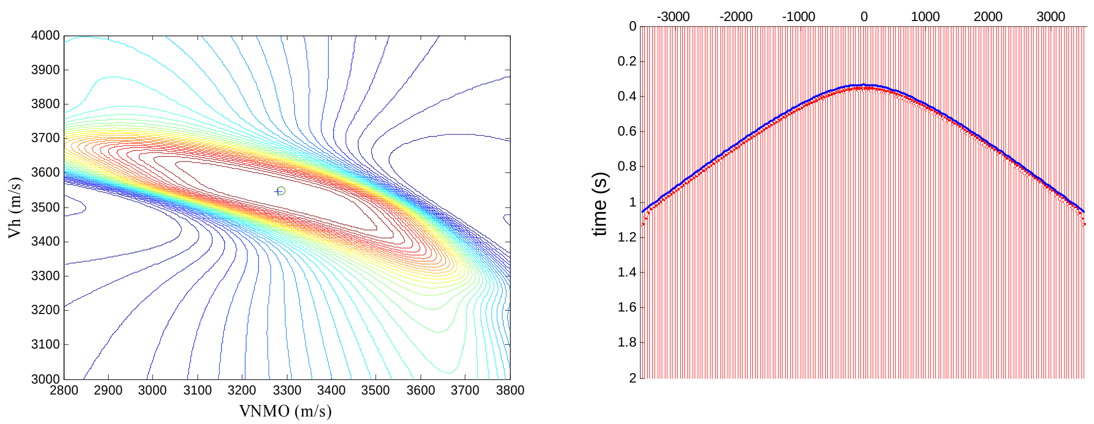
ESTIMATION OF THOMSEN'S ANISOTROPY PARAMETERS

For simplicity, we consider a series of single-layer case in order to determine how both actual anisotropy parameters and spread length affect the estimation of anisotropy parameters. The input CMP gather for anisotropy-parameter estimation contains a single reflection from a flat interface. The depth of this interface is 500 m. Vertical P- and S-wave velocities above the reflector are 3000 m/s and 1500 m/s, respectively. The values of ε are fixed at 0.2, 0.1, and 0.0, respectively, and those of δ range from –0.2 to 0.2 at increments of 0.02.

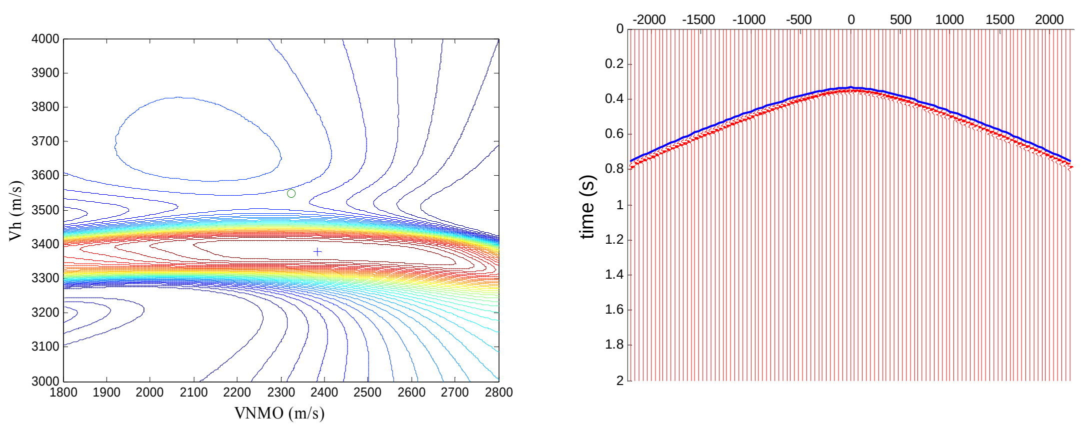
Using equations (1), (3), (5), and (8), we can pick up effective coefficients A_2, A_4, V_{NMO}, V_{h} and S, and then obtain anisotropic parameters ε and δ by using equations (2), (4), (6), and (10) through a Dix-type differentiation procedure (here, vertical P-wave velocity α_0 is known from well logs, check-shots, or VSP). Semblance scanning is employed to estimate effective coefficients.



(a)



(b)



(c)

FIG. 2. Semblance plots using nonhyperbolic approximation for anisotropic parameters: (a) $\varepsilon = 0.2$, $\delta = 0.2$; (b) $\varepsilon = 0.2$, $\delta = 0.1$; (c) $\varepsilon = 0.2$, $\delta = -0.2$.

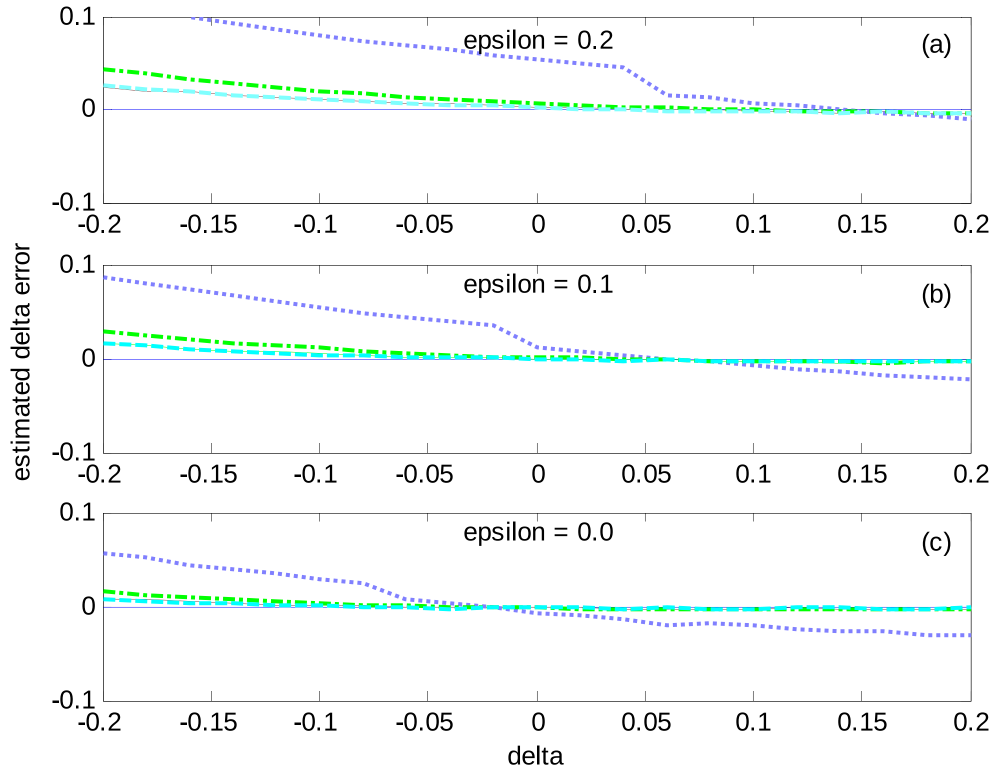


FIG. 3. The error in estimated δ plotted against true δ when offset/depth = 1.0. Blue solid line: the exact anisotropic parameters; purple dotted line: hyperbolic traveltime inversion; green dash-dot line: the modified three-term Taylor-series inversion; red solid line: the shifted hyperbolic inversion; cyan dashed line: nonhyperbolic inversion.

Taking nonhyperbolic inversion as an example, by differentiation of equations (2) and (10), the errors in the estimated parameter δ and ε is given by

$$\Delta\delta = \frac{\sqrt{1+2\delta}}{\alpha_0}\Delta V_{\mathrm{NMO}}, \; \Delta\varepsilon = \frac{\sqrt{1+2\varepsilon}}{\alpha_0}\Delta V_{\mathrm{h}} \tag{11}$$

Therefore, the errors in the estimates of the anisotropy parameter δ and ε depend not only on the accuracy of the picked NMO velocity but also on the subsurface model parameters (α_0, ε, and δ).

Figure 2 shows the semblance plots using the nonhyperbolic approximation. On the left, the symbol ○ represents the true values of V_{NMO} *and* V_{h} while + represents the estimated values. On the right, the actual seismic arrival times (red) are plotted together with the traveltime curves (blue) calculated using the estimated velocities from the left. The close fit of actual events with those calculated from the approximated velocities shows that semblance analysis is capable of estimating traveltime quite well. It can also be seen from Figure 2 that the deviations of the estimated values from the true values depend on the subsurface anisotropy parameters.

Figures 3 shows the errors in estimated δ plotted against true δ when offset/depth = 1.0, ε values are 0.2, 0.1, or 0.0, and δ ranges from –0.2 to 0.2 in increments of 0.02. From Figure 3 it appears that:

i) the smaller the value of $(\varepsilon - \delta)$, the higher the accuracy of the estimated δ value;

ii) the estimated values deviate greatly from the true values when $|\varepsilon - \delta| > 0.2$;

iii) the nonhyperbolic and the shifted-hyperbolic estimations are better than the three-term Taylor-series method, which in turn is better than hyperbolic estimation.

Figure 4 shows the errors in estimated ε plotted versus true δ when offset/depth = 2.0, ε values are 0.2, 0.1, and 0.0, and δ ranges from –0.2 to 0.2 in increments of 0.02. From Figure 4 we can see that only nonhyperbolic inversion is able to estimate parameters ε with any accuracy.

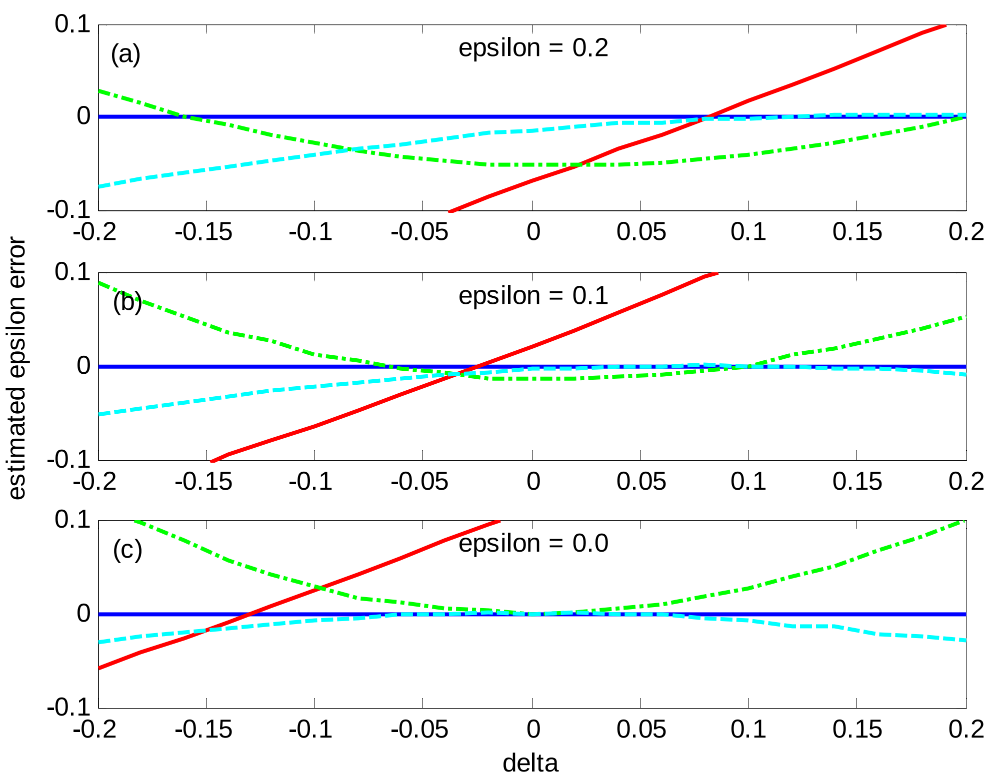


FIG. 4. The error in estimated ε plotted against true δ when offset/depth = 2.0. Green dash-dot line: the three-term Taylor-series inversion; red solid line: the shifted hyperbolic inversion; cyan dashed line: nonhyperbolic inversion.

AN EXAMPLE FOR LAYERED VTI MEDIA

Table 1 demonstrates the model parameters for a four-layer model. Note that all ($\varepsilon - \delta$) values in Model I are less than 0.2. The only difference between Model II and Model I is that the value of ($\varepsilon - \delta$) in the second layer is larger than 0.2.

Figure 5 shows estimated anisotropy-parameter values (dashed lines) and actual values (solid lines). These estimations from multilayer VTI media also demonstrate that the estimated interval anisotropy parameters are very close to the true parameter values. Only when (ε − δ) is larger than 0.2 do the estimated interval parameter values depart significantly from the true value.

Table 1. Model parameters for layered VTI media

Thickness (m)	α_0 (m/s)	β_0 (m/s)	Model I ε, δ	Model II ε, δ
500	2800	1400	0.20, 0.10	0.20, 0.10
500	3000	1500	0.15, 0.08	0.20, - 0.20
500	3200	1600	0.10, 0.04	0.10, 0.04
500	3500	1750	0.08, 0.02	0.08, 0.02

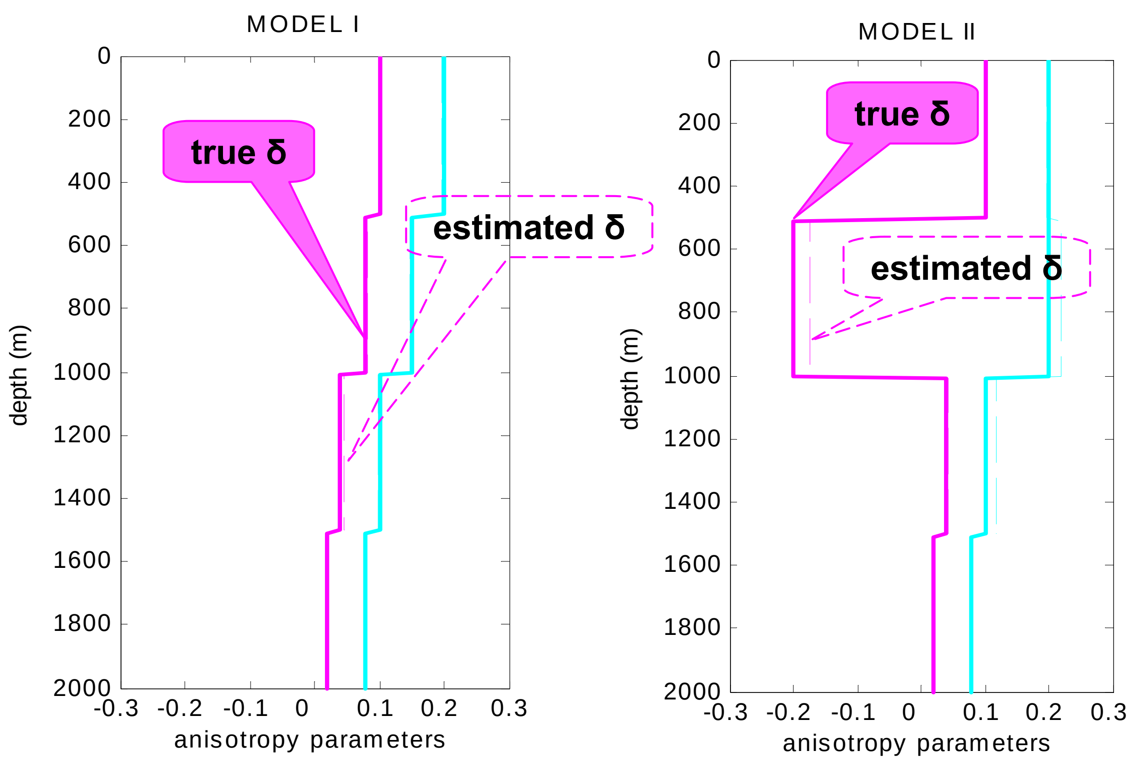


FIG. 5. Estimated anisotropy parameters for (a) Model I, and (b) Model II. Estimated δ (dashed magenta line) and ε (dashed cyan line); True δ (solid magenta line) and ε (solid cyan line);

CONCLUSIONS

The accuracy of the estimated anisotropic parameter δ depends not only on the accuracy of the picked NMO velocity but also on the value of $(\varepsilon - \delta)$. The smaller the value of $(\varepsilon - \delta)$ and the value of ε, the higher the accuracy of estimated δ. The results of the four traveltime inversions by semblance analysis for the seismic examples demonstrate that the nonhyperbolic and shifted-hyperbolic estimations are better than the three-term Taylor-series method which, in turn, is better than the hyperbolic estimation. Only nonhyperbolic inversion can be used to estimate accurately the anisotropy parameter ε.

ACKNOWLEDGEMENTS

We would like to thank the sponsors of the CREWES Project for their support.

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