Automatic selection of reference velocities for recursive depth migration

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ABSTRACT

Wave equation depth migration methods such as phase-shift plus interpolation, extended split-step Fourier, or Fourier finite difference plus interpolation require a limited set of reference velocities for efficient wavefield extrapolation through laterally inhomogeneous velocity models. In basic implementations, reference velocities are selected as either a linear or a geometric progression spanning the range of model velocities. However, it is unlikely that the model velocities are distributed linearly or geometrically. If the reference velocities can be distributed statistically to more closely approximate the actual distribution, the accuracy of the extrapolation step can be improved. In this paper, we present a modification to a previously published algorithm for statistical selection of reference velocities (Bagaini et al. 1995).

Key features of our automatic reference velocity selection algorithm are 1) division of the velocity distribution into clusters 2) entropy based statistical control to determine the minimal number of reference velocities required within a cluster, 3) a novel 'greedy search' that selects reference velocities within each cluster at or near peaks in the probability distribution, and 4) calculation of a single reference velocity if the velocity range within each cluster is less than a minimum threshold. We show that our method is superior to Bagaini et al.'s method, which includes only step (2) above, and - in place of step (3) - selection of reference velocities by piecewise constant interpolation of the probability distribution of the underlying velocities. Our automatic velocity selection algorithm can be run using a single parameter – the approximate desired velocity step expressed as a percentage – although the maximum step and minimum threshold can be specified if the defaults are not suitable. The automatic velocity selection algorithm is implemented as part of a prestack PSPI depth migration of the 2-D Marmousi model. The resulting images are clearly superior to images created using a linear or geometrical distribution of a similar number of reference velocities. The algorithm is suitable for 3-D data, where the tradeoff between accuracy and efficiency is more pronounced.

INTRODUCTION

Wave equation depth migration methods such as phase-shift plus interpolation (Gazdag and Sguazzero, 1984), extended split-step Fourier (Kessinger, 1992), or Fourier finite difference plus interpolation (Biondi, 2002) require a limited set of reference velocities for efficient wavefield extrapolation through laterally inhomogeneous velocity models. All of these authors suggests that a geometric distribution can produce a good set of reference velocities. Kessinger (1992) mentions that, by empirical testing, a percentage velocity increment of 15% provides a good compromise between accuracy and efficieny. Another standard method calculates a linear distribution of velocities (e.g. Ferguson and Margraves, 2002), which may have an advantage of increasing the accuracy of higher-angle wavefield propagation with depth.

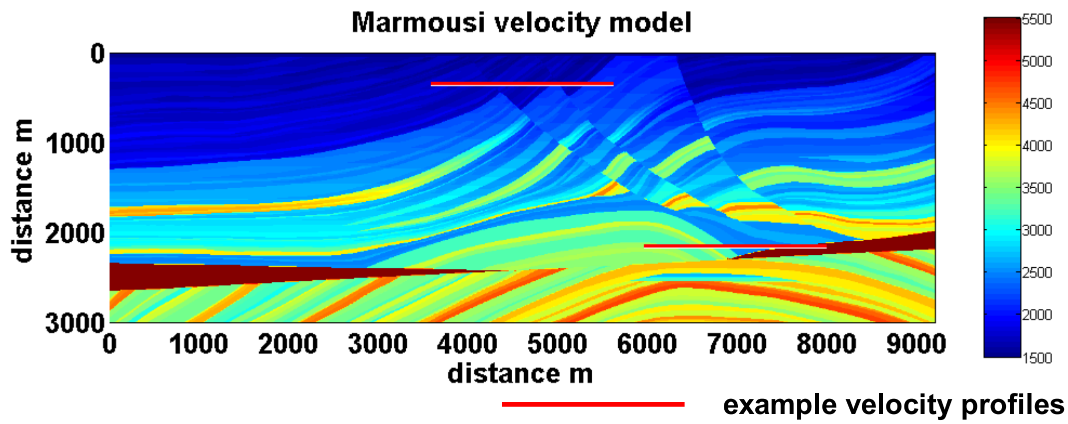


FIG. 1. The Marmousi velocity model. Red lines mark velocity profiles used in this study.

Bagaini et al. (1995) propose a statistical method for optimal selection of reference velocities. The optimal choice of reference velocities will be a balance between accuracy and efficiency. In this study, we do not determine a quantitative method for evaluating optimality. Instead, we compare the selected reference velocities against the probability distribution of the velocities over the range for the profile. Figure 1 is a colour contour of the Marmousi velocity model. The red lines indicate profiles used in this study.

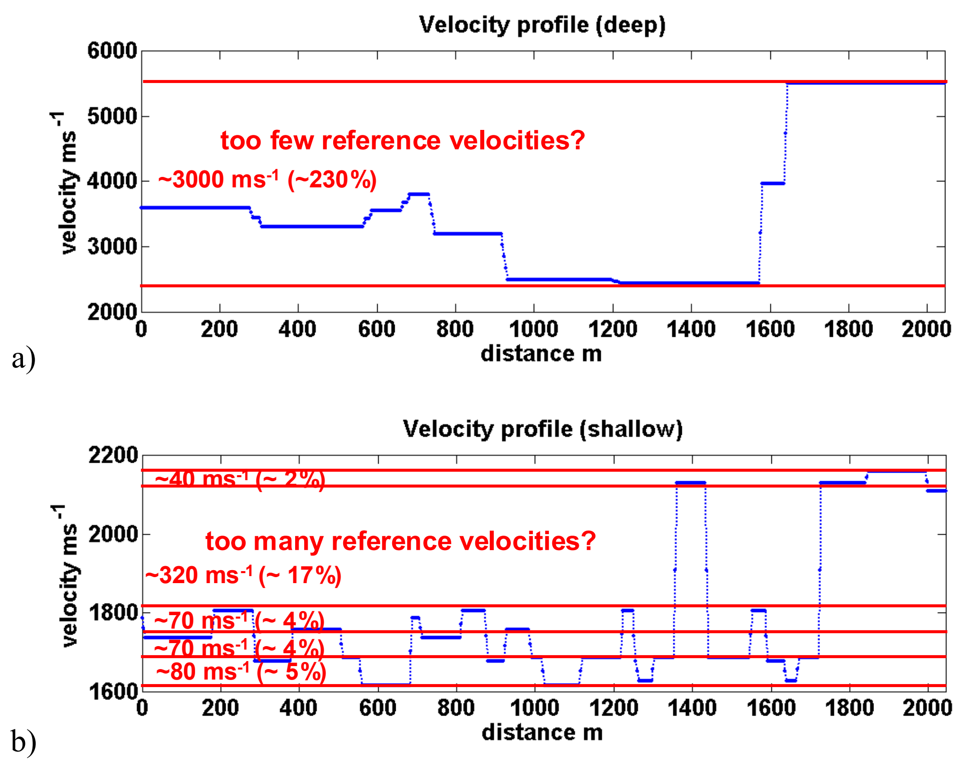


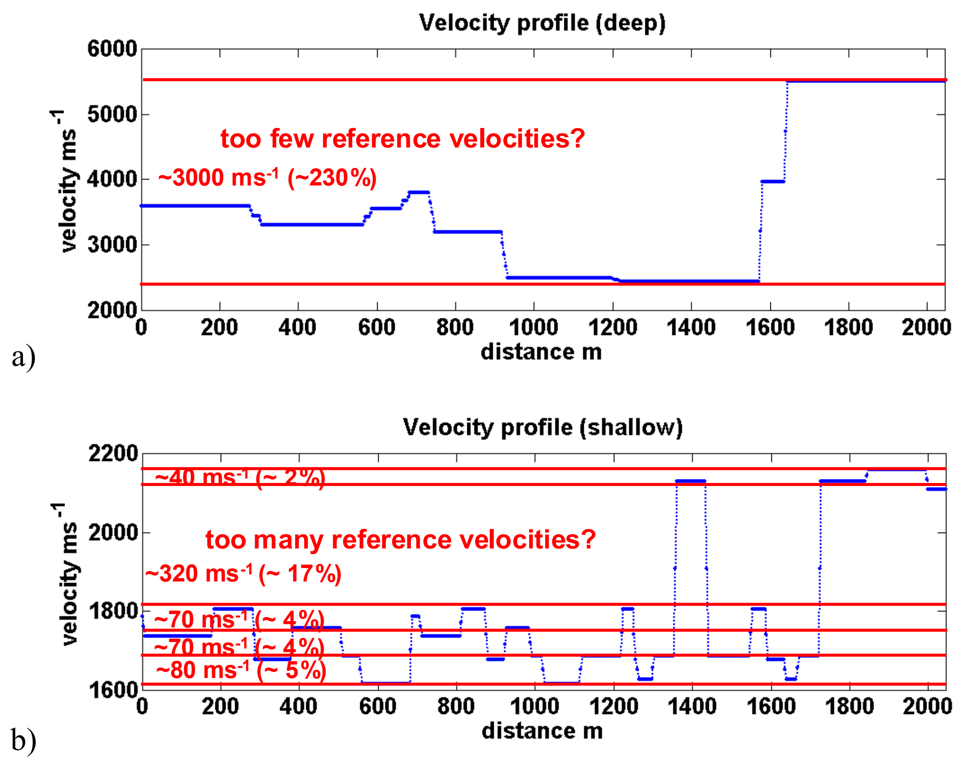
FIG. 2. How should reference velocities be chosen for wavenumber-space domain extrapolators such as PSPI? Empirical testing (Kessinger, 1992) suggests that an increment of approximately 15% provides a reasonable compromise between accuracy and efficiency. In a) selection of the bounding velocities creates too large an increment (230%), whereas in b) a careful selection of velocities that closely matches steps in the velocity profile may produce too many velocities (steps of 2%, 4%, 4%, and 17%).

Keeping Kessinger's empirical percentage increment of 15% in mind, we can easily determine when we might have too few velocities (Figure 2a). However, a good fit to a velocity profile, as shown in Figure 2b, might not satisfy Kessinger's empirical increment. How then should we select velocities to create a best fit to the underlying probability disribution?

In this paper, we review the method of Bagaini, show that their reference velocities are not necessarily near the peaks in the probability distribution of the velocities over a layer, and propose a new statistical method that produces a more optimal distribution of reference velocities. We test our method against Bagaini's method, and against the linear and geometric methods, using velocity profiles from the Marmousi acoustic model (Versteed and Grau, 1991).

LINEAR AND GEOMETRIC METHODS

The linear method is described as follows: 1) choose an approximate velocity spacing dV, 2) Calculate nV=round((v_{max}-v_{min})/dV) and 3) determine references velocities as equal increments of v_{step}=(v_{max}-v_{min})/nV. However, what is a good choice for dV? One option is a value close to Kessinger's empirical percentage. A question remains, however: Will the selected increment be optimal reasonable for both low and high velocities? Figure 4 illustrates the geometric method applied to a Marmousi velocity profile.



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The geometric method is described as follows: 1) choose an appropriate percentage step v_{prcnt}, 2) calculate $v(i) = (1+v_{prcnt})*v(i-1)$. 3) The percentage can be adjusted by solving a simple logarithmic equation if reference velocities are desired at both the minimum and maximum velocities. As mentioned previously, Kessigner (1992) recommends v_{prcnt}=0.15. Figure 4 illustrates the geometric method applied to the deeper Marmousi velocity profile.

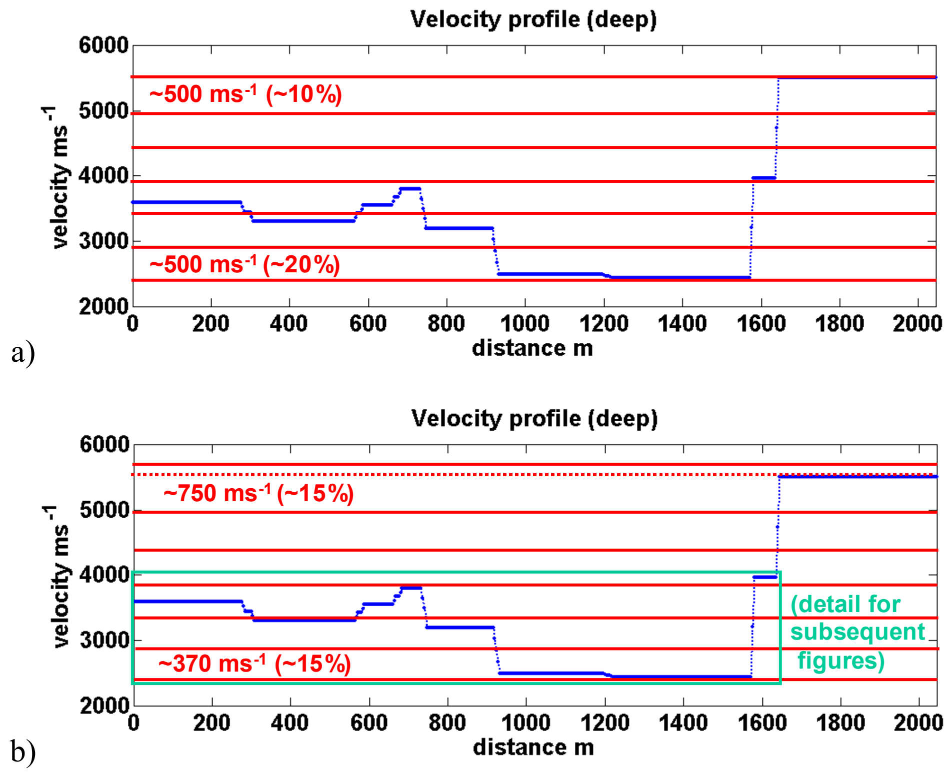
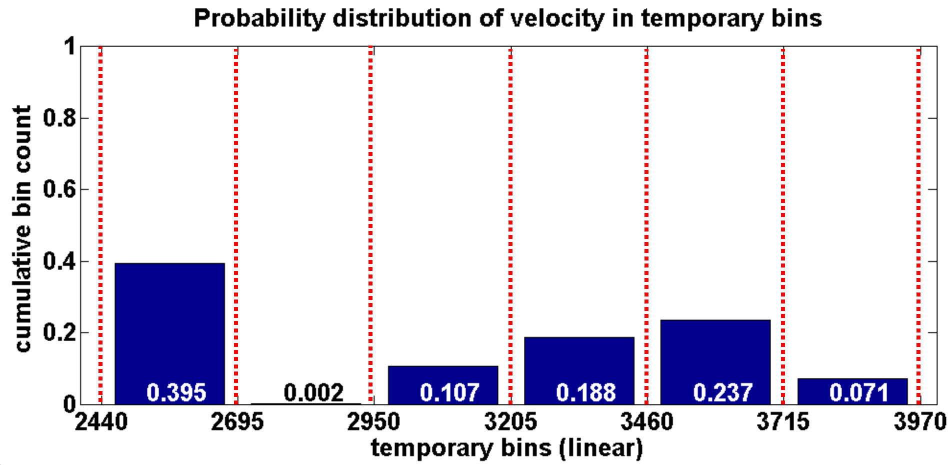


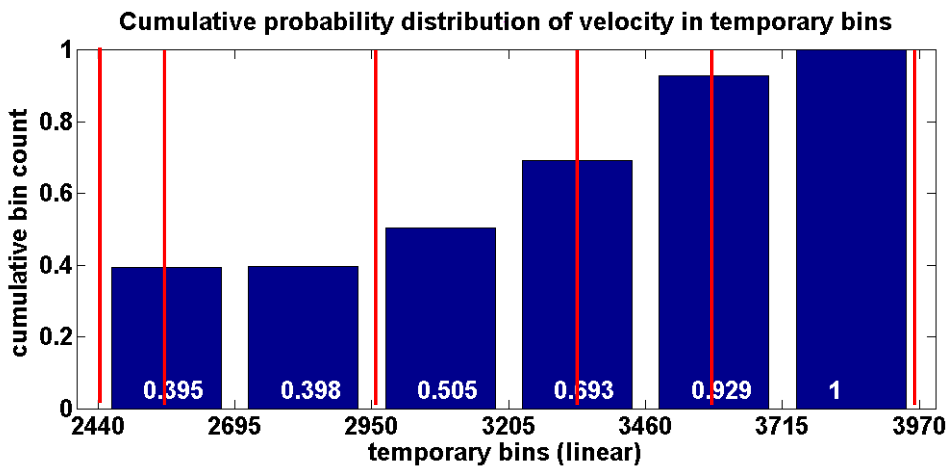
FIG. 3. Standard methods for selecting reference velocities include a) a linear interpolation between the minimum and maximum velocities within a layer or cluster, and b) a geometric interpolation based on a fixed percentage increment from the minimum velocity. Empirical testing (Kessigner, 1992) suggests that an increment of approximately 15% provides a reasonable compromise between accuracy and efficiency. The green box in figure b) shows the velocity profile used in Figure 4.

THE METHOD OF BAGAINI ET AL. (1995)

The method of Bagaini et al. (1995) can be described as follows: 1) choose a preliminary dv (geometric?); 2) determine equally spaced bins over v_{min}:v_{max} (e.g. Figure 4a, where nB_{temp} = 6)bin the velocities to give probability density P_i, where $\Sigma P_i = 1$, 4) determine theoptimal number of bins nB_{opt} by statistical entropy $S=\Sigma P_i \log P_i$, where nB_{opt} = round(exp(S)+0.5) (e.g. nB_{opt} = 5 in Figure 4); 5) •calculate the cumulative probability distribution Y_i, where Y_i = S P_i - each optimal bin will hold 1/nB_{opt} of the cumulative probability distribution (e.g. 0.2 in Figure 4b); 6) start at v_{min} and linearly interpolate the position of the reference velocities from the temporary bin boundaries (e.g. at 0.2, 0.4, 0.6, 0.8 in Figure 4b).



a)



b)

FIG. 4. Bagaini et al's (1995) method for selecting reference velocities. a) the velocities are binned to create a discrete probability distribution. An optimal number of reference velocities nB_{opt} is calculated using a measure of statistical entropy. b) the discrete cumulative probability distribution is divided into nB_{opt} equal increments by linearly interpolating from the bin boundaries.

Is this method optimal? A linear increment of the cumulative probability distribution does not necessarily produce reference velocities on or near the peaks in the probability distribution. In the worst case scenario, none of the reference velocities will fall on peaks and the extrapolation through that layer will require large interpolations.

OUR NEW 'GREEDY' PEAK SEARCH METHOD

We propose a new method that specifically searches for peaks in the probability distribution. We use Bagaini's measure of statistical entropy to determine an optimal number of reference velocities. In all likelihood, however, the number of peaks in the probability distribution will not match the statistically optimal number of number of reference velocities. Our method uses a variety of techniques to position the reference velocities closest to the peaks. In short, we collapse the closest peaks, starting with the peaks of lowest probability density. A new reference velocity is determined as a weighted distance between the two original peaks, except at the ends where the values of the minimum and maximum velocities are preserved as the bounding reference velocities.

We also use a clustering algorithm to divide the velocity profile up into clusters that might be separated by a much larger percentage difference than acceptable. Within each cluster, we can apply any of the above specified methods. The general clustering method is described as follows:

1) sort the velocities in the profile

2) select clusters where the percentage increment in sorted velocities exceeds some maximum value

3) each i^{th} cluster is now defined by a minimum velocity v_i^m and maximum velocity v_i^M. Starting with the fist cluster (lowest velocity), determine

   a. If $v_i^M / v_i^m \leq 1 + v^{\%thresh}$, then $v_i^m \approx v_i^M$ and this cluster can safely be extrapolated at the single velocity $v_i^{avg} = \left(v_i^m + v_i^M\right)/2$. Set all velocities within these bounds to the average velocity $v_{xy}^{in} \in \left[v_i^m, v_i^M\right] = v_i^{avg}$, add this velocity to the velocity vector $v_k^{use} = v_i^{avg}$, and increment $k = k + 1$.

   b. If $c_i^M / c_i^m \leq 1 + c_{\max}^{\%}$, then the bounding velocities for the cluster v_i^m and v_i^M are a valid step without any intermediate velocities

   c. If $c_i^M / c_i^m > 1 + c_{\max}^{\%}$ there will be at least three velocities in the cluster (including the bounding velocities). Use the linear, the geometric, or the Bagaini et al. (1995) measure of statistical entropy to determine an optimal number of bins in the cluster.

4) Depending on the method chosen in 3), calculate the reference velocities within the cluster and cycle through all clusters determined in 2)

In Figure 5, we compare the selection of reference velocities within a cluster. The 'greedy' peak search method shown in Figures 5c and 5f does a better job of locating the reference velocities near the peaks in the probability distribution.

MARMOUSI EXAMPLE

We applied the various techniques to the select reference velocities for PSPI migration of the Marmousi data set. The results of the PSPI migration using the clustering algorithm with the Bagaini method are shown in Figure 6. Figure 6a is the bandlimited reflectivity calculated as using gradients of the velocity and density model. The modified Bagaini method obviously produces an excellent image of Marmousi (Figure 6b). Even the elusive target zone at depth (Figure 6d) compares well with the reflectivity (Figure 6d). This research is ongoing. More results will be presented at the meeting

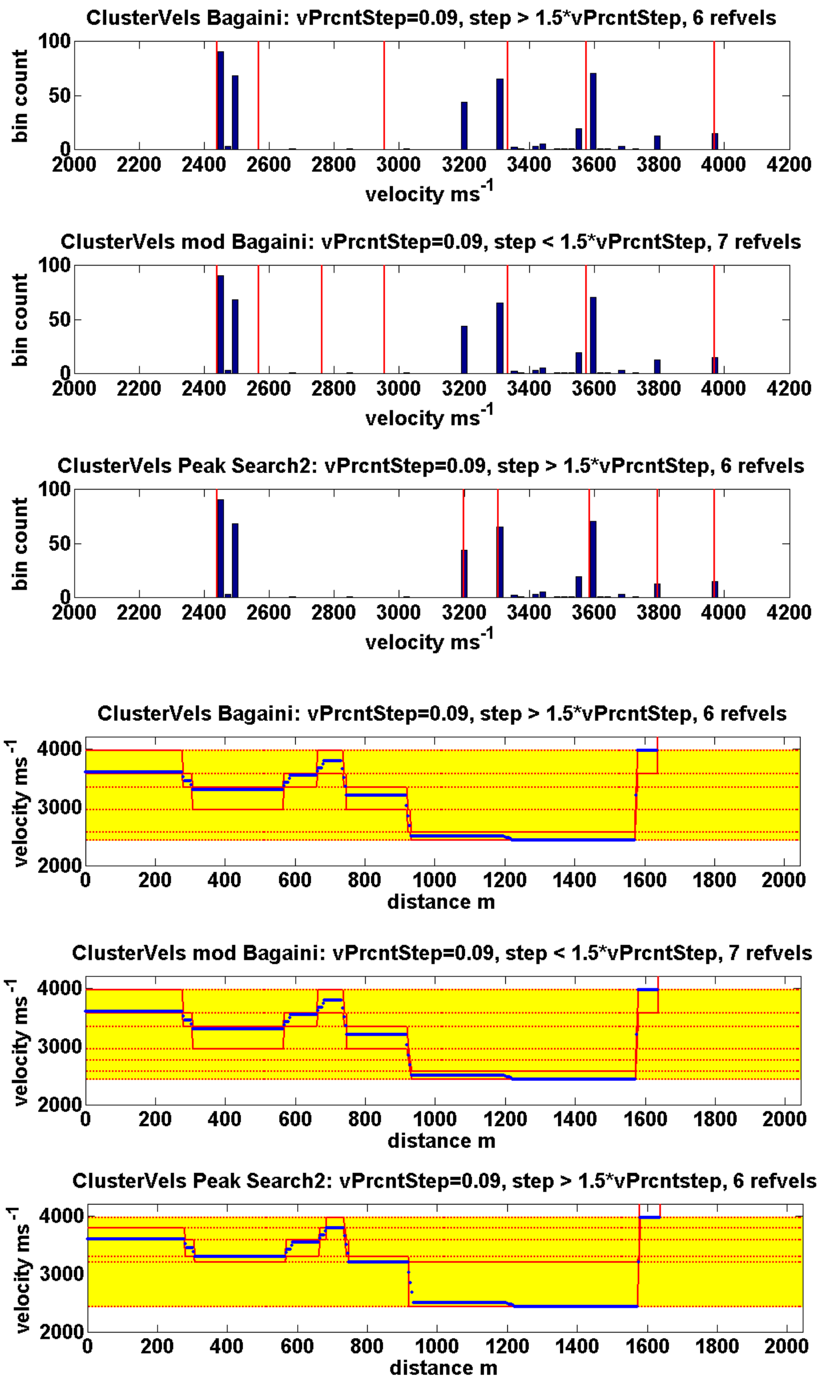


FIG. 5. a-c) Probability distribution of velocities within a cluster from a selected layer of the Marmousi (blue bars) and selected reference velocities using various methods (red lines). d-f) the velocity profile with distance (blue dotted line), selected reference velocities (red dotted lines) and bounding velocities (red lines). Note that in a) and d) the reference velocities by the statistical entropy method of Bagaini et al (1995) are not necessarily at the peaks in the distribution. In b) and e), large gaps are linearly interpolated. In c) and f) the new ‘greedy’ peak search method selects reference velocities at the peaks in the probability distribution. The new method should produce more accurate wavefield extrapolation for the same computational cost, and hence better images.

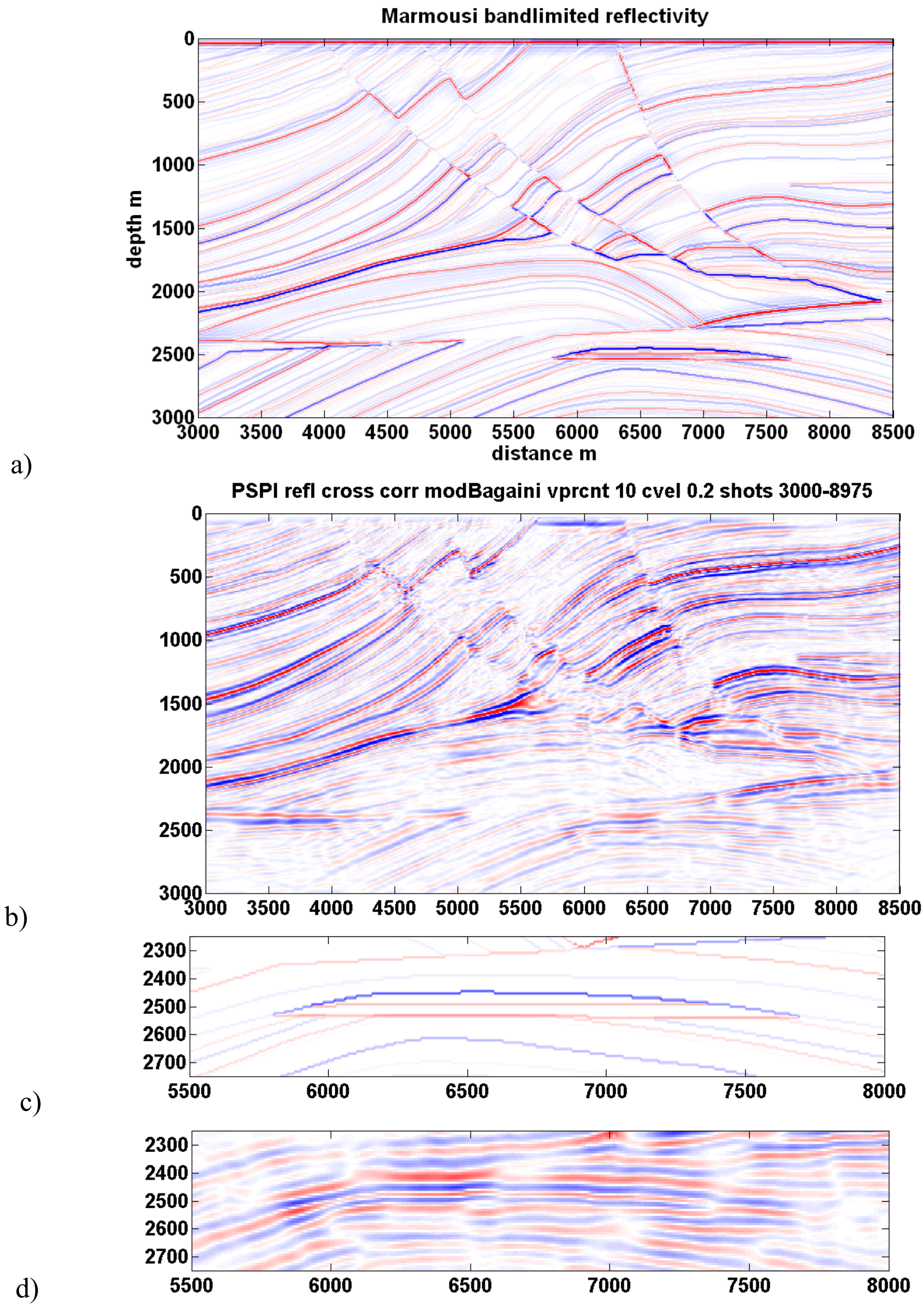


FIG. 6. a) Bandlimited reflectivity for Marmousi acoustic model. b) Depth image by PSPI migration with cross-correlation imaging condition using modified method of Bagaini et al (1995), where reference velocities are selected using clustering and statistical entropy within clusters. c) and d) Zoom of full bandwidth reflectivity and PSPI image at the target zone. The target is accurately imaged, although the limited bandwidth of the data results in a ringy appearance.

CONCLUSIONS

An optimal selection of reference velocities within a depth layer can maximize the accuracy and efficiency of wavenumber/spatial domain wavefield extrapolators such as PSPI. A linear or geometric distribution does not take into account the underlying distribution of the velocities. Hence, large interpolations between reference wavefields may be required, which can result in reduced accuracy and a poor image. The statistical method of Bagaini et al. (1995) distributes the reference velocities based on equal intervals of the cumulative probability distribution of the velocities within a layer. On average, this will reduce the size of the interpolations, but may still produce a reference velocity distribution that requires some large interpolations. We propose a modification to the method of Bagaini et al., whereby the velocities within a layer are first separated into clusters divided by large gaps in the distribution, an optimal number of velocities within a cluster is chosen based on Bagaini et al.'s method of statistical entropy, a 'greedy' search algorithm is used to select reference velocities within each cluster as close as possible to the peaks in the probability distribution, and as weighted averages between smaller pairs of peaks if there are more peaks than the optimal number of velocities. A single reference velocity is selected for clusters or layers where the spread of velocities is less than a specified threshold.

The new 'greedy' peak search method was tested on the Marmousi acoustic data set using a PSPI wave-equation prestack shot-record migration algorithm. The depth images from the new peak search method are clearly more accurate than those produced by a geometric distribution. Surprisingly, a linear distribution is shown to produce reasonable images in the shallow section that are similar to the new algorithm and much better than thosee produced using a geometric distribution. At present, we do not have a good explanation for this result. In the shallow section, the peak search method and the Bagaini et al. method yield similar images. The benefits of our new method are most pronounced in the deeper section, where focusing and positioning are better than any of the methods used for comparison.

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