# A practical implementation of FOCI — FORFOCI depth migration

R. Maier, G. Margrave, W. Sun, and M. Lamoureux

#### ABSTRACT

FOCI (Forward Operator Conjugate Inverse) is an implementation of explicit wavefield extrapolation for seismic depth migration in the space-frequency domain. FOCI adopts several techniques to improve the stability and efficiency. FORFOCI is the C and FORTRAN 95 implementation of FOCI. FORFOCI takes portibility and efficiency as priorities.

#### **INTRODUCTION**

FORFOCI is a full implementation of FOCI. It includes routines for linear systems, matrices, signal processing, etc.

FORFOCI is designed to be flexible, efficient and portable. To attain these goals, FOR-FOCI is written in C and FORTRAN 95 in an objected-based structure. The I/O parts are written in C, the migration part in FORTRAN 95, making use of FORTRAN's superior speed in numerics. Contact between the two parts (C and FORTRAN) is minimal, easing adaptation to larger processing systems.

The heart of FOCI is to implement explicit wavefield extrapolation for seismic depth migration in the space-frequency domain with special techniques to improve the stability and efficiency as detailed in (Margrave et al., 2006). Consider the n-dimensional GPSPI fomula in the space-frequency domain,

$$\psi(\boldsymbol{x}_T, z + \Delta z, \omega) = \frac{1}{(2\pi)^{n-1}} \int_{\mathbb{R}^{n-1}} \psi(\boldsymbol{u}_T, z, \omega) W(\boldsymbol{x}_T - \boldsymbol{u}_T, k(\boldsymbol{x}_T), \Delta z) d\boldsymbol{u}_T, \quad (1)$$

where  $\psi(\boldsymbol{x}_T, z, \omega)$  is the space-frequency domain wavefield,  $\omega$  is the temporal frequency,  $\boldsymbol{x}_T$  denotes the transverse spatial direction,  $\boldsymbol{u}_t$  is the transverse spatial direction of integration,  $\Delta z$  is the depth step, and W is the wavefield extrapolation operator

$$W(\boldsymbol{x}_T - \boldsymbol{u}_T, k(\boldsymbol{x}_T), \Delta z) = \frac{1}{(2\pi)^{n-1}} \int_{\mathbb{R}^{n-1}} \hat{W}(\boldsymbol{k}_T, k(\boldsymbol{x}_T), \Delta z) \exp(-i\boldsymbol{k}_T \cdot (\boldsymbol{x}_T - \boldsymbol{u}_T)) d\boldsymbol{k}_T,$$
(2)

with

$$\hat{W}(\boldsymbol{k}_T, k(\boldsymbol{x}_T), \Delta z) = \exp(ik_z(k(\boldsymbol{x}_T), \boldsymbol{k}_T))\Delta z, \qquad (3)$$

and

$$k_{z}(k(\boldsymbol{x}_{T}), \boldsymbol{k}_{T}) = \begin{cases} \sqrt{k^{2}(\boldsymbol{x}_{T}) - k_{T}^{2}}, k^{2}(\boldsymbol{x}_{T}) > k_{T}^{2} \\ i\sqrt{k_{T}^{2} - k^{2}(\boldsymbol{x}_{T})}, k^{2}(\boldsymbol{x}_{T}) \le k_{T}^{2}, \end{cases}$$
(4)

and

$$k(\boldsymbol{x}_T) = \frac{\omega}{v(\boldsymbol{x}_T)},\tag{5}$$

where v is the velocity.

## THE FOCI METHOD

There are three key components to the FOCI method:

- 1. Operator stabilization by Wiener filtering;
- 2. The use of dual operators to reduce evanescent filtering;
- 3. spatial downsampling of the lower frequencies to increase operator accuracy and decrease run time.

### **Stability Design**

The wavefield extrapolation operator W has infinite length, with a sharp transition between the wave-like domain and the evanescent domain, decaying very quickly in the evanescent domain. See Figure 1 and refer to (Margrave et al., 2006).



FIG. 1. A plot of  $|\hat{W}|$  in the wavenumber domain for some fixed velocity

An approximation of W with compact support may lead to an unstable extrapolator. FOCI utilizes a stabilizing Wiener filter to enhance the stability in a computationally fast and efficient way. The FOCI extrapolator is designed as follows

$$W_F(\Delta z) = \bar{W}_I(\Delta z/2) * W(\Delta z/2), \tag{6}$$

where  $W_F$  is the FOCI extrapolator,  $\Delta z$  is the depth step and  $\overline{W}_I$  is the conjugate inverse of the windowed W with compact support.  $W_I$  also has compact support computed by a least square approximation

$$W_I(\Delta z/2) * W(\Delta z/2) = \mathcal{F}^{-1}(|\hat{W}(\Delta z/2)|^{\eta}), \tag{7}$$

where  $0 \le \eta \le 2$  is an adjustable parameter and  $\mathcal{F}^{-1}$  is the inverse Fourier transform.

# **Spatial Downsampling**

FOCI is frequency adaptive. Data are resampled in order to keep more data in the wavelike region.

# FORFOCI

FORFOCI is designed as follows:

- 1. Easily extendable
- 2. Flexible I/O design
- 3. Operator overload design
- 4. Code rendered re-usable through "kind of" and "part of " hierarchies

FORFOCI consists of two parts

- 1. CFOCI— written in C, the general I/O module which reads in SEGY data, generates parameters for FORFOCI, then stacks the migrated data and export the data.
- 2. FORFOCI —written in Fortran 95, which takes the shot data and carries out the wavefield extrapolation using the FOCI algorithm.

FORFORCI has the following properties:

- 1. Object oriented. The full package is built in different modules, such as the MATH MODULE, the SEISMIC MODULE, the GABOR MODULE, the FOCI MODULE, etc.
- 2. Most FORFOCI routines are overloaded and can handle different types of data.
- 3. Portability. FORFOCI also takes portability into account.

## **Input and Output**

All I/O and parameter settings are listed in a single text file.

The FOCI command takes hence exactly one argument, the parameter file name. Without an argument it will write out a template which, saved as a file, can be edited and used as an input file.

Velocities may be specified either in ASCII or SEGY format.

Seismic data are expected in SEGY format. Shots may be selected by sequence number.

Output of migrated shot gathers and optional stacks is in SEGY format.

## Example

Migrated Marmousi data, input data are 240 shots at 96 traces with end-on shooting. The parameters were similar to those used in Figure 6 of (Al-Saleh and Margrave, 2006). The trace spacing was 12.5 m, dt = 4 ms,  $\eta = 0.7$  and forwards and backwards operator lengths were 21 and 31, respectively. Figure shows the result of the migration done with the converted version of FOCI.



FIG. 2. Example of a migrated Marmousi data set as described in the text.

## REFERENCES

Al-Saleh, S. M., and Margrave, G., 2006, Enhancing the foci extrapolation method by using a weighted least-squares approach: CREWES Research Report, **17**.

Margrave, G., Geiger, D., Al-Saleh, W., and Lamoureux, M., 2006, Improving explicit seismic depth migration with a stabilizing wiener filter and spatial resampling: Geophysics, **71**, No. 3, 111–120.