Finite-difference elastic modelling below a structured free surface

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ABSTRACT

This paper shows experiments using a unique method of implementing a structured free surface boundary for a finite-difference model. The method employs the same concepts as the implicit solutions used for finite-difference formulations posed in a recursive manner. The displacements required across the free surface boundary are then found by a deterministic method where structure is locally flat, and by an implicit method where the structure is sloped, proceeding from low to higher elevations.

INTRODUCTION

When a finite-difference model is begun, one of the first choices that must be made is where the boundaries of the model space will be. With the exception of well to well models, most choices will have the surface of the earth as a top boundary. This is in contrast to the other three boundaries, which we wish would go away, and would put at infinity if we could.

The top boundary of the model must then represent a 'real' boundary, which is almost perfectly represented as a free surface because the air above the surface can transmit only tiny amounts of energy in the range characterizing seismic waves. This condition can be simulated by assuming displacements across the boundary which are not zero, but which when used in equations spanning the boundary result in zero stresses. This means zero shear stress and zero compressional stress. These conditions ensure that body waves are reflected from the surface accurately, but more noticeably, that the surface can propagate surface 'Rayleigh' waves.

An improvement in the realism of a finite-difference model would allow a structured free surface, simulating particular acquisition conditions. Here, the actual top of the model could still be flat, but the boundary conditions on the top would be irrelevant because the structured free surface would prevent any energy from entering and leaving the real model area.

THE FLAT FREE SURFACE BOUNDARY CONDITIONS

The method for doing finite-difference modelling near a free surface boundary is given in the author's PhD thesis (Manning, 2008). The relevant stress equations may be found in Levander (1988). They are

$$\sigma_{xz} = \mu\left(\frac{\partial U_z}{\partial x} + \frac{\partial U_x}{\partial z}\right) = 0 \text{ , and} \quad 1)$$

$$\sigma_{zz} = (\lambda + 2\mu)\frac{\partial U_z}{\partial z} + \lambda\frac{\partial U_x}{\partial x} = 0 \text{ ,} \quad 2)$$

where the first equation sets the shear stress to zero, and the second equation sets the compressional stress to zero. These equations are used to project from the main body of displacements across the boundary, where particular displacements are required to ensure zero stress conditions.

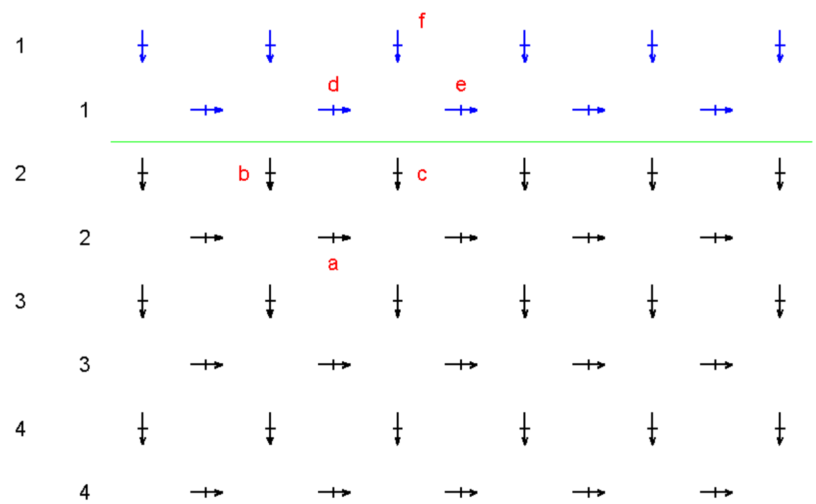

FIG. 1. The staggered grid displacement positions required for second order finite-difference calculations near an external boundary (the green line). The black arrows mark displacements which will be calculated by the standard equations. The blue arrows mark the cross boundary displacements required as input for the standard equations. The red letters show how the two rows of blue displacements may be projected from the internal displacements by assuming free surface conditions.

The practical use of equation 1 is illustrated with the *abcd* block in Figure 1. It requires that *d* must be made equal to *a*+*c*-*b*. This means that the *abcd* block may be twisted, but not distorted.

Similarly, equation 2 applies to the *cdef* block. The displacement at *c* is already available, and the displacements at *d* and *e* may be calculated from equation 1. Equation 2 then requires that *f* be made equal to $c + (e - d)\lambda/(\lambda + 2\mu)$. If the medium was a fluid ($\mu = 0$) it would mean maintenance of a constant area. In an elastic medium the displacement at *f* compared to *c* is not changed quite as much.

STRUCTURED FREE SURFACE BOUNDARY CONDITIONS

An example of a structured free surface boundary is given by the blue line in Figure 2. The staggered grid of the main body of the model is shown with black arrows, and the cross boundary displacements required to update at the edge black arrows are shown with red arrows.

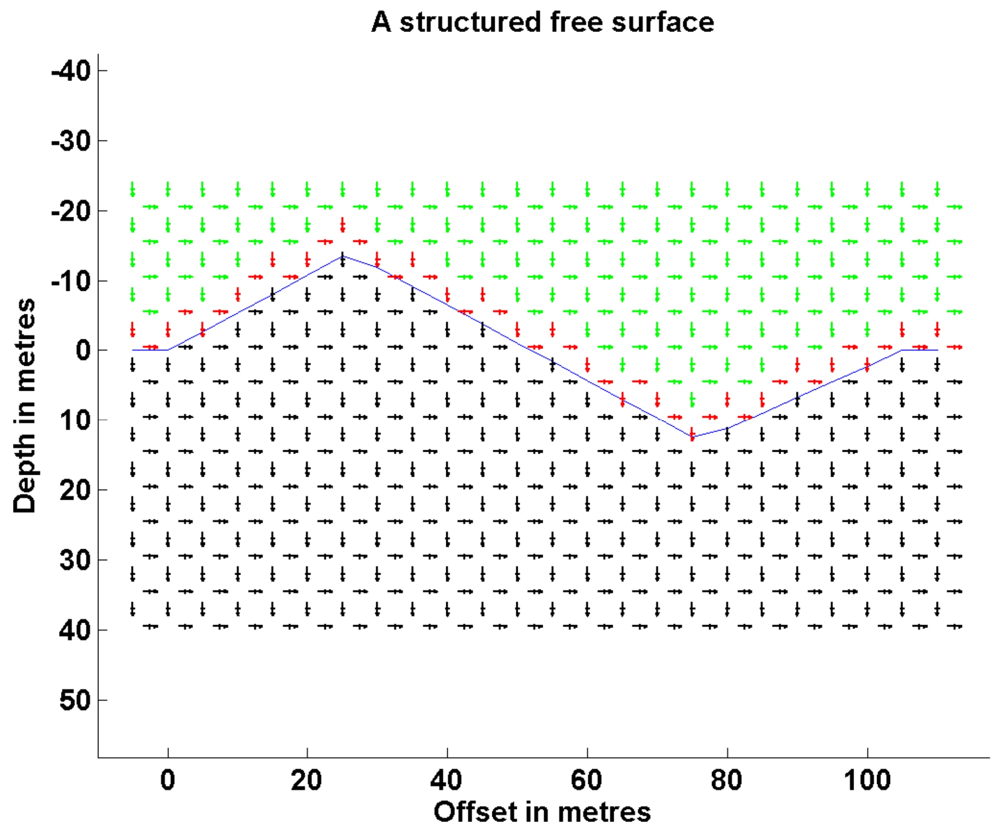

FIG. 2. An example of a structured free surface. The main body of the model is shown with black arrows for displacements. The red arrows show the cross boundary displacements required for the standard equations. Displacements at the green arrows are irrelevant to the main part of the model.

Displacements at the red arrows cannot in general be projected from the main body displacements by the use of equations 1 and 2. However, equation 1 may be used to calculate the vertical displacements at particular points, for example at offsets 5 and 75 metres. Similarly, equation 2 may be used to project the horizontal displacements at offsets of 42.5 and 112.5 metres.

Further examination of Figure 2 will show that the displacements determined by these first projections may be used, along with other main body displacements, to determine further displacements. For example, the projected vertical displacement at 75 metres provides part of the scaffold to project the horizontal displacements at 72.5 and 77.5 metres. Continuing with this process will allow all the required (red) displacements to be projected.

This process may appear to be time dependent, but in reality the displacements are simultaneous, where the solutions must be determined by a succession of operations. This type of solution may be found in the branch of finite-difference modelling that uses implicit equations. The recursive equations used there can only be solved by using boundary conditions, and projecting from these boundaries throughout the model. This type of solution is described in Strikwerda (2004), section 3.5.

TEST CASES

The algorithm described above has been tested for some of the more simple cases. An example is shown in Figure 3, where the slope of the free surface is 1 in 6. The surface wave here is quite compact and perpendicular to the surface, properties to be expected in a surface wave above a half space.

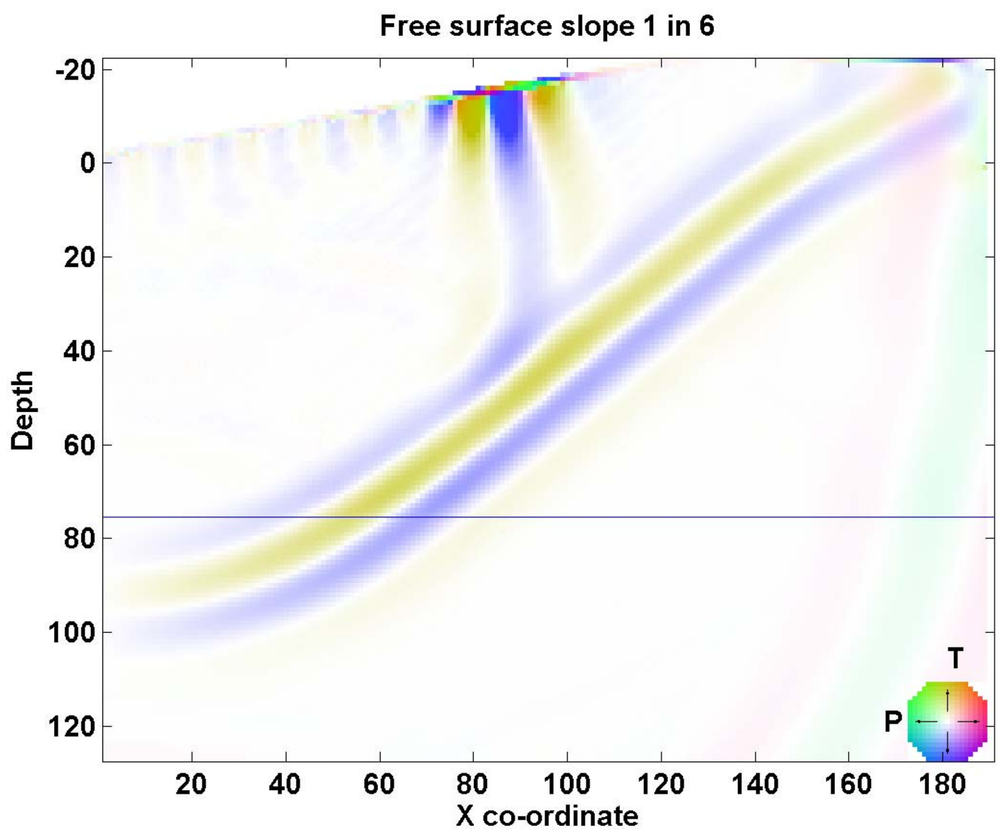

FIG. 3. This Figure shows a satisfactory model for a free surface sloped by 1 part in 6. The irrelevant displacements above the free surface have been zeroed out, but the projected displacements have been included. The surface wave at top-centre is perpendicular to the surface, as expected.

A second test of a steeper slope is shown in Figure 4. The slope in this Figure is 1 in 5, and the results are not encouraging. It appears that the evenly spaced steps of the free surface here cause instabilities that tend to reinforce.

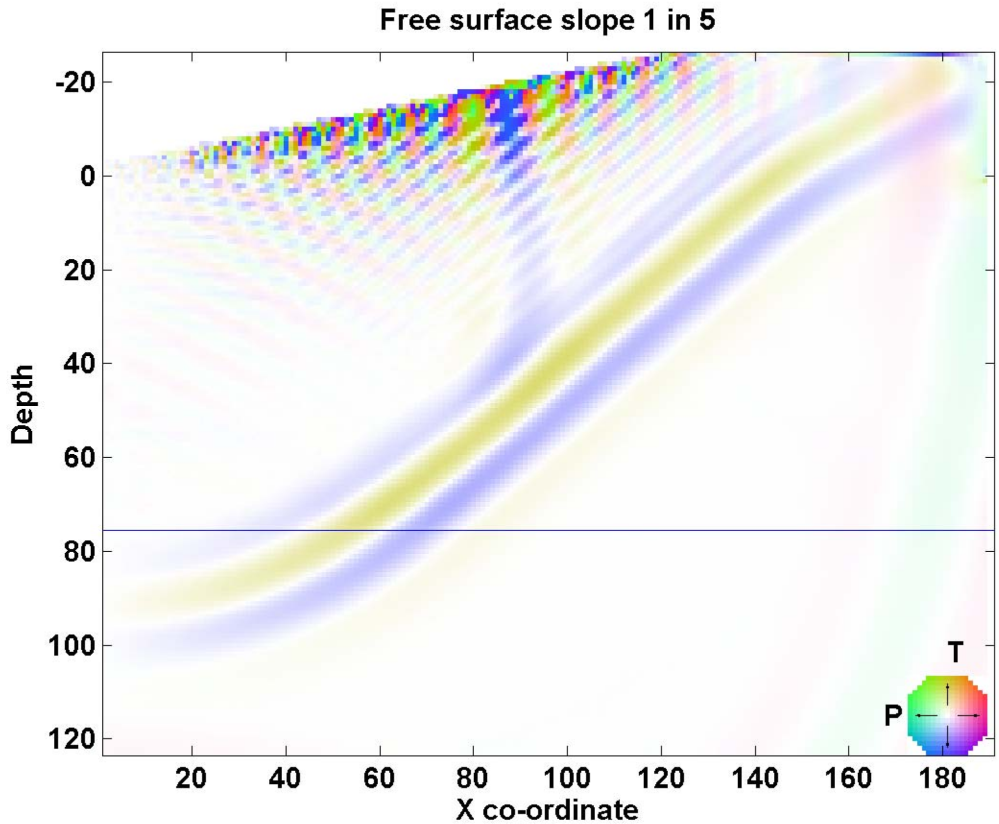

FIG. 4. A model with a free surface slope of 1 part in 5. This shows that the algorithm introduced here has some limitations. The periodic nature of the constant slope seems to reinforce the instabilities generated from the surface.

The next case is a test of the method on a feature sometimes encountered with seismic acquisition, an abrupt rise in elevation along the line. The rise here is 5 metres, sloped over ten metres from 50 to 60 metres in X on Figure 5. The small elevation change has made a significant change in the surface wave compared to the flat free surface case shown in Figure 6.

Another way to compare these cases is with the use of the simulated seismic data acquired at the surface. The horizontal data (in line) is displayed for the flat free surface case in Figure 7, and for the 5 metre step case in Figure 8. Here too, the differences are very noticeable.

DISCUSSION OF THE STEP CASE

The major differences seen in the free surface 5 metre step case compared to the flat case is worth special comment. The step height of 5 metres was chosen because in a half space, with the frequencies and velocities used, it is a depth that divides the energy of a surface wave into two almost equal parts. Still, it is difficult to explain the results of this test.

It appears that the step causes a second surface wave/shear body wave combination to be created which is advanced from the combination which occurs in the flat surface case, and still occurs with this case also. Thus, in Figure 8, the major events of the first breaks (the pressure wave/shear wave combination) and the surface wave from Figure 7 occur, and a second lagging first break appears, along with a second surface wave event which leads the established one. Perhaps when the pressure wave/shear body wave combination at the surface is broken up at the step, it can not be reinitiated without also initiating another surface wave.

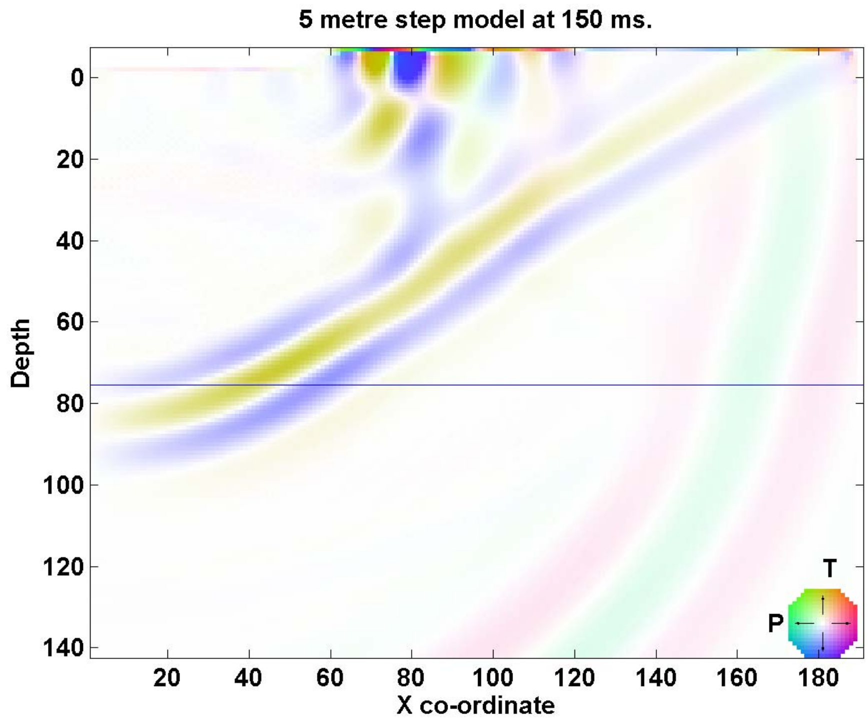

FIG. 5. The 5 metre step model wave field after propagation for 750 ms. The step here is 5 metres vertical over 10 metres horizontal, extending from 50 to 60 metres. Instability here has not been a significant problem. The step has affected the relationship between the converted shear wave and the surface wave as seen with the flat free surface in Figure 6.

The difficulty for the purposes of this report is that the software here has as yet not been extensively tested for consistency of results, and has not been compared at all with field data.

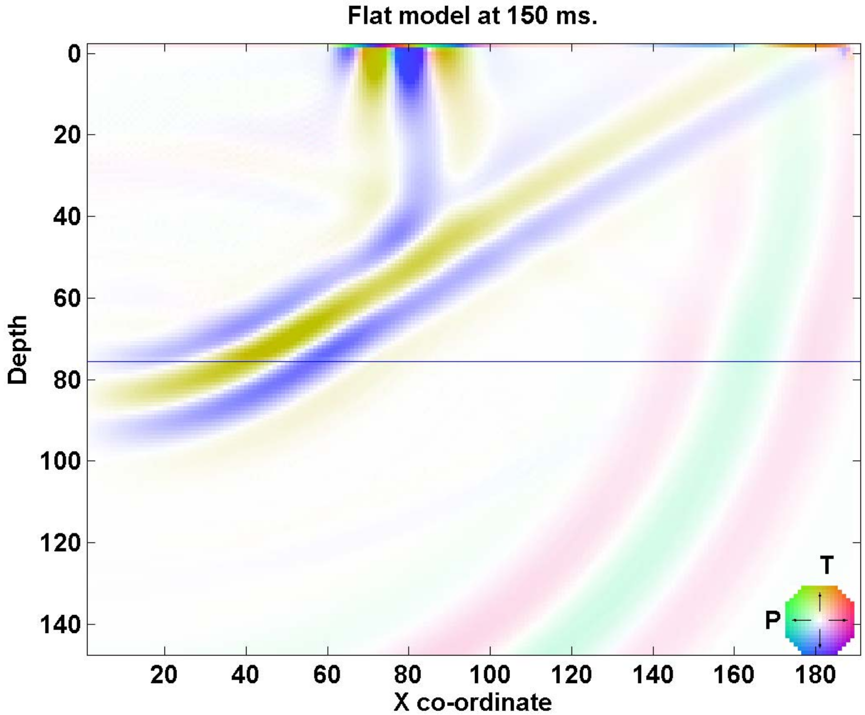

FIG. 6. The flat free surface model at 750 ms., to compare with Figure 5. The smooth relationship between the shear wave and the surface wave may be seen. The shear wave, the surface wave, and the pressure wave were all coincident at the surface earlier in time near X = 0.

CONCLUSIONS

The software presented with this report has shown some interesting results, with the following cautions:

- Extended steeper slopes tend to create instabilities as a boundary.
- Results must be further tested for consistency, and realistic models compared with real data
- As with the CREWES finite-difference modelling package, the general boundaries need improvement.

REFERENCES

Levander, A. R., 1988, Fourth-order finite-difference P-SV seismograms: Geophysics, **53**, No. 11, 1425-1436.

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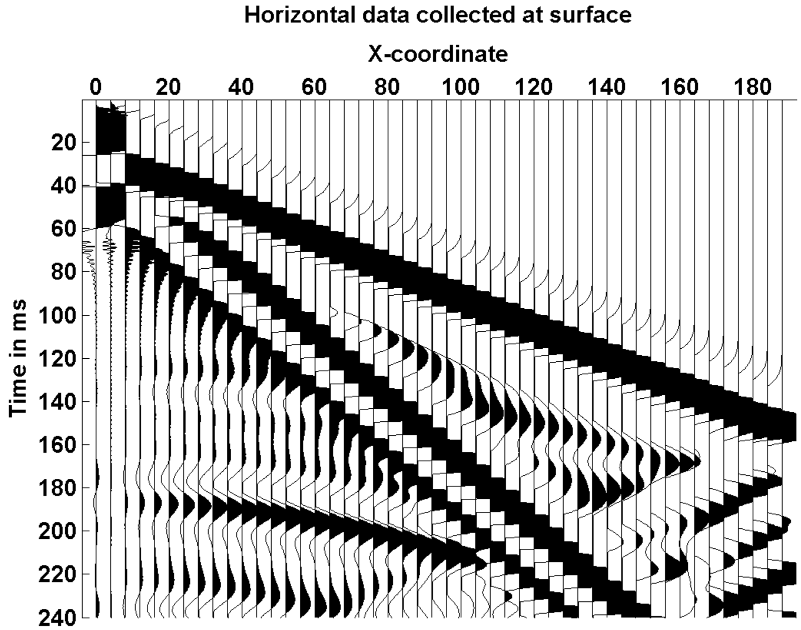

FIG. 7. The horizontal seismic data collected at the free surface of the flat model. The consistent nature of the first break pressure/shear event, and the steeply dipping surface wave event, are clear. The deeper events propagating from right to left are reflections from the right boundary.

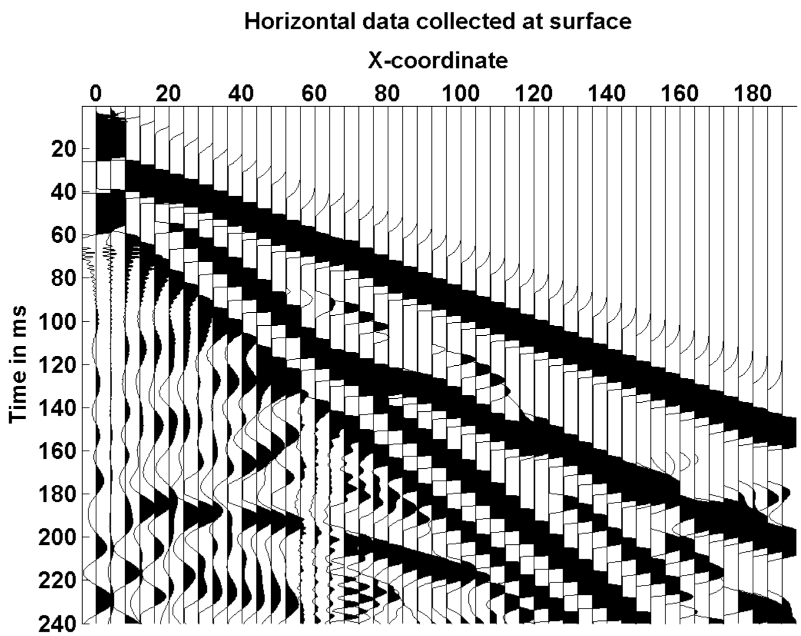

FIG. 8. The seismic source gather from the surface of the 5 metre step model. The step seems to have caused a lagged shear wave event parallel to the first breaks, and an advanced surface wave event connected to it.