Microseismic sensitivity for four receivers on a square grid

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ABSTRACT

There are a number of analytic solutions for estimating the location of a microseismic event from the first arrival clock-times at three or four receivers. This paper evaluates the sensitivity of four receivers on a square grid with respect to errors in the location of the receivers, jitter on the clock-times, and error in the velocity.

INTRODUCTION

The traveltime equations for raypaths between a source at (x_0, y_0, z_0) and four arbitrarily located receivers at (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), and (x_4, y_4, z_4) are:

$$\begin{aligned}(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2&=v^2(t_{01}-t_0)^2\\(x_2-x_0)^2+(y_2-y_0)^2+(z_2-z_0)^2&=v^2(t_{02}-t_0)^2\\(x_3-x_0)^2+(y_3-y_0)^2+(z_3-z_0)^2&=v^2(t_{03}-t_0)^2\\(x_4-x_0)^2+(y_4-y_0)^2+(z_4-z_0)^2&=v^2(t_{04}-t_0)^2\end{aligned}, \tag{1}$$

where v is the (constant) velocity, t_0 is the clock-time of the source event and t_{01}, t_{02}, t_{03}, and t_{04}, are the clock-times of the event at the corresponding receivers. These equations are the starting point of the Apollonius solution (Bancroft and Du 2006 and 2007). If we assume the four receivers are coplanar on a square grid, and the side of the square has a dimension h, then the equations in (1) can be simplified to

$$\begin{aligned}x_0^2+y_0^2+z_0^2&=v^2\left(t_{01}-t_0\right)^2\\\left(x_0-h\right)^2+y_0^2+z_0^2&=v^2\left(t_{02}-t_0\right)^2\\x_0^2+\left(y_0-h\right)^2+z_0^2&=v^2\left(t_{03}-t_0\right)^2\\\left(x_0-h\right)^2+\left(y_0-h\right)^2+z_0^2&=v^2\left(t_{04}-t_0\right)^2\end{aligned}. \tag{2}$$

The source time t_0 is be defined by

$$t_0=\frac{t_{01}^2-t_{02}^2-t_{03}^2+t_{04}^2}{2\left(t_{01}-t_{02}-t_{03}+t_{04}\right)}, \tag{3}$$

which is totally independent of the geometry. The source coordinates x_0, y_0, and z_0 are computed from

$$\begin{aligned}x_0&=\left\{v^2\left[2t_0\left(t_{02}-t_{01}\right)-\left(t_{02}^2-t_{01}^2\right)\right]+h^2\right\}/2h\\y_0&=\left\{v^2\left[2t_0\left(t_{03}-t_{01}\right)-\left(t_{03}^2-t_{01}^2\right)\right]+h^2\right\}/2h,\\z_0&=sqrt\left[v^2\left(t_{01}-t_0\right)^2-\left(x_0^2+y_0^2\right)\right]\end{aligned} \tag{4}$$

where the sign of z_0 can be chosen to suit the dimensions of the project.

SENSITIVITY ANALYSIS

We choose to investigate the sensitivity of the source location by perturbating all the controlling parameters. Although the receiver locations are not part of solving equations (3) and (4), they do define the traveltimes from the source to the receivers. We start with the four corners on a square with sides of length $h = 10$ m, and the lower left corner at the origin. The location of the microseismic source is defined at (30, 20, -50), and the velocity $v = 2000$ m/s. We perturb the parameters by adding random noise to the coordinates, times and velocities. Using the perturbed parameters, we compute the source to receiver traveltimes, then use these times to estimate the source location. This procedure is repeated for 100 trials. The results of the trials are modified by removing the upper and lower 10% of the outliers. Plots are made of these results, with the source a blue star "*****", the receiver a green plus sign "**+**", and the estimated location as red circles "**o**".

The standard deviations (SD) of the corresponding noise that is added to the parameters are chosen so that the distribution or the source estimates are approximately the same for each parameter that is tested.

RESULTS

Perturbations in the receiver locations

We start the first trial by adding Gaussian noise to the x and y coordinates with standard deviations (SD) of 0.1m for the each coordinate. The z coordinate is left unperturbed. The resulting estimates of the source location are plotted in Figure 1, which contains two side views, a map view (plan), and a perspective view.

The x and y coordinates are left unperturbed, but the vertical axis z is perturbed with a SD = 0.1 m giving the results in Figure 2.

Perturbations in the traveltimes

The receiver coordinates are then left unperturbed, but the traveltimes to each receiver are perturbed by adding jitter (noise in time) of 0.01 ms. Note, the distance from the source to the receivers is approximately 60 m, and the traveltimes are close to 30 ms. The results are plotted in Figure 3.

Perturbations in the velocity

The velocities were not perturbed in a random manner, but varied from $0.4v$ to $1.4v$ in increments of 0.1. The results are in Figure 4.

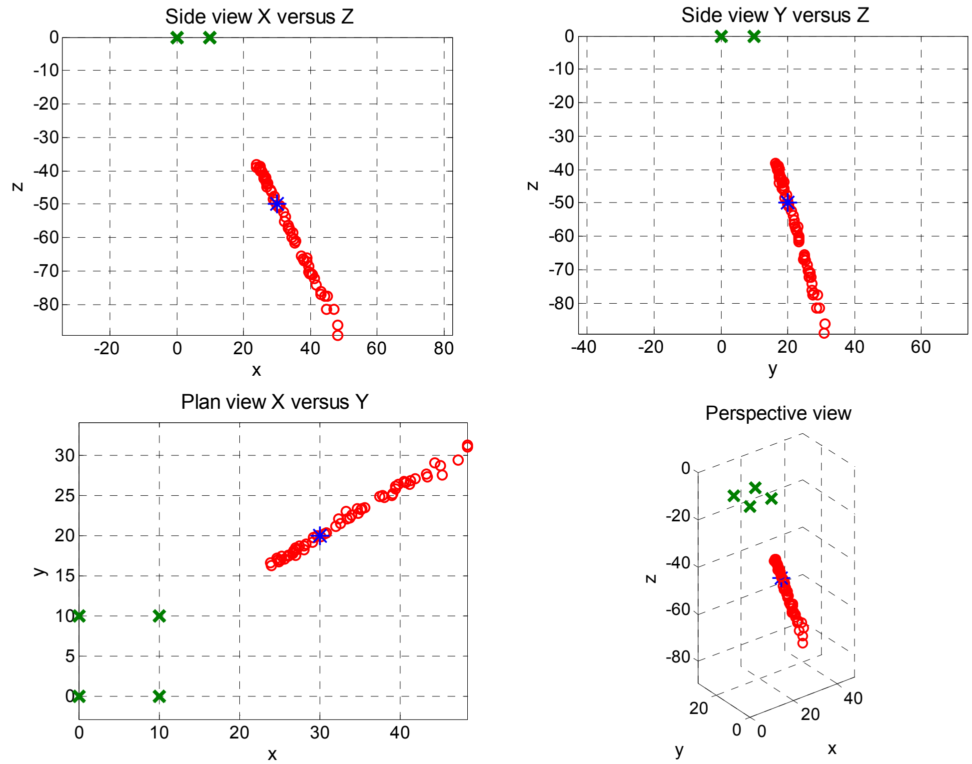

FIG. 1 Plot of source sensitivity when the *x* and *y* coordinates of the receivers are perturbed by 1% or a SD = 0.1 m.

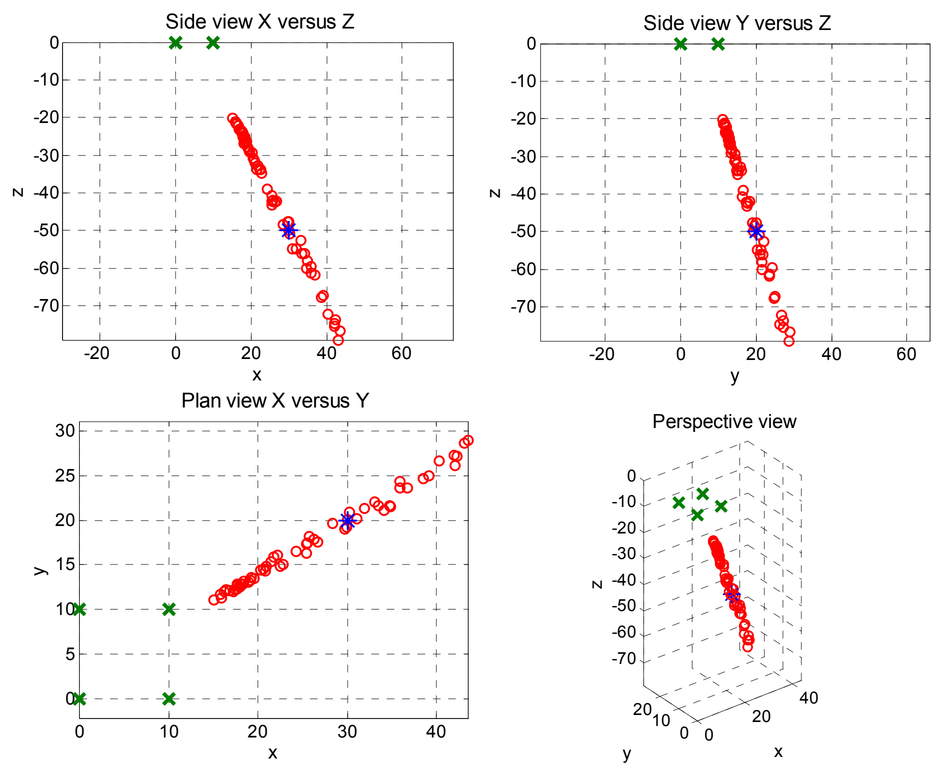

FIG. 2 Plot of source sensitivity when the *z* coordinate of the receivers is perturbed by 1% or a SD = 0.1 m.

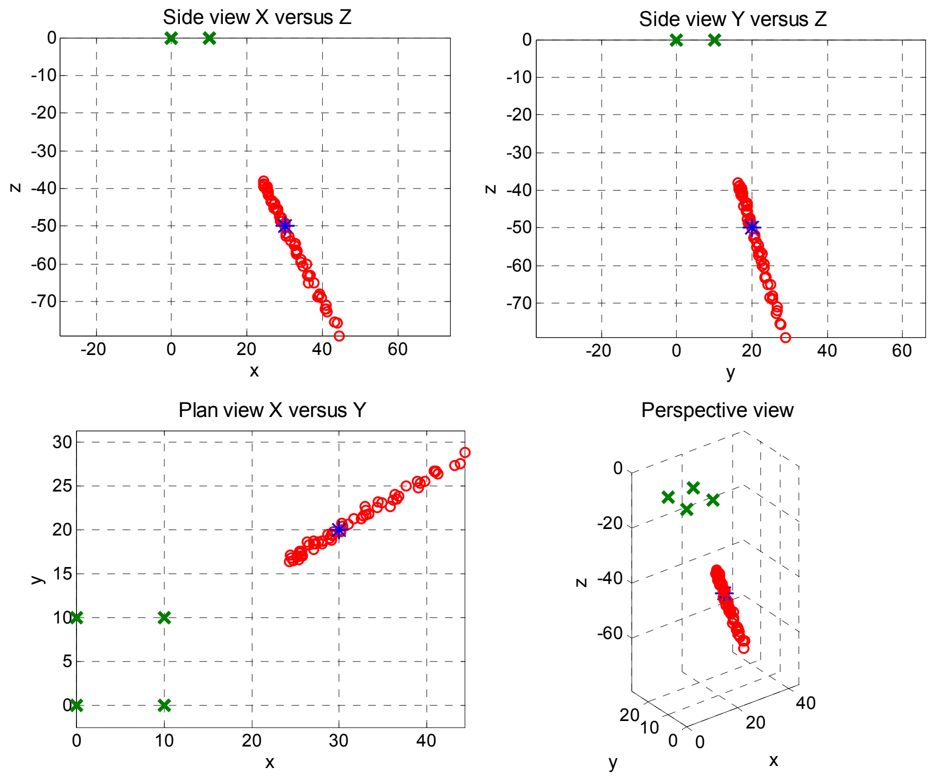

FIG. 3 Plot of source sensitivity when the traveltime from the source to the receivers is perturbed with a jitter that has a SD = 0.025 ms.

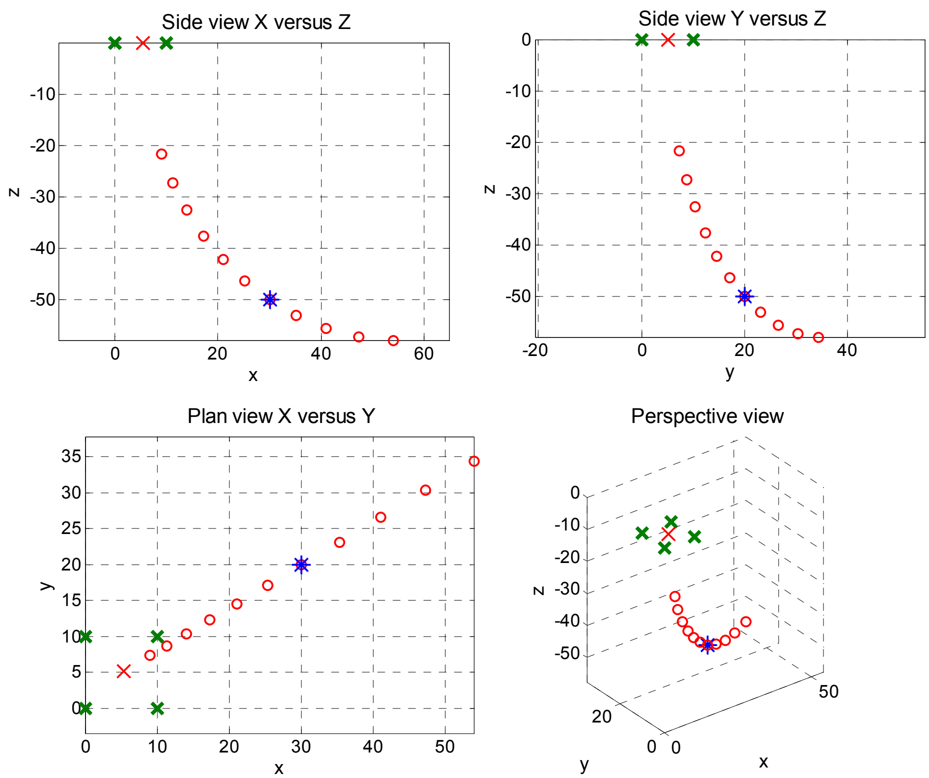

FIG. 4 Plot of the estimated source locations when the velocity is incremented from 0.4v to 1.4v.

DISCUSSION OF RESULTS

When the receiver location and the traveltimes were perturbed, the distribution of the source estimates tend to fall along a line that extends to the center of the receivers. The size of this spread is discouraging as in practical applications, there is only one measurement of the traveltimes from a microseismic event, and its error can be quite large. However, the linear distribution of energy provides a significant benefit. We can now fit a line from the center of the receivers to an estimated source location that will pass close to the actual source. We assume that the receivers are part of a much larger grid of receivers and that there will be many of these lines, and all will point in the direction of the source location. Simple linear algebraic solutions exist for finding a source location (Han et al. 2010).

When the velocity was varied, the distribution of the estimated source locations is not linear but curved. This curve extends from a location (x_{min}, y_{min}) near the center of the receivers. Using this point and a reasonable velocity to define another point, an analytic expression may be developed to define this curve. The intersection of these curves from a grid of receivers may not only help provide a source location, but may aid in refining the velocity model.

The location of a point in the plane of the receivers will have a low velocity as the distances to the receiver will be minimal. This velocity, v_{min}, and the location (x_{min}, y_{min}) can be computed from the following equations that are derived in the Appendix. We first define the coefficients a, b, and c using

$$\begin{aligned} a &= (t_2^2 - t_1^2)^2 + (t_3^2 - t_1^2)^2 \\ b &= -2h^2 (t_2^2 + t_3^2) \\ c &= 2h^4 \end{aligned} \quad , \tag{5}$$

where $t_{i=1,2,3,4}$ are the raypath traveltimes after t_0 has been removed from the receiver clock-times; then v_{min} is defined using the results of equation (5),

$$v_{\min} = \left[\frac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a} \right]^{1/2} . \tag{6}$$

Given v_{min} we compute the locations x_{min} and y_{min} from

$$\begin{aligned} x_{\min} &= \frac{h^2 - v_{\min}^2 (t_2^2 - t_1^2)}{2h} \\ y_{min} &= \frac{h^2 - v_{\min}^2 (t_3^2 - t_1^2)}{2h} \end{aligned} . \tag{7}$$

The location of (x_{min}, y_{min}) is also plotted in Figure 4 and identified by the red plus sign "+". The estimated velocity was v_{min} = 242.99 m/s, and the location x_{min} = 5.369, and y_{min} = 5.221.

SOFTWARE

The software in this report was developed with MATLAB.

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\2010-Matlab\MicroSeisLocation\
FourReceiversOnSquareXYZrecNoise.m
FourReceiversOnSquareJitterStability.m
FourReceiversOnSquareVaryVelocity.m
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ACKNOWLEDGEMENTS

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CREWES: The Consortium for Research in Elastic Wave Exploration Seismology

REFERENCES

Bancroft, J.C., 2009, Sensitivity measurements for locating microseismic events, CREWES Research Report Vol. 21.

Han, L., and Bancroft, J. C., 2010, Nearest approaches to multiple lines in *n*-dimensional space, CREWES Research Report Vol. 22.

Bancroft, J.C. and Du, X., 2007, Traveltime computations for locating the source of micro seismic events and for forming gridded traveltime maps, 69th EAGE Conference, London, England

Bancroft, J.C. and Du, X., 2007b, Traveltime computations for locating the source of micro seismic events and for forming gridded traveltime maps, CSPG CSEG Convention, Calgary, Alberta, Canada.

Bancroft, John C., 2006, Locating microseismic events and traveltime mapping using locally spherical wavefronts, CREWES Research Report

John C. Bancroft, and Xiang Du, 2006, The computations of traveltimes when assuming locally circular or spherical wavefronts, SEG International Convention, New Orleans

APPENDIX

We have four coplanar receivers that are located on the corners of a square, and derive the location of a point (x_{min}, y_{min}) that is coplanar with the receivers which has the same raypath traveltimes $t_{i=1,2,3,4}$ as a microseismic event, but a much lower velocity v_{min}.

We start with equation , set $z = 0$, and updated raypath times giving

$$\begin{aligned} x_{\min}^2 + y_{\min}^2 &= v^2 t_1^2 \\ (h - x_{\min})^2 + y_{\min}^2 &= v^2 t_2^2 \, . \\ x_{\min}^2 + (h - y_{\min})^2 &= v^2 t_3^2 \end{aligned} \tag{8}$$

We drop for now the *min* subscript, then subtract lines 1 from 2, and line 1 from 3,

$$\begin{aligned} h^2 - 2xh &= v^2 (t_2^2 - t_1^2) \\ h^2 - 2yh &= v^2 (t_3^2 - t_1^2) \end{aligned} , \tag{9}$$

and solving for x and y,

$$x = \frac{h^2 - v^2(t_2^2 - t_1^2)}{2h}$$
$$y = \frac{h^2 - v^2(t_3^2 - t_1^2)}{2h} \qquad (10)$$

We now need to solve for v, and start by substituting the values from equations (10) into the first line of equation (8)

$$\frac{\left[h^2 - v^2(t_2^2 - t_1^2)\right]^2}{4h^2} + \frac{\left[h^2 - v^2(t_3^2 - t_1^2)\right]^2}{4h^2} = v^2 t_1^2 . \qquad (11)$$

Squaring the square bracketed terms [ ] and sorting for v we get

$$v^4\left\{(t_2^2 - t_1^2)^2 + (t_3^2 - t_1^2)^2\right\} - v^2 2h^2\left\{t_2^2 + t_3^2\right\} + 2h^4 = 0 , \qquad (12)$$

which is a quadratic equation with v^2. Defining

$$a = (t_2^2 - t_1^2)^2 + (t_3^2 - t_1^2)^2$$
$$b = -2h^2(t_2^2 + t_3^2) \quad , \qquad (13)$$
$$c = 2h^4$$

we have

$$v_{\min}^2 = \frac{-b \pm \sqrt{\left(b^2 - 4ac\right)}}{2a} . \qquad (14)$$

Choose the smallest positive value for v_{min}, it is substituted into equations (10) to define x_{min} and y_{min}.