Least squares AVF inversion

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ABSTRACT

INTRODUCTION

There are a number of examples in geophysics literature reporting dispersion anomalies in seismic reflection data associated with highly absorptive targets. There is a frequency-by-frequency (AVF) method of inverting frequency dependent absorptive reflectivity for Q developed by (Innanen, 2011). This method requires as input, an estimate of the spectrum of the absorptive reflection coefficient. In other CREWES reports (for example, see Bird et al., 2011; Bird and Innanen, 2011) the authors explored the effects that a wavelet, random noise, and proximal seismic events will have on the spectrum of a target absorptive reflection coefficient. Recommendations for implementing AVF inversion in the presence of these effects were made but it was found that the result of AVF inversion was typically inconsistent and unreliable. It is because of this that we develop a method here of casting the AVF inverse problem as a least-squares minimization problem. The opportunity for optimization and regularization offered by a least squares approach may bring stability to the output of AVF inversion in the presence of wavelet, noise, proximal events, etc. Further the angle dependence of reflection coefficients is studied in the least squares framework and the effect of the wavelet is taken into account by utilizing the estimate of the wavelet in the least squares algorithm as opposed to deconvolving the trace prior to implementation of AVF inversion. This approach of utilizing the wavelet estimate in the least squares algorithm may also help stabilize the AVF inversion result.

LEAST-SQUARES AVF INVERSION AT NORMAL INCIDENCE AND NO WAVELET

We start with the expression for the linearized absorptive reflection coefficient for an elastic overburden overlaying a highly attenuative target given by (for derivation see Innanen, 2011)

$$R(\omega) = -\frac{1}{2}a_Q F(\omega) + \frac{1}{4}a_C, \tag{1}$$

Where the reflection coefficient, $R(\omega)$, is complex and a function of frequency and a_Q and a_C are the perturbation parameters in Q and acoustic seismic velocity respectively. It is also worth noting that $F(\omega)$ is a known function. The AVF inverse problem is to determine a_Q or a_C from measurements of $R(\omega)$. By differencing the reflection coefficient across frequencies the perturbation in Q can be solved for (Innanen, 2011; Bird et al., 2010a). In concurrent CREWES reports (Bird et al., 2011) attempts to implement AVF inversion on a target absorptive reflection coefficient in the presence of a wavelet, random noise, and numerous proximal events. It was found that the inversion result was often unreliable and inaccurate in the presence of these seismic data phenomena. For this reason, we cast the AVF inverse problem in a least-squares formalism in the hope of stabilizing the results of

AVF inversion. By looking at 1 we can discretize $R(\omega)$ into N values and re-write in matrix form,

$$\begin{pmatrix} R(\omega_1) \\ R(\omega_2) \\ \vdots \\ R(\omega_N) \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{2}\mathrm{F}(\omega_1) \\ \frac{1}{4} & -\frac{1}{2}\mathrm{F}(\omega_2) \\ \vdots & \vdots \\ \frac{1}{4} & -\frac{1}{2}\mathrm{F}(\omega_N) \end{pmatrix} \begin{pmatrix} a_C \\ a_Q \end{pmatrix}$$

Equation 2 highlights the fact that solving for a_C and a_Q is an overdetermined problem. To cast the problem as a least-squares minimization problem we can view 2 in the form

$$d = Gm, \tag{3}$$

where the data vector d is $R(\omega)$, G is the operator and the model vector m is a_C and a_Q now, least squares minimizing the error between d and Gm solving for m yields

$$m = (G^T G)^{-1} G^T d, \tag{4}$$

This is the least squares solution for a_C and a_Q. To test the effectiveness of this approach modeling was performed for a series of absorptive reflections. The modeling is performed for an impulsive plane wave incident upon a planar boundary separating an elastic overburden with a highly absorptive target. Synthetic traces were generated for this experiment in which Q was allowed to vary from 10 to 105. The spectrum of the traces were then obtained using an FFT and4 was implemented to solve for a_C and a_Q for each of these traces. Figure 1 shows an example trace in which Q of the target was 40. Figure 2 shows the spectrum of this absorptive reflection and Figure 3 and Figure 4 show the accuracy of the inversion of a_C and a_Q for all of these experiments. Notice in Figure 4 that the inversion for a_Q fails at low Q values. This is to be expected since we linearized the expression for the absorptive reflection coefficient by assuming small a_Q (for example, see Bird et al., 2010a; Innanen, 2011). And as Q becomes small the assumption of a small a_Q fails.

LEAST-SQUARES AVF INVERSION INCLUDING OFFSET AND A WAVELET

In the previous section we cast the AVF inverse problem in a least-squares formalism for normal incidence data with not wavelet. We can extend this idea to include angle dependent data and a source wavelet. We first extend the normal incidence formula in equation 1 to oblique incidence (Innanen, 2011, for derivation, see) given by the expression

$$R(\omega,\theta) = \left(-\frac{1}{2}a_Q F(\omega) + \frac{1}{4}a_C\right)\left(1 + \sin^2\theta\right), \tag{5}$$

where θ is the angle of incidence and $R(\omega,\theta)$ is the linearized, frequency dependent and angle dependent reflection coefficient. In the same way as 1 was recast in a least squares framework so too can 5. We start by rewriting 5, for a given angle of incidence θ_m as

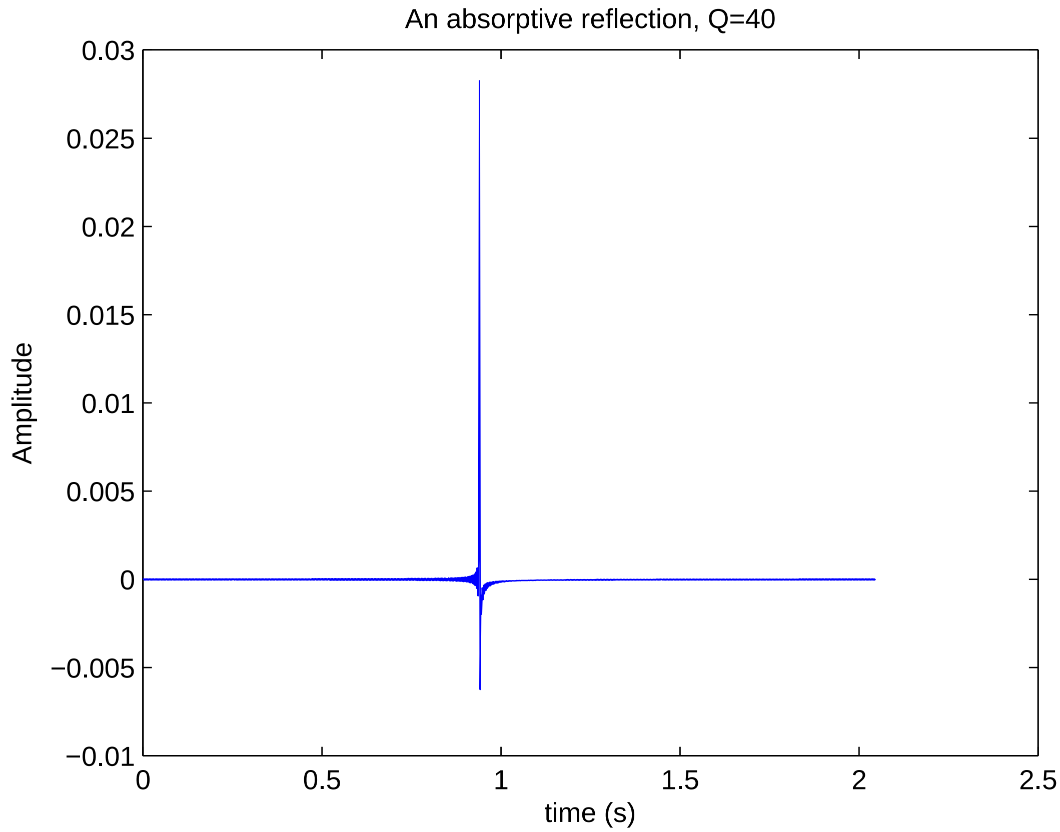

FIG. 1. A synthetic seismic trace with a single absorptive reflection and no source wavelet. Q of the target is 40.

$$\begin{pmatrix} R(\omega_1,\theta_m) \\ R(\omega_2,\theta_m) \\ \vdots \\ R(\omega_N,\theta_m) \end{pmatrix} = \begin{pmatrix} \frac{1}{4}\left(1+\sin^2\theta_m\right) & -\frac{1}{2}\mathrm{F}(\omega_1)\left(1+\sin^2\theta_m\right) \\ \frac{1}{4}\left(1+\sin^2\theta_m\right) & -\frac{1}{2}\mathrm{F}(\omega_2)\left(1+\sin^2\theta_m\right) \\ \vdots & \vdots \\ \frac{1}{4}\left(1+\sin^2\theta_m\right) & -\frac{1}{2}\mathrm{F}(\omega_N)\left(1+\sin^2\theta_m\right) \end{pmatrix} \begin{pmatrix} a_C \\ a_Q \end{pmatrix}$$

Further, we can include the effect of a source wavelet. In order to include a source wavelet, we need only multiply the reflection coefficient with the wavelet in the frequency domain. Since 1 and 5 are written as functions of frequency, we can simply re-write 5 as

$$\tilde{R}(\omega,\theta) = R(\omega,\theta)S(\omega) = \left(-\frac{1}{2}a_Q F(\omega) + \frac{1}{4}a_C\right)\left(1+\sin^2\theta\right)S(\omega), \tag{7}$$

where $S(\omega)$ is the spectrum of the source wavelet and $\widetilde{R(\omega,\theta)}$ is reflection coefficient multiplied by the source wavelet. This $\tilde{R}(\omega,\theta)$ will now represent our data vector d in 3. To cast equation 7 as a least-squares inverse problem we first re-write as

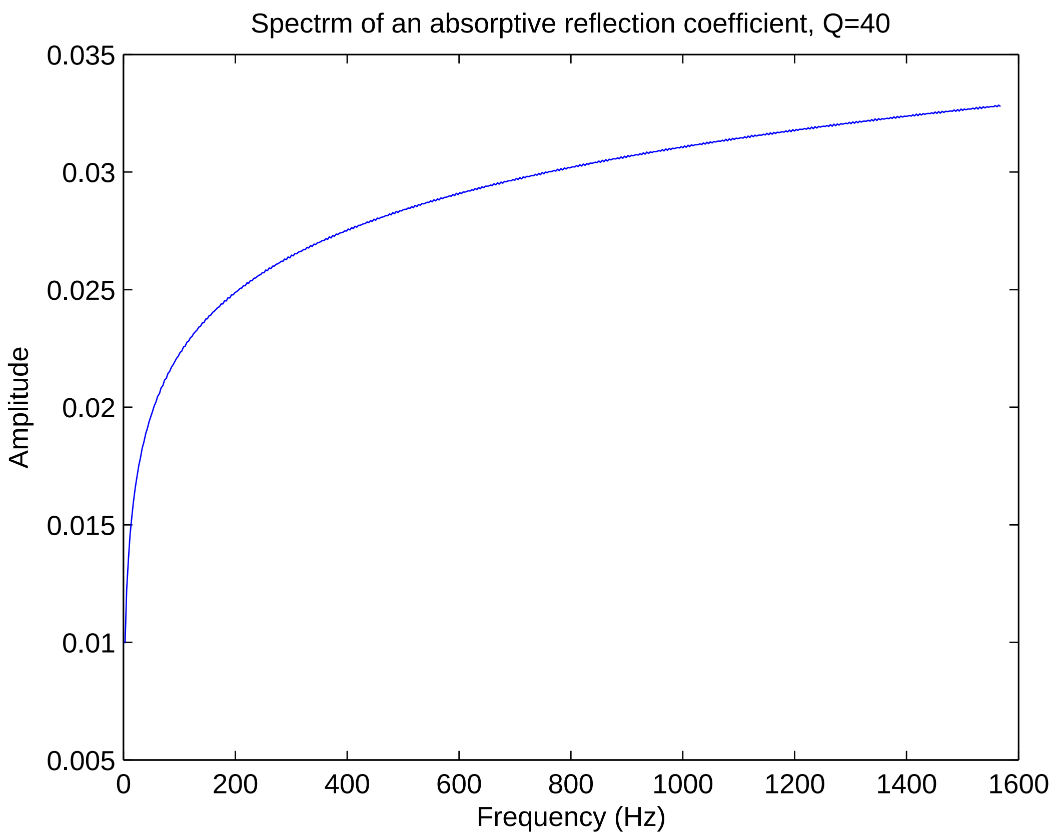

FIG. 2. The spectrum of the absorptive reflection shown in Figure 1.

$$\begin{pmatrix} \tilde{R}(\omega_1,\theta_m) \\ \tilde{R}(\omega_2,\theta_m) \\ \vdots \\ \tilde{R}(\omega_N,\theta_m) \end{pmatrix} = \begin{pmatrix} \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_1) & -\frac{1}{2}F(\omega_1)\left(1+\sin^2\theta_m\right)S(\omega_1) \\ \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_2) & -\frac{1}{2}F(\omega_2)\left(1+\sin^2\theta_m\right)S(\omega_2) \\ \vdots & \vdots \\ \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_N) & -\frac{1}{2}F(\omega_N)\left(1+\sin^2\theta_m\right)S(\omega_N) \end{pmatrix} \begin{pmatrix} a_C \\ a_Q \end{pmatrix}$$

Now we let

$$d = \begin{pmatrix} \tilde{R}(\omega_1,\theta_m) \\ \tilde{R}(\omega_2,\theta_m) \\ \vdots \\ \tilde{R}(\omega_N,\theta_m) \end{pmatrix}, \tag{9}$$

and

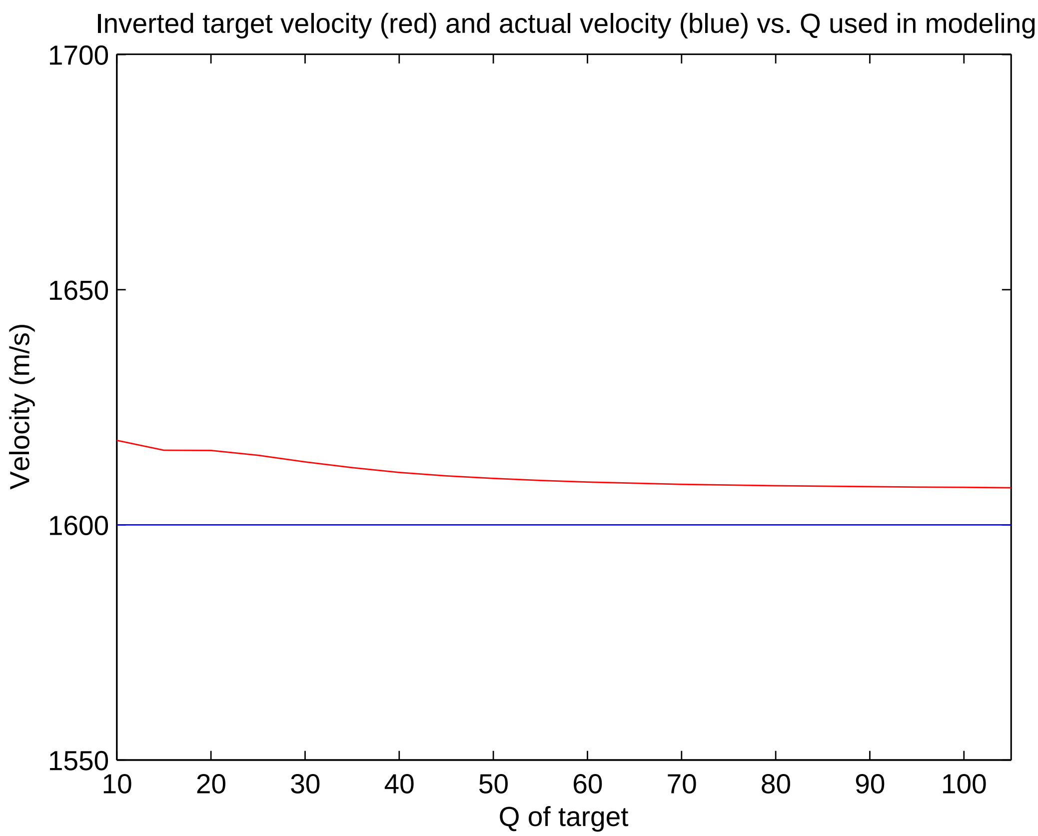

FIG. 3. Comparison of the inversion of target velocity (red) and the actual target velocity (blue) vs. the Q used in the modeling.

$$G = \begin{pmatrix} \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_1) & -\frac{1}{2}F(\omega_1)\left(1+\sin^2\theta_m\right)S(\omega_1) \\ \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_2) & -\frac{1}{2}F(\omega_2)\left(1+\sin^2\theta_m\right)S(\omega_2) \\ \vdots & \vdots \\ \frac{1}{4}\left(1+\sin^2\theta_m\right)S(\omega_N) & -\frac{1}{2}F(\omega_N)\left(1+\sin^2\theta_m\right)S(\omega_N) \end{pmatrix}, \tag{10}$$

and finally let

$$m = \begin{pmatrix} a_C \\ a_Q \end{pmatrix}, \tag{11}$$

which allows us to write the least-squares solution for m as

$$m = (G^T G)^{-1} G^T d, \tag{12}$$

To test this least-squares AVF inversion framework, angle dependent reflection coefficients were calculated and a minimum phase wavelet was convolved with the reflection

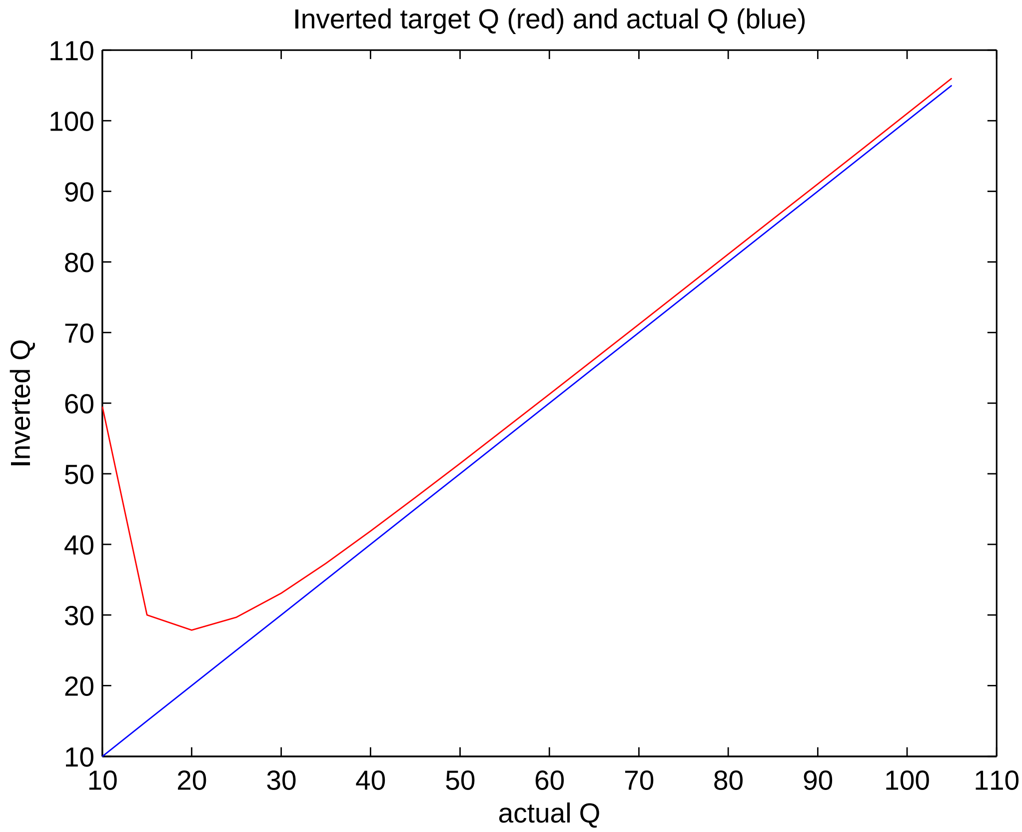

FIG. 4. Comparison of the inversion of target Q (red) and the actual target Q (blue) vs. Notice the inversion fails at low Q. This is due to linearization error.

coefficients. Figure 5 shows an example of the seismic wavelet convolved with a reflection coefficient. Of course on real seismic data we will not have the luxury of knowing the exact wavelet. Therefore a standard weiner deconvolution code from the CREWES matlab toolbox was used to obtain an estimate of the wavelet and that was implemented in the operator (equation 10) prior to inversion. Figure 6 shows the inversion result for the wavespeed, the red line corresponds to the inversion result and the blue line is the actual wavespeed. Notice that the inversion fails at high angles of incidence, this is due to the fact that 5 is a linearized expression obtained by expanding around small $\sin^2\theta$. Figure 7 shows the inversion result for Q in red and the actual Q in blue, again the failure of the inversion at high angles of incidence is due to the linearization error. Figure 6 and Figure 7 show that this least squares approach has the ability to invert for the wavespeed and Q of the target for angles of incidence up to about 40 degrees. This is a promising result as it shows that an estimate of the wavelet using standard deconvolution codes can be used to obtain reliable estimates of target Q.

LEAST-SQUARES AVF INVERSION USING THE FST AS INPUT

In order to implement this least-squares AVF approach on real seismic data we must a method of time-frequency decomposition to estimate the local spectrum of the target reflection coefficient. We have shown in previous papers (Bird et al., 2010b,a) that a fast S-transform algorith (FST) provides reliable estimates of the local spectra of reflection events and so it should prove straightforward to reformulate the least squares AVF approach to accept input using the FST. To test the effectiveness of this least-squares approach to AVF inversion using the FST to estimate the spectrum of the absorptive reflection coefficient, absorptive reflections were modeled at normal incidence and with no wavelet. The FST was then used to estimate the local spectrum of the absorptive reflection and then the least squares approach was used to invert for Q of the target. The procedure was repeated for a large number of Q values and the results are shown in Figure 8. In Figure 8 we see the actual target Q shown in blue and the inverted value shown in red, the inversion values are reasonably accurate especially at lower Q values. For future work, the effect of a wavelet, noise, and numerous events will need to be studied to validate the least-squares AVF inversion approach.

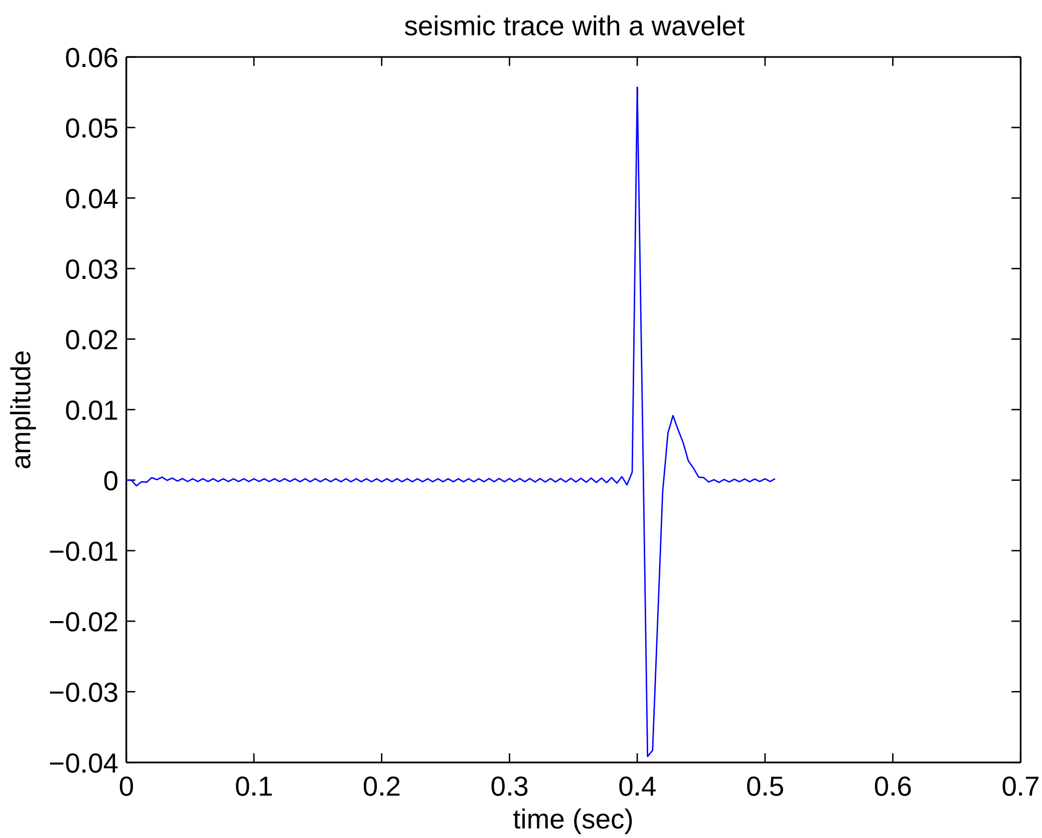

FIG. 5. A reflection coefficient convolved with a wavelet.

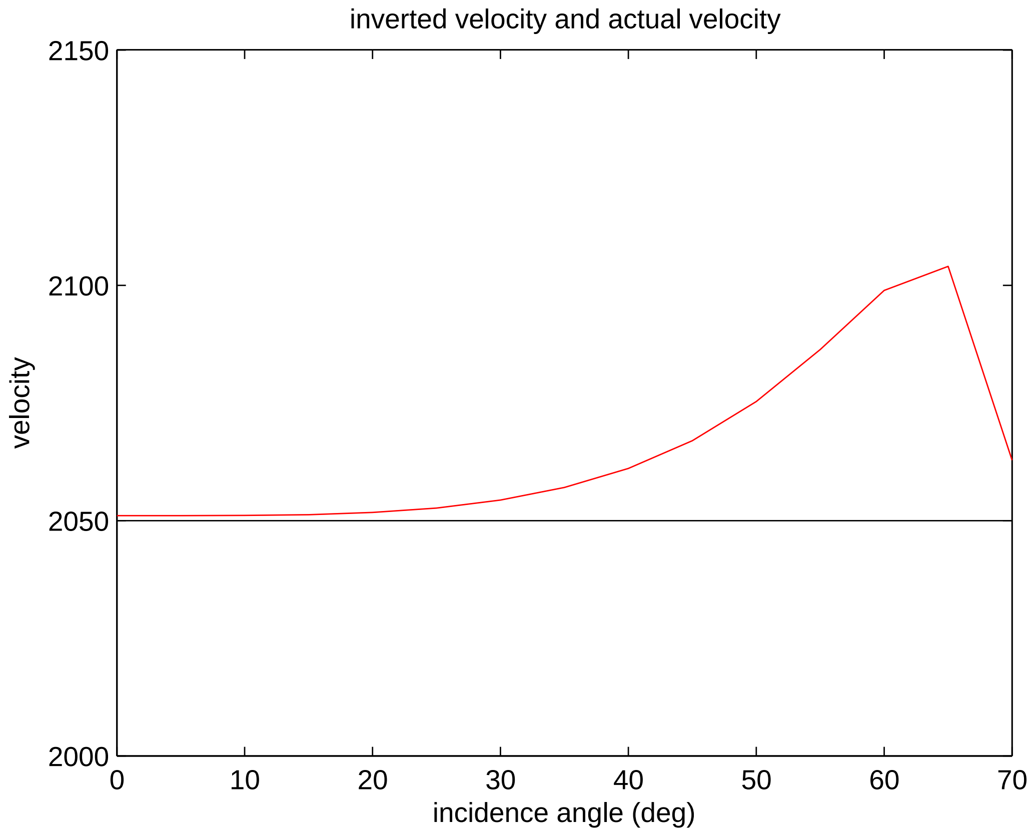

FIG. 6. Comparison of the inversion of target Q (red) and the actual target Q (blue) vs. Notice the inversion fails at low Q. This is due to linearization error.

CONCLUSIONS

In this paper, the AVF inverse problem was recast as a least sqaures minimization problem and this least squares approach was adapted for angle dependent reflections and a seismic wavelet. In concurrent CREWES reports (Bird et al., 2011; Bird and Innanen, 2011) it was shown that AVF inversion was inconsistent in the presence of noise, numerous reflection events, and a wavelet. The hope of this least-squares approach is to stabilize AVF inversion in the presence of these phenomena. It was shown that for incidence angles up to about 40 degrees reliable estimates of both wavespeed and Q were obtainable, even in the presence of a source wavelet. Further, it was shown that using standard deconvolution codes to obtain an estimate of the wavelet, a reliable inversion result is achieved. Any hope of implementing AVF on seismic field data will require an estimate of the local spectrum of seismic reflections using time-frequency decomposition. We tested this least-squares AVF inversion using the FST estimate of an absorptive reflection coefficient as input. It was found that the inversion results were reasonably accurate. For future work, we will extend this least squares AVF approach on more complex models which include noise, wavelet, and numerous reflection events.

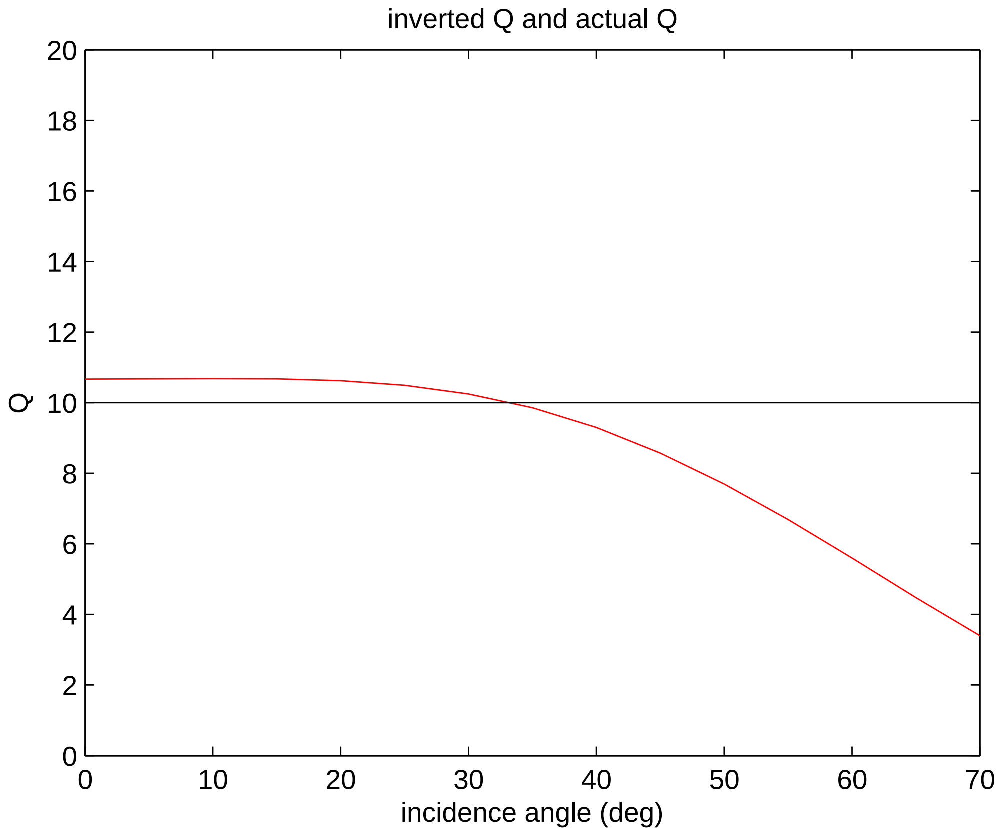

FIG. 7. Comparison of the inversion of target Q (red) and the actual target Q (blue) vs. Notice the inversion fails at low Q. This is due to linearization error.

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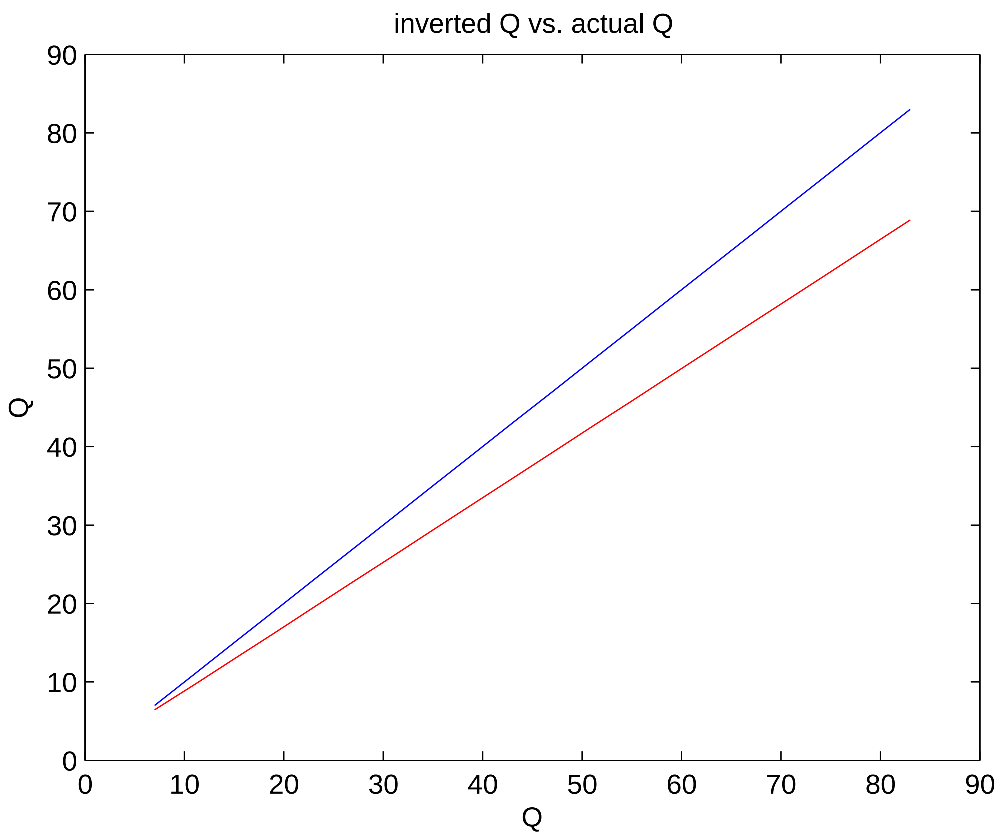

FIG. 8. Comparison of the inversion of target Q (red) and the actual target Q (blue) using the FST to estimate the spectrum of the reflection coefficient