Comments on stacking noisy data

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ABSTRACT

Stacking assumes that we have seismic traces that are contaminated with noise, and that the sum of similar traces divided by the number of traces will reduce the noise by the square-root of the number of traces. This stacking process assumes the standard deviation (SD) of the noise is similar in each trace. In applications where the SD of the noise varies from trace to trace, some form of weighing each trace by the inverse of its SD may improve the overall quality of the stack. In prestack migration gathers, where the noise is much greater than the data in a trace, balancing the amplitudes by dividing by the square-root of the fold may improve the stack.

INTRODUCTION

When using the equivalent offset method (EOM) of prestack migration, common scatterpoint (CSP) gathers are formed for every migrated trace. All input traces within the migration aperture are summed into the offset bins of each CSP gather. The fold is recorded for each sample in every trace and can be quite high. It has become standard practice to divide by this time varying fold.

The fold of the data varies considerably and increases from zero, at zero offset, to a maximum at the maximum source receiver offset. It then decreases with increased offset. Even when the noise on the input traces has a constant SD, the noise in the offset bins will vary with the square-root of the fold. Stacking these traces, after moveout correction, will not produce the optimum stack.

I propose that dividing the traces in the offset bins by the square-root of the fold will balance the noise in the offset bins to produce an optimum stack of the signal energy. This conjecture is demonstrated with real noisy data, and is based on the "Pythagorean Theorem of Statistics" where, for independent random variables, the stack with have a variance equal to the sum of the individual variances.

Forming a CSP gather

Figure 1 shows a prestack volume of the input data where the x axis is the common midpoint (CMP) location, the h axis is the source receiver half offset, and the vertical axis represents the time of the data. The small black dots on the surface represent the trace locations for a number of left sided shot records. We desire to form one CSP gather identified by the vertical plane. All input traces are summed into the bins of the gather at a simplified equivalent offset defined by

$$h_e = \sqrt{x^2 + h^2} \, . \tag{1}$$

This is equivalent to defining a radius from the zero offset of the gather to the location of the offset trace. The input trace in then summed into the bin with an offset defined by

that radius as illustrated by the curved black lines. Zero-offset traces (green dots) are summed in along with the offset traces (in blue).

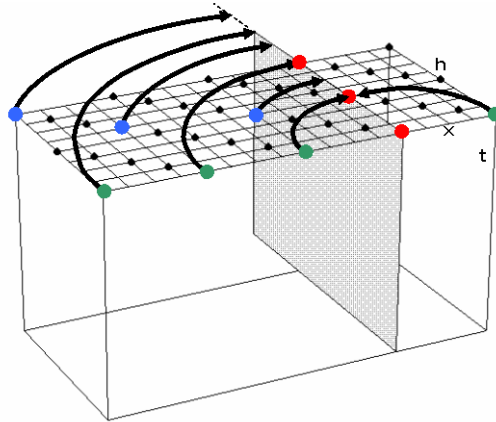This figure has been included to illustrate the fold that develops in the CSP gather.

FIG 1 Prestack volume illustrating the movement of prestack traces into a CSP gather.

The fold

At near offset, there are few traces that will sum into the offset bins. As the offset increases, the arc that contains the offset traces increases proportionally with the offset, i.e. the fold increase proportionally to the offset. At the maximum source-receiver offset h_{max}, the arc has maximum length and the fold will be the largest. There are many traces that will sum into the gather at equivalent offsets well beyond h_{max} as illustrated. The length of the arc will contain fewer input traces, and the fold will start to decrease and tend to $\pi/4$ of the maximum value. At some large offset, the time of the useful data in the input trace will tend to the maximum time, at which point the fold will become zero.

An example of a raw CSP gather is shown in Figure 2 where the time of the useful data in each offset has been limited by a parameter such as a maximum dip, or limited to a minimum single moveout value. The offset varying fold is displayed in Figure 3. The data in this example were quite sparse, creating an oscillating effect in the fold.

The time varying nature of the fold is illustrated in Figure 3 by the four coloured lines that represent the fold at 25%, 50%, 75% and 100% of the maximum time.

THE DATA

The data are from north east British Columbia and were recorded with multicomponent receivers. The line is approximately 13 km long, with about 1300 stations located at 10 m intervals. There were 220 shots, located at every 6th station or at a 60 m interval. The line rolled in and out, and had a maximum source/receiver offset of 1,600 m, or a half offset h of 800 m. The fold is approximately 27 in the center of the line. We chose the radial component to process converted wave data. The quality of the data is quite poor as illustrated by a central shot record in Figure 4. The data in this figure has been preprocessed up to the point prior to prestack migration.

The data were prestack migrated using EOM to produce the CSP gather illustrated in Figure 2 with the fold shown in Figure 3. Note the fold is highest round 800 m, then falls off to a stable level. This corresponds to an amplitude build up in the CSP gather. This gather was formed with the maximum dip levels set to taper from 80 to 90 degrees.
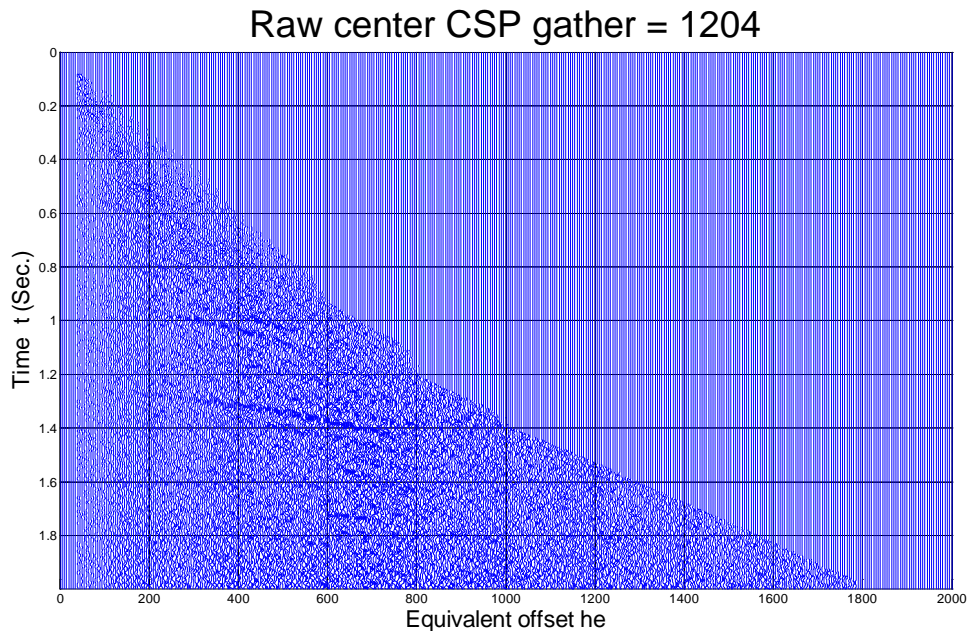

FIG 2 A raw CSP gather that corresponds to the fold in Figure 2.
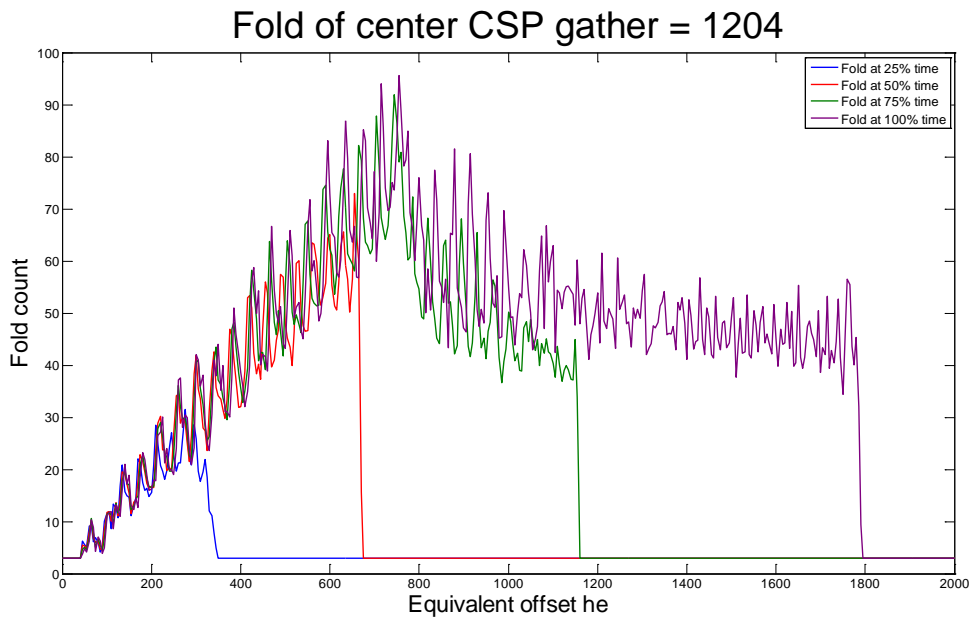

FIG 3 Offset varying fold for the above CSP gather

With this information we now proceed to illustrate the effect of dividing the CSP gather by the fold.
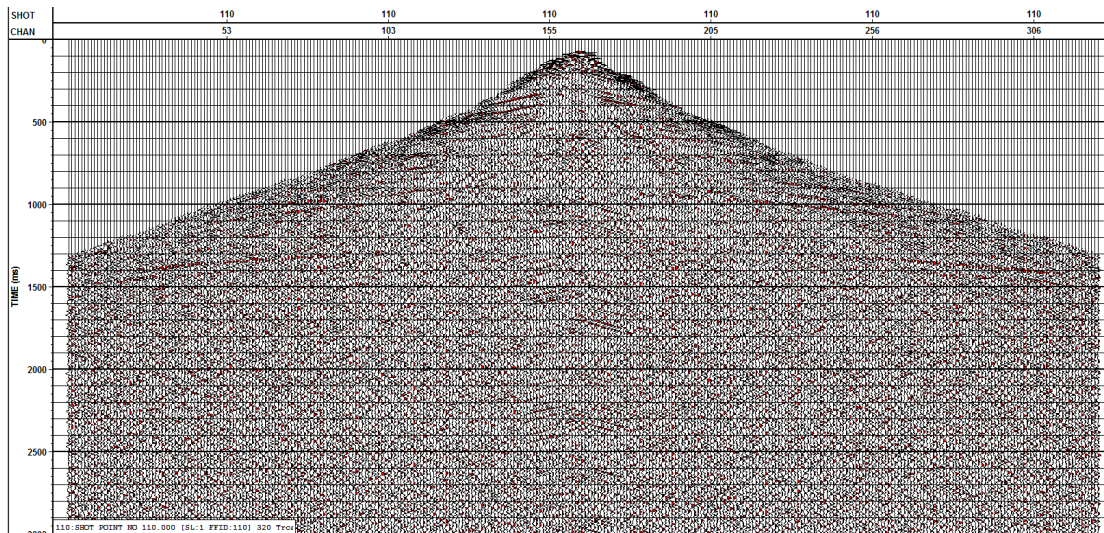
FIG 4 Central shot record, preprocessed standard processing up to the point of stacking.

NORMALIZING THE FOLD IN A CSP GATHER

In noise free data, it would be logical to divide the data in a CSP gather by the fold at each sample. However, the data we are using is dominated by noise, and there may be a more effective method to obtain a better stack.

Summing noisy traces, with equal SDs, produces a new noisy trace with a SD that is the square-root of the number of input traces.

In least squares problems, the maximum likelihood solution is obtained by weighting the data by the inverse of the standard deviation (assuming Gaussian noise). This normalizes the SD in the observations. Similarly, when stacking data, normalizing the SDs of each trace will maximize the signal-to-noise ratio of the stack.

The data in the CSP gather has noisy traces with folds that vary significantly. I suggest that the noise in each trace has a SD that is proportional to the square-root of the fold. Dividing by the square-root of the fold should normalize the noise in each trace to balance the noise in each offset bin to produce an optimum stack.

The following data is similar to the previously illustrated data, but the gathers have been dip limited to 40 degrees. A gamma of 2.2 was also used for an initial estimate of the converted wave velocity. The following three CSP gathers illustrate the raw CSP gather, the CSP gather with full fold normalization, and the third with square-root fold normalization.
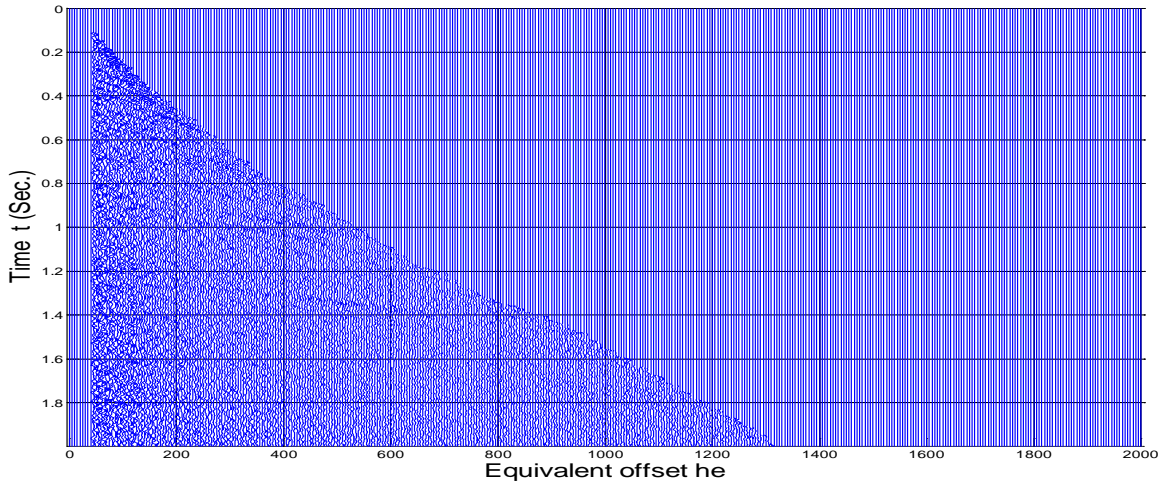
FIG 5 Raw CSP gather

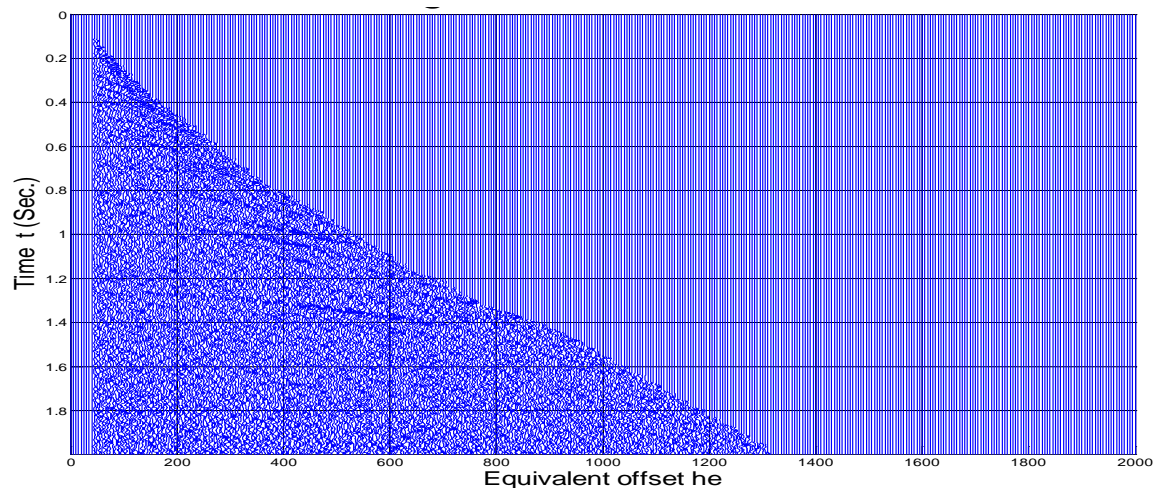FIG 6 CSP gather with full fold division

Fig 7 CSP gather with square-root fold division.

Note how the square-root fold division of the noisy traces produces a more balance amplitude.

Corresponding fold values for the full and square-root fold are shown below. These examples have less offset corresponding to the reduced size of the maximum offset in the gathers that were limited by dip.
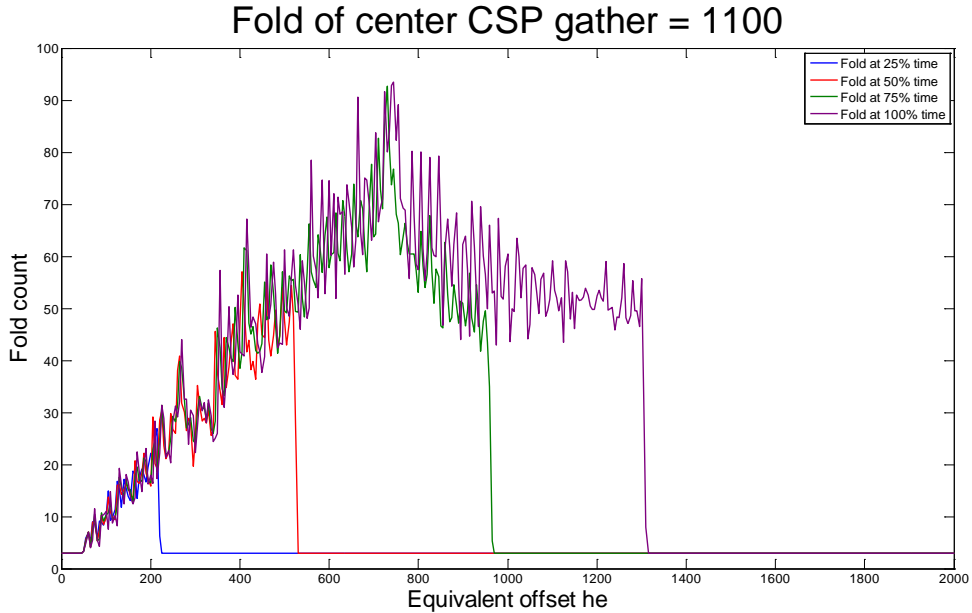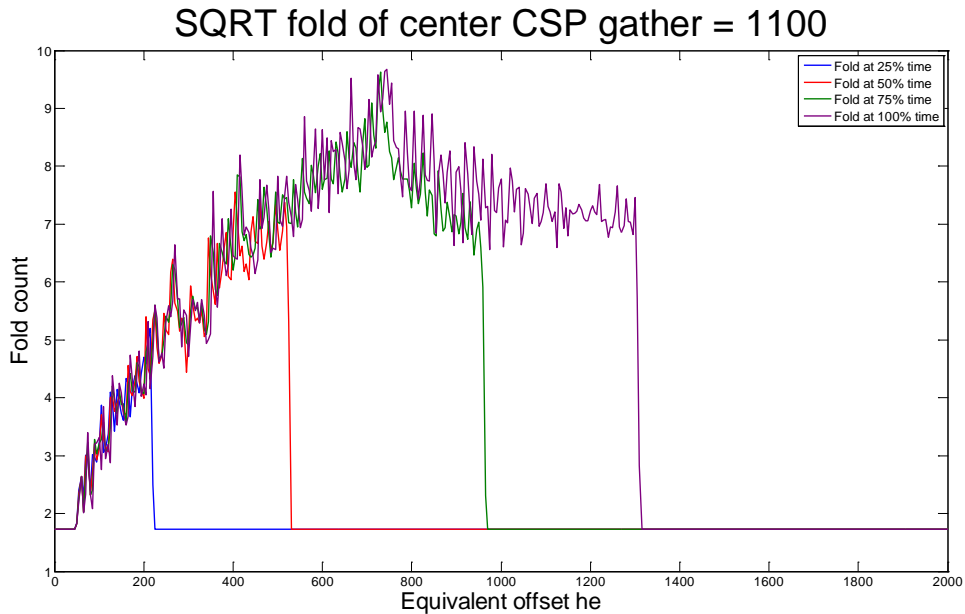

FIG 8 Full fold of the CSP gather.

FIG 9 Square-root of the fold.

Stacks of the data from CSP gathers represent the prestack migration of the data and are shown below in Figures 10 and 11.

EOM stack2 with offset fold division, method = 4 Vt = 3
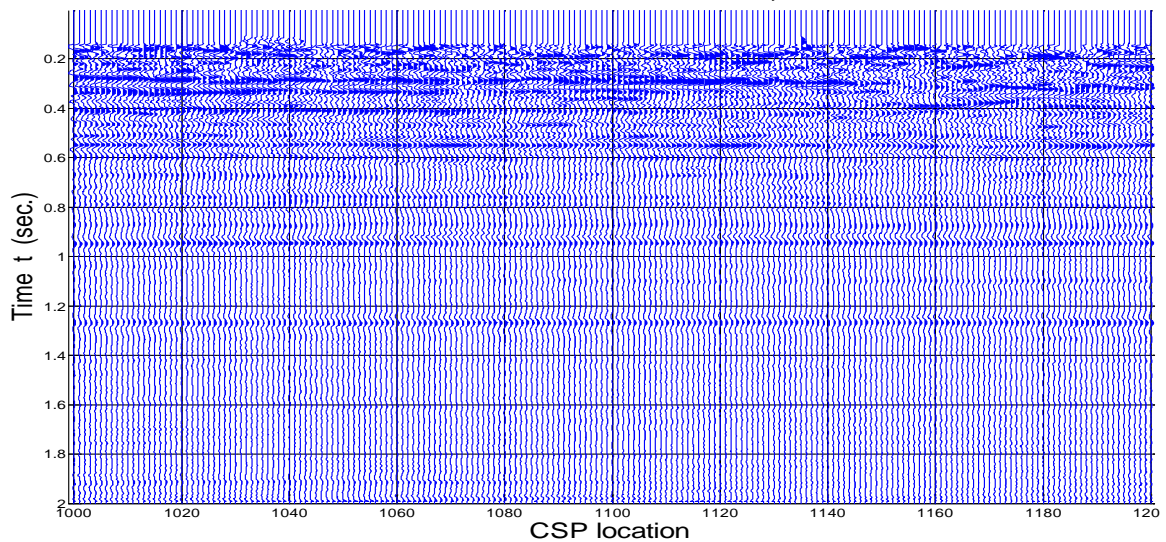

FIG 10 Prestack migration with full fold division.

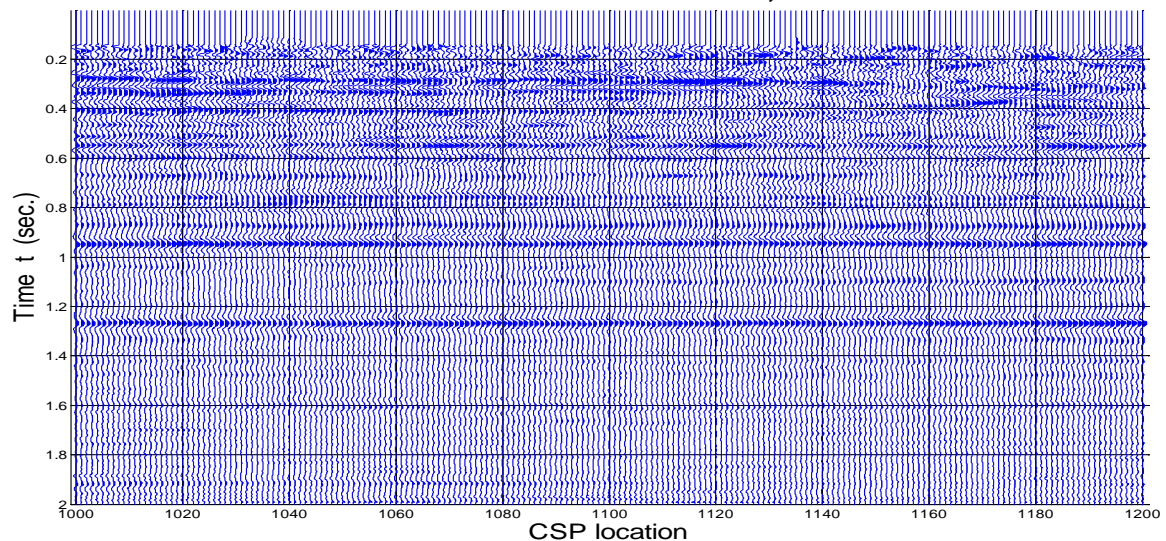## EOM stack2 with offset fold division, method = 4 Vt = 3

Fig 11 Prestack migration with square-root fold division.

The prestack migration with the square root fold division shows better overall amplitude scaling, but this difference could be removed with appropriate scaling or an AGC. The overall amplitude of each migration was chosen for the shallow data, but inspection of the two scaled gathers shows that theylook similar in that range, and not too much of a difference is to be expected.

COMMENTS AND CONCLUSIONS

A closer inspection of the data around 1.0 s at similar amplitudes did not appear to show any differences in the quality of the migrations. This was surprising and will continue to be investigated.

One benefit of the amplitude balanced gathers may be in the velocity analysis using the semblance plots. However, if the semblance algorithm includes an AGC, then there will be little difference.

The fold division is applied before moveout correction. It may be possible to apply moveout correction to the fold information and carry it through to the final stacking algorithm. This also should be investigated.

ACKNOWLEDGEMENTS

APPENDIX

The following are examples of moveout that has been applied to the CSP gather. The velocity for the moveout correction in Figure A1 used V_s, modified with a gamma of 2.2and a mute corresponding to a dip range of 25 to 30 degrees was used. In Figure A2, one iteration of V_{c2} wav used with a dip limit of 70 to 80 degrees.
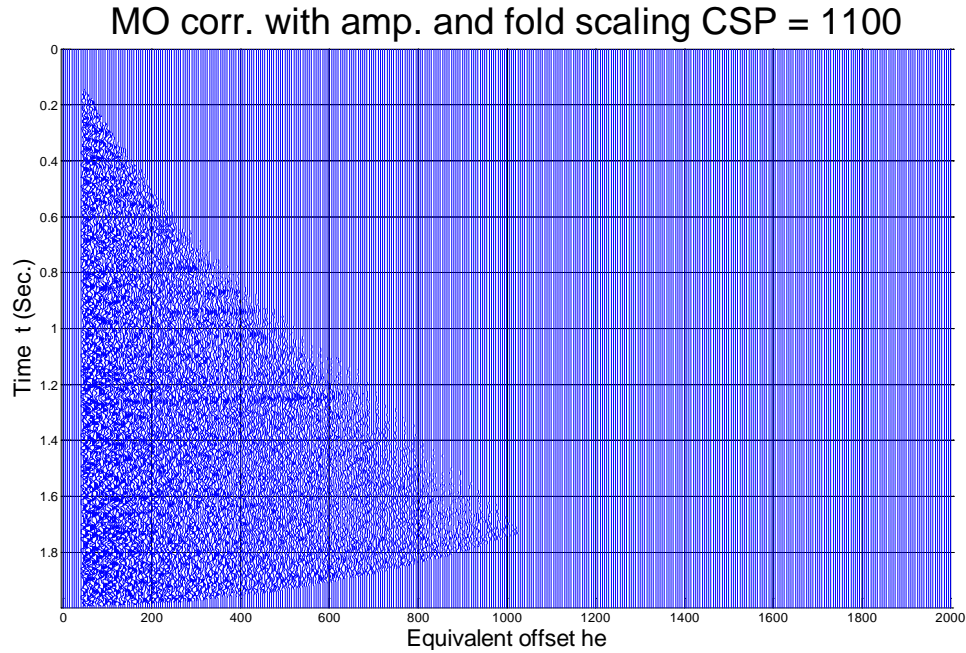

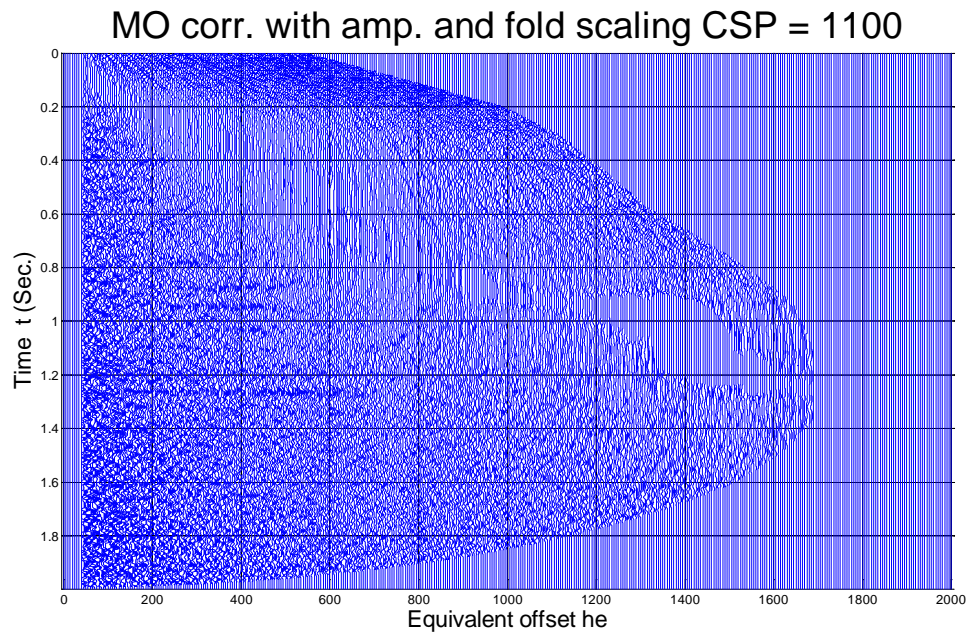FIG A1 A CSP gather with moveout correction, dip limit 25 - 30 degrees using V_{p1}.

FIG A2 A CSP gather with moveout correction, dip limit 70 - 80 degrees using V_{p2}.