Hagedoorn's +/- method and interferometric refraction imaging

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ABSTRACT

Hagedoorn's plus/minus method involves the calculation of sums and differences of first arrivals in refraction data sets. These quantities can be used to estimate subsurface velocities and the depths of curved refractors. Since seismic interferometry is, in a sense, a formal way of adding and subtracting traveltimes, it follows that the +/- method should be expressible in the form of an interferometric procedure. An algorithm for refractor imaging, expressed in terms of interferometric calculations on forward and reverse shot records, is derived heuristically and tested with a simple synthetic data set.

INTRODUCTION

Hagedoorn's plus/minus method (e.g., Telford et al., 1990) is a means by which two coincident shot records, acquired in opposing directions, can be analyzed to determine the depth-vs.-position of a curved refracting interface. The "plus term", in particular, assists in this by sifting from the two shot records those portions which have common paths to a given receiver. Those common paths have a simple relationship with the depth of the interface below the receiver, and this information can be teased out of the shot records.

It is perhaps not surprising, then, that interferometry (Claerbout, 1968; Wapenaar et al., 2010), which is the science of shared wave paths, can be used to recreate a version of the plus/minus method. The argument is maximally simple: Hagedoorn's method is driven by sums and differences of first arrivals; we should be able to reproduce it with correlations of normal or time-reversed versions of the shot records.

This paper is organized as follows. First, we review the salient features of Hagedoorn's method in its standard form, including the key quantities *delay time*, *plus term*, and *minus term*. Second, we point out that cross correlating slightly altered forms of the forward and reverse shots accomplishes the key steps of Hagedoorn's method, normally accomplished by hand. The outputs, which we refer to as the *plus field* and the *minus field*, are data sets of the same size as the forward and reverse shots, but whose first arrivals coincide with Hagedoorn's plus and minus terms respectively. We point out that (1) a re-scaling of the time axis of the plus-field transforms it into an image of the subsurface reflector, and (2) the cross correlations carried out in determining the fields were in fact sequences of 1D interferometric calculations.

REVIEW: HAGEDOORN'S +/- METHOD

Hagedoorn's plus/minus method is applied to refraction problems involving undulating refractors, whose depth is to be determined as a function of lateral position (along with the velocities above and below the refractor). In Figure 1 is a rough schematic diagram of the survey and its +/- interpretation. A suite of geophones is arrayed between two shot points, which we will refer to as F for forward and R for reverse (referring to the directions the wave energy is shot into the geophones).

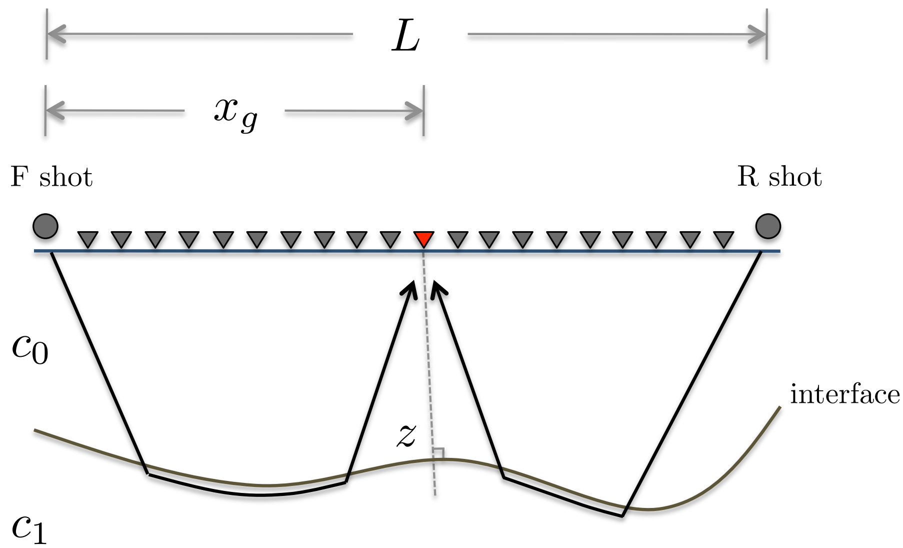

FIG. 1. Schematic representation of Hagedoorn's +/- method. Two shots on either end of the geophone spread are ignited. Sums and differences of the refraction arrival times can be used to determine c_0, c_1, and the normal depth to the interface $z(x_g)$ within the plus-minus window.

The method involves the idea of *delay times*, which are defined for the beginning and end of the ray path of all source-receiver pairs. For instance, the receiver delay time δ_{xg} at x_g, which is the same for both forward and reverse shots, is given by

$$\delta_{xg} = \frac{z(x_g)\cos\theta_c}{c_0}, \tag{1}$$

where θ_c is the critical angle and z is the refractor depth normal to the interface beneath the geophone x_g. In terms of delay times a travel time formula is expressed as

$$\tau(x_g) = \frac{x_g}{c_1} + \delta_s + \delta_{xg}, \tag{2}$$

where δ_s is the delay time at the source. The forward shot then has the traveltime equation

$$\tau_F(x_g) = \frac{x_g}{c_1} + \delta_F + \delta_{xg}, \tag{3}$$

and the reverse shot has

$$\tau_R(x_g) = \frac{L - x_g}{c_1} + \delta_R + \delta_{xg}. \tag{4}$$

Hagedoorn's plus term is defined as

$$\tau^+(x_g) = \tau_F(x_g) + \tau_R(x_g) - \tau_{rc}, \tag{5}$$

where τ_{rc} is the *reciprocal time*, or the travel time from one shot to the other. Hagedoorn's minus term is similarly defined to be

$$\tau^-(x_g) = \tau_F(x_g) - \tau_R(x_g) - \tau_{rc}. \tag{6}$$

Substituting equations (3)–(4) into equation (6), we see that the minus term

$$\tau^{-}(x_g) = \left(\frac{2}{c_1}\right) x_g + \delta_F - \delta_R - \frac{L}{c_1} - \tau_{rc} \tag{7}$$

has a slope proportional to the reciprocal of the velocity of the lower medium. Hence using the direct wave and the minus term to two velocities c_0 and c_1 can be determined. Substituting equations (3)–(4) into equation (5) we find that

$$\tau^{+}(x_g) = 2\delta_{xg}. \tag{8}$$

Finally, substituting equation (1) into equation (9), we see that by scaling the plus term we may generate a continuous estimate of the normal depth to the interface as a function of x_g:

$$z(x_g) = \left(\frac{c_0}{2\cos\theta_c}\right) \tau^{+}(x_g). \tag{9}$$

The x_g aperture within which this estimate can be formed, known as the *plus-minus window*, is determined by the crossover distances of the forward and reverse shots, and is an interior fraction of the space between the two shots as illustrated in Figure 1.

THE PLUS-MINUS METHOD AND INTERFEROMETRY

Since the +/- method consists of summation and differencing of first arrivals, the process can be significantly automated through crosscorrelation of the two data sets. We point out that doing so makes Hagedoorn's +/- method appear as a sequence of 1D interferometric calculations.

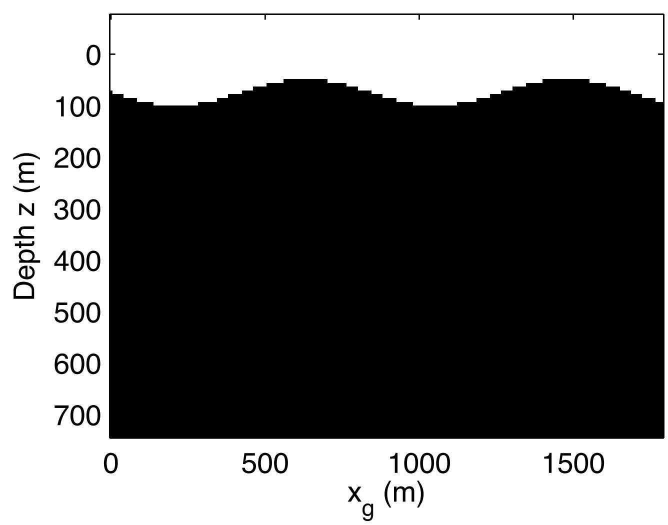

FIG. 2. Synthetic model with one curved refractor.

The plus term and the plus field as a sequence of interferometric calculations

Let the refraction data from the forward and reverse shots be $D^F(x_g, t|x_F)$ and $D^R(x_g, t|x_R)$ respectively. Further, consider

$$\hat{D}^R(x_g, t|x_R) = \int dt' D^R(x_g, t'|x_R)\delta(t' + \tau_{rc}) \tag{10}$$

to be the reverse shot data set, delayed by the reciprocal time. Our interest is in the sum of first arrival travel times in these data sets at common x_g locations. Notice that if we form

$$\begin{aligned} D^{+}(x_g, t|x_F, x_R) &= \int d\tau D^F(x_g, \tau|x_F)\hat{D}^R(x_g, t-\tau|x_R) \\ &= D^F(x_g, t|x_F) \otimes \hat{D}^R(x_g, -t|x_R), \end{aligned} \tag{11}$$

i.e., correlate D^F with the time reverse of $\hat{D}^R$, the first arrivals in the result will have times that correspond to the desired output. So, although $D^{+}(x_g, t|x_F, x_R)$ is a full data set of the same size as the two input refraction data sets, it is essentially a direct calculation of Hagedoorn's plus term. We will refer to $D^{+}(x_g, t|x_F, x_R)$ as the "plus-field". This is a sequence of 1D interferometric quantities, in the spirit of the original form due to Claerbout (1968).

The minus term and the minus field as a sequence of interferometric calculations

Continuing, if we form

$$\begin{aligned} D^{-}(x_g, t|x_F, x_R) &= \int d\tau D^F(x_g, \tau|x_F)\hat{D}^R(x_g, t+\tau|x_R) \\ &= D^F(x_g, t|x_F) \otimes \hat{D}^R(x_g, t|x_R), \end{aligned} \tag{12}$$

i.e., cross-correlate the two data sets (with the reverse data delayed by the reciprocal time), then the first arrivals in the output will have arrival times which are equal to Hagedoorn's minus term. We will refer to $D^{-}(x_g, t|x_F, x_R)$ as the "minus-field".

Forming an image with Hagedoorn's interferometric + term

The plus-field calculated in equation (11) is a 2D function, varying with x_g and time t. Transforming the time axis to a depth z axis using equation (9), and plotting the output of equation (11) in the (x_g, z) plane constitutes an image, wherein the first arrival is coincident with the refractor within the +/- window.

SYNTHETIC EXAMPLE: IMAGING AN UNDULATING REFRACTOR

A one interface model

In Figure 2 a synthetic model used to test the idea of "interferometric refraction imaging", embodied in equations (9) and (11), is illustrated. Two shot records of data will be taken on the measurement surface $z = 0$, and an attempt will be made to recover the depth and character of the undulating refractor.

Synthetic refraction data

In Figures 3a-b the two shot records are illustrated. Direct arrivals, reflections, and refracted arrivals are all noted. The refractions deviate from the linear events expected for planar targets. In order to focus on the first arrivals we pick a linear mute on both shots, shown in yellow. The data after muting are illustrated in Figures 3c-d.

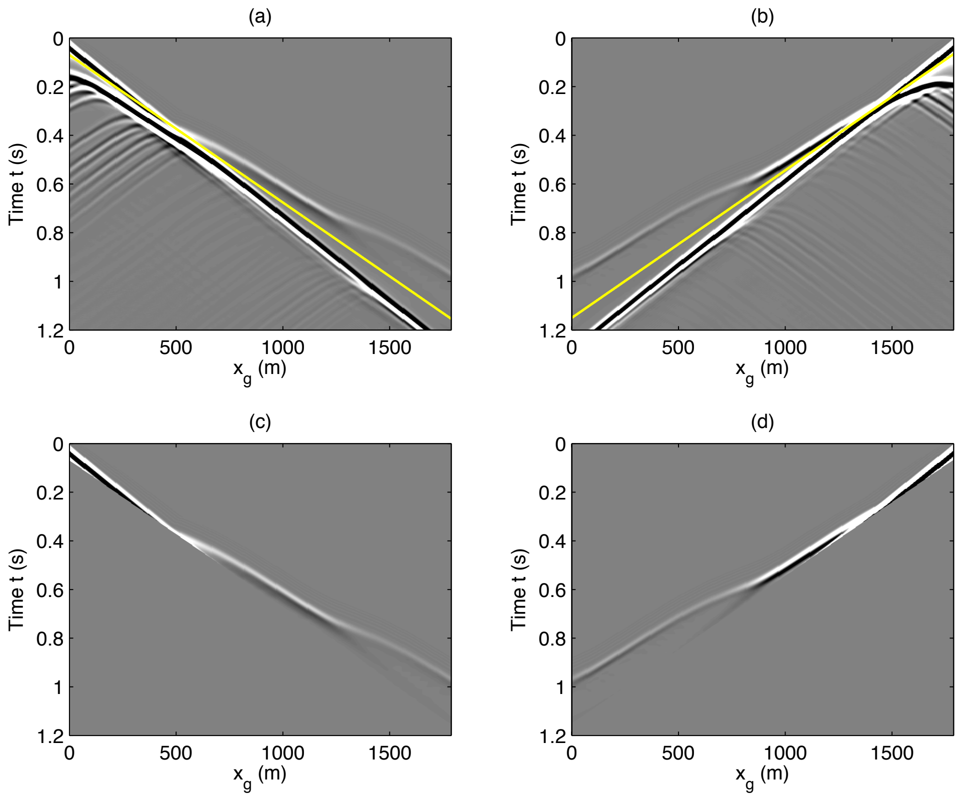

FIG. 3. Synthetic refraction data taken over the model in Figure 2. (a)-(b) The forward and reverse shots with all events, including the direct, reflected and refracted arrivals. A linear mute is illustrated in yellow. (c)-(d) The forward and reverse shots after muting, with first arrivals retained.

In Figure 4 the muted forward and reverse shots are displayed together, as they typically are prior to application of the plus-minus method. The plus and minus terms can be calculated by hand by drawing a vertical line at the desired x_g value, and reading off the travel times τ_F and τ_R. Thereafter they can be summed or differenced. We will instead automate this procedure and calculate the plus and minus fields.

Plus and minus fields

We next take the forward and reverse shots illustrated in Figures 3c-d, and use them as input to the interferometric calculations embodied by equations (11)–(12). In Figure 5a the plus field is illustrated; in Figure 5b the minus field is illustrtated.

Imaging with the plus field

We use the processed data in Figure 5a to form a refraction image, by scaling the time axis to depth as per equation (9). The result is shown in Figure 6a, with the actual model repeated for comparison next to it Figure 6b.

The plus-minus window is judged to lie roughly between $x_g = 500$m and $x_g = 1500$m. Within this window, a first arrival is picked on the image (Figure 6c, red dashed line). This pick is superimposed on the actual model in Figure 6d. Qualitatively, the character of the

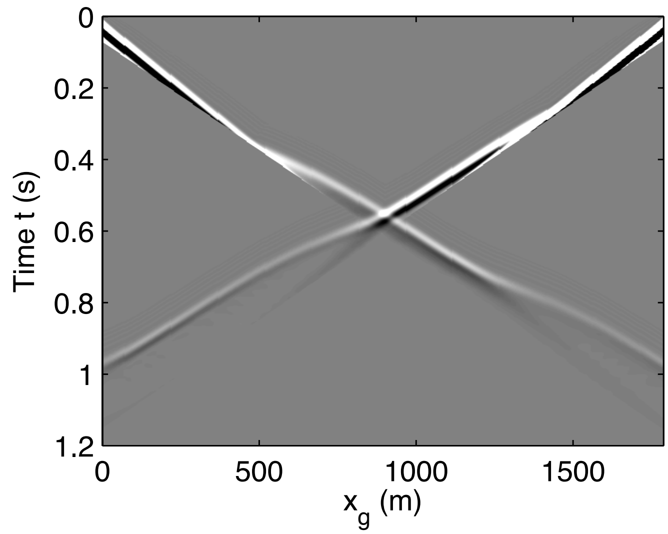

FIG. 4. The two refraction data sets overlain in the manner of the plus/minus method.

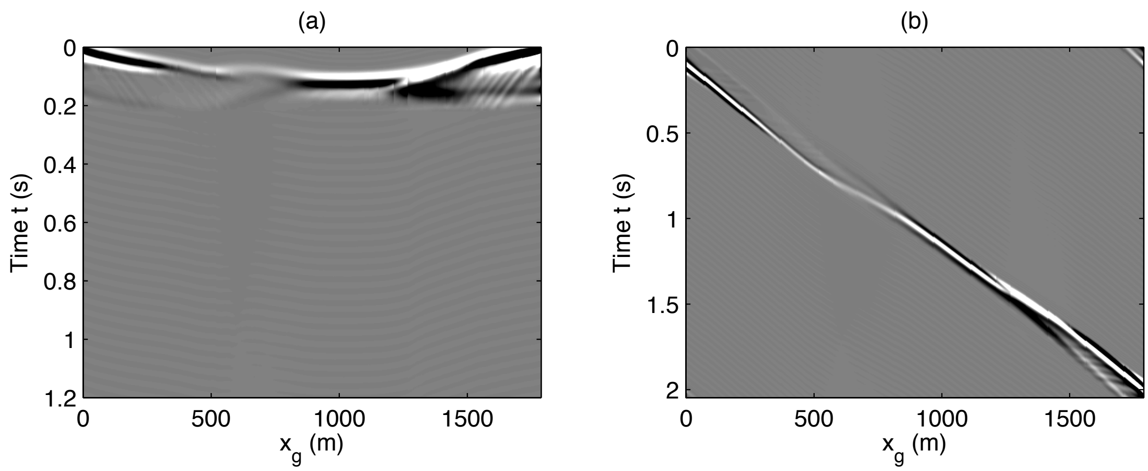

FIG. 5. The plus and minus fields as calculated using equations (11) and (12). (a) The plus field, in which the time of first arrival at each x_g coincides with Hagedoorn's + term. (b) The minus field, in which the time of first arrival at each x_g coincides with Hagedoorn's - term.

sinusoidal interface is captured, with some significant error noted in the depth. Some is to be expected, as (1) the plus-minus method assumes a very low level of interface undulation, and (2) the calculated depth $z(x_g)$ is the depth *normal to the interface*, whereas the model plot is the actual spatial distribution of the medium properties. As the curvature of the interface increases, these two quantities deviate from each other, so we should not expect a precise recovery of the black model with the red line.

CONCLUSIONS

The observation that since the plus-minus method involves sums and differences of travel times, something like interferometry is going on in carrying it out, appears to be borne out. An interferometric calculation of the "plus field", the full data set version of the plus term, forms a sensible, if not perfect, refractor image emerges from the plus-field when the depth is scaled properly.

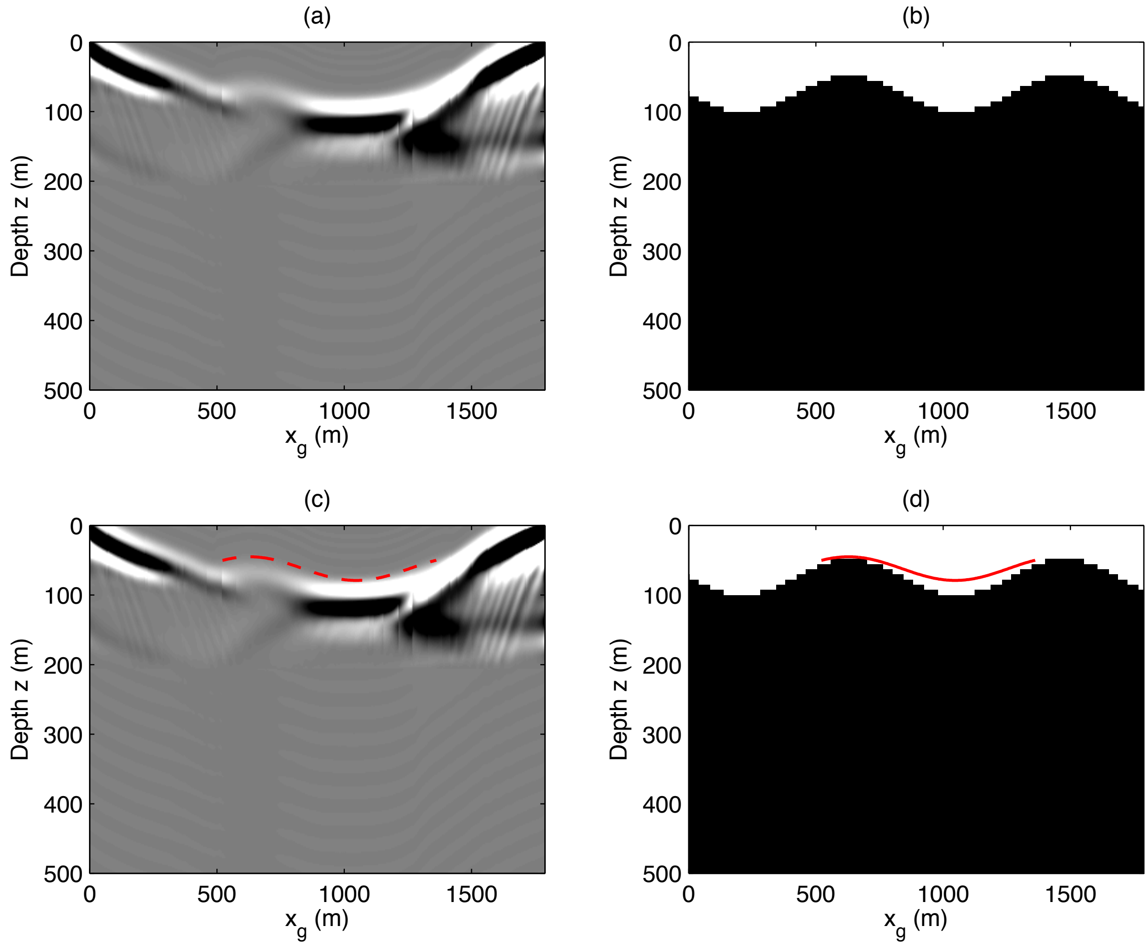

FIG. 6. Imaging with the interferometric plus field. (a) The plus field is plotted agains the scaled depth axis in equation (9). (b) The original model is plotted for comparison. (c) Within the +/- window the first arrival is picked (red dashed line). (d) The picked line is re-plotted against the actual model (red solid line).

One interpretation of this procedure is that in calculating the plus field, we construct a notional fixed-offset reflection data set, in which a wave propagates a distance of $2z(x_g)$ with the vertical slowness $\cos\theta_c/c_0$. And, in scaling the time axis to a depth axis, we perform a rudimentary migration of that data set through this medium. It may be that the direction to push this in the future is to pursue this interpretation further, such that more sophisticated migration procedures could be used.

ACKNOWLEDGMENTS

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REFERENCES

Claerbout, J. F., 1968, Synthesis of a layered medium from its acoustic transmission response: Geophysics, **33**, 204–2091.

Telford, W. M., Geldart, L. P., and Sheriff, R. E., 1990, Applied Geophysics: Cambridge University Press, second edn.

Wapenaar, C. P. A., Draganov, D., Snieder, R., Campman, X., and Verdel, A., 2010, Tutorial on seismic interferometry: part 1 - basic principles and applications: Geophysics, **75**, No. 5, 75A195–75A209.