Using the Sharpe Hollow Cavity model to investigate power and frequency content of explosive pressure sources

Christopher C. Petten* and Gary F. Margrave †

ABSTRACT

The nature of elastic waves emitted near the source of an explosive pressure source, such as dynamite, are difficult to predict due to the non-linear behaviour of the subsurface in the presence of an explosion. This problem is often referred to as the "melting problem" in Geophysics and can cause significant limitations when designing exploration surveys around dynamite. The Sharpe Hollow Cavity Model, derived by Joseph A. Sharpe in 1949, assumes that the non-linear region of the explosion process can can be treated as a hollow spherical region containing an arbitrary pressure pulse acting on its walls. Outside of this spherical region, the subsurface behaves in a linear fashion and thus their he motion of the elastic waves propagating in this region can be predicted using the elastic wave equation. This paper explores this model and some of the theoretical predictions set forth regarding the amount of charge that is used. It was found that the power of an explosion increases in a cubic fashion with the cavity radius and the dominant frequency of the emitted waveform decreases with increased charge size. However, it was also observed that the high frequency content of the explosion is diminished with smaller charge sizes and increased burial depth.

INTRODUCTION

The production of elastic waves via explosive pressure sources is an important topic of research in geophysical exploration. Out of all of the tools available for seismic wave exploration, dynamite is one of the cheaper means of producing compressional waves making it a very valuable tool in geophysical surveys. However, one of the major drawbacks to dynamite is the lack of understanding of the initiation process involved with the explosion. The nature of elastic waves emitted near the source of a dynamite explosion is not linear and does not propagate in a manner that can be predicted by the elastic wave equation (Sharpe 1949). In order to optimize the use of dynamite in exploration seismology, it is of utmost importance to understand the nature of the emitted waves and and be able to predict how they will behave when they are detected. The frequency content and the power of a travelling wave are the two primary factors that affect the ability to resolve features in the subsurface. Theoretical models that could be used to predict these two factors would be useful for developing surveys in which dynamite was the initiation source of the elastic waves.

In 1949 Joseph A. Sharpe developed a theoretical model for explosive pressure sources which is referred to as the Sharpe Hollow Cavity Model (SHCM). The SHCM assumes that a buried explosive source can be modelled as hollow cavity containing an arbitrary pressure pulse that acts on the inside of the cavity. The size of the cavity is directly proportional to the amount of explosive that is present and elastic waves are assumed to be non-linear in

*ccpetten@ucalgary.ca
†margrave@ucalgary.ca

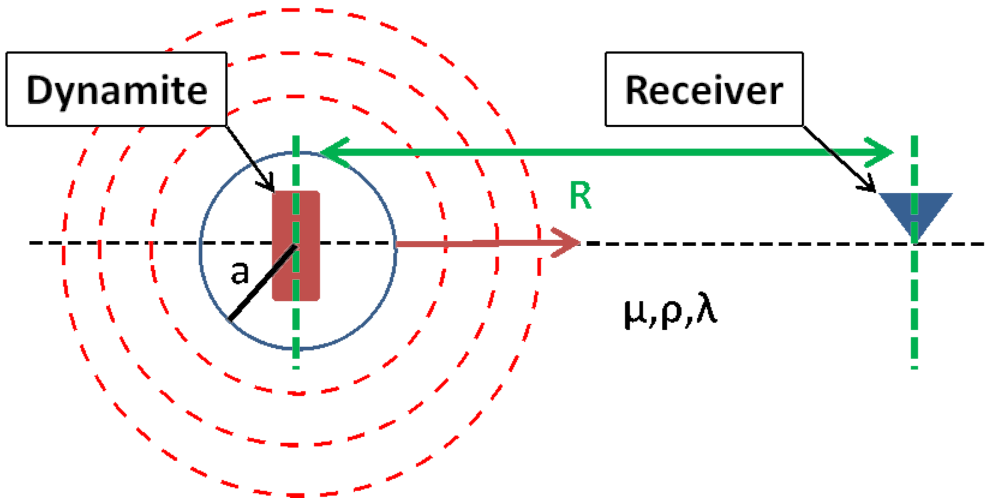

FIG. 1. Graphical depiction of the Sharpe Hollow Cavity Model. The source and receiver are assumed to be at zero depth in an infinite half space with parameters μ, ρ, and λ. The emitted waves (shown in red) propagate directly from the walls of the cavity.

this region, and thus their behaviour cannot be predicted using the elastic wave equation. However, if the receiver is located at a distance that is significantly larger than the cavity radius, the waves travelling in the region outside of the cavity can be assumed to follow the elastic wave equation. The SHCM model is quite robust and due to the spherically symmetric nature of the model, it would be an extremely useful tool for survey design using dynamite if it can accurately predict the behaviour of waves emitted from an explosive source. This paper investigates the SHCM and explores some theoretical predictions set forth by this model.

THOERY

A graphical depiction of the SHCM can be found in Figure 1. The source and the receiver are located at zero depth in an infinite half-space with Lamé parameters λ and μ; the emitted waves (represented by the red dashed lines) emanate from the walls of the cavity (represented by the solid blue line surrounding the dynamite). It is important to note that the waves emitted by the explosion are assumed to be strictly compressional due the spherical nature of the proposed model, any deviation from this type of symmetry would greatly reduce the viability of this model in practice. In this case, the pressure that acts on the cavity walls is assumed to be spherically symmetric since dynamite the waves emitted by an explosive source are mostly compressional (Aki and Richards, 1984). Additionally, dynamite is buried in a bulbous, water-filled cavity with a cylindrical hole that leads to the surface. The spherical approximate is most likely justified as long as the the cylindrical hole leading the center of the cavity is small compared to that of the bulbous water-filled regions such that waves do not propagate upwards through a cylindrical column.

In a spherically symmetric medium the pressure will be uniformly distributed over the inner walls of the cavity. Therefore, polar coordinates are more appropriate for this case establishing the following boundary condition for the pressure acting on the cavity wall:

$$\frac{\partial^2(r\Phi)}{\partial t^2} = v^2\frac{\partial^2(r\Phi)}{\partial r^2}, \tag{1}$$

since the polar components v and w are zero due to the symmetry of the problem. The boundary condition for the pressure pulse must vanish at the walls of the cavity, which can be accomplished by imposing the following condition:

$$p(t) = -\left[(\lambda + 2\mu)\frac{\partial u}{\partial r} + 2\lambda\frac{\mu}{r}\right]_{r=a} \tag{2}$$

where u is the particle displacement, and $p(t)$ is the time-dependent pressure pulse. The left right side of Equation 2 is the radial component of the stress which must be of equal magnitude and opposite sign to the pressure that acts on the cavity walls (Sharpe, 1949). To simplify the problem, a homogeneous medium allows for the approximation of the Lamé constants be equal. Expressing the Lamé constants in terms of the p-wave velocity, Equation 2 can be rewritten as:

$$p(t) = -\rho v^2\left[\frac{\partial u}{\partial r} + \frac{2}{3}\frac{u}{r}\right]_{r=a} \tag{3}$$

where ρ is the density of the medium and v is the wave velocity. In this case, since the explosive source is assumed to emit only compressional wave v will represent p-wave velocity from now on.

Since the cavity is spherical, a harmonic solution representing a wave diverging from the surface of a spherical cavity is most desirable (Sharpe, 1949). One such solution is as follows:

$$\Phi = \frac{1}{r}e^{-in\tau} \tag{4}$$

where τ is known as the retarded time. The retarded time is the time required for a wave to travel from the center of the cavity to the walls, which is expressed mathematically as:

$$\tau = t - \frac{r-a}{v} \tag{5}$$

where r is the distance from the center of the source shown in Figure 1, a is the radius of the cavity, and t is the time required for the wave to travel the distance r. A major advantage of this particular solution shown in Equation 4 is the fact that it is divergent and so it should, in theory, account for the divergence of a spherically expanding wave front which is characteristic of p-wave propagation that is observed in practice. This solution should therefore produce a decreasing amplitude with increased distance from the source, making it a viable model for p-waves emanating from the walls of the cavity in the SHCM.

It would be useful to be able to to reduce Φ in Equation 4 to something that can be solved analytically when it is substituted into the wave equation shown in Equation 1. Any function that is independent of r and t multiplied by the R.H.S of Equation 4 is a solution for Φ. Similarly, a sum of such solutions would also be a solution for Φ and so Equation 4 can be rewritten as:

$$\Phi = \frac{1}{2\pi r}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(n)p(\gamma)e^{in(\gamma-\tau)}\,dn\,d\gamma \tag{6}$$

which represents a viable solution to the equation of motion shown in Equation 1 (Sharpe, 1949). Note that the solution proposed in the above equation if and only if it is harmonic, and so $A(n)$ has the value of:

$$A(n) = \frac{(a/\rho)}{n^2 + 4inv/3a - 4v^2/3a^2} \tag{7}$$

where ρ is the density, v is the p-wave velocity, n is the harmonic, and a is the cavity radius (Arfken, 1985). Substituting $A(n)$ into the solution in Equation 4 and forming $u = \partial\Phi/\partial r$ along with the boundary condition given in Equation 3, yields the expression:

$$\Phi = \frac{a}{2\pi\rho r} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p(\gamma) e^{in(\gamma - \tau)}\, dn\, d\gamma}{n^2 + 4inv/3a - 4v^2/3a^2} \tag{8}$$

where $p(\gamma)$ represents the pressure pulse. This expression is a formal solution to the wave equation in polar coordinates when an arbitrary pressure pulse is applied to the cavity.

Now that a formal solution to the wave equation has been found, the next step is to introduce and arbitrary pressure pulse into the solution for Φ in order to produce a displacement that results from applying said pressure. For simplicity, a unit pressure pulse acting on the walls of the cavity would be most desirable in this case such that

$$p(t) = p_o e^{-\alpha t} \text{ for } t \geq 0, \tag{9}$$

and

$$p(t) = 0 \text{ for } t < 0, \tag{10}$$

where α is a positive constant, and p_o represents the initial and highest pressure applied to the cavity wall. If this pressure form is then substituted into Equation 8, the resulting expression can be evaluated using the Residue Theorem which yields the result:

$$\Phi = \frac{ap_o/\rho r}{(\omega/\sqrt{2} - \alpha^2) + \omega^2} \left[-e^{-\alpha t} + e^{-\omega\tau/\sqrt{2}} \left\{ \left(1/\sqrt{2} - \alpha/\omega \right) \sin\omega\tau + \cos\omega\tau \right\} \right]. \tag{11}$$

In this case, ω is the angular frequency corresponding to the oscillating term in the solution shown in the above equation, and has the form:

$$\omega = \frac{2\sqrt{2}v}{3a}. \tag{12}$$

Since a homogeneous medium has been assumed, ω can also be approximated as the angular frequency of the resulting waves emanating from the surface of the cavity (Sharpe, 1942). Finally, using the solution for Φ shown in Equation 11 in the displacement equation, $u = \partial\Phi/\partial r$, allowing α to go to zero to create a uniform pressure pulse, and solving for the far=field terms results in an expression for the particle displacement resulting from the SHCM:

$$u = \frac{a^2 p_o}{2\sqrt{2}\mu r} e^{-\omega\tau/\sqrt{2}} \sin\omega\tau, \text{ for } \tau \geq 0 \tag{13}$$

where a is the cavity radius in meters, p_o is a uniform pressure pulse, μ is the rigidity of the medium in Newtons per meter, τ is the retarded time, and ω is the angular frequency of

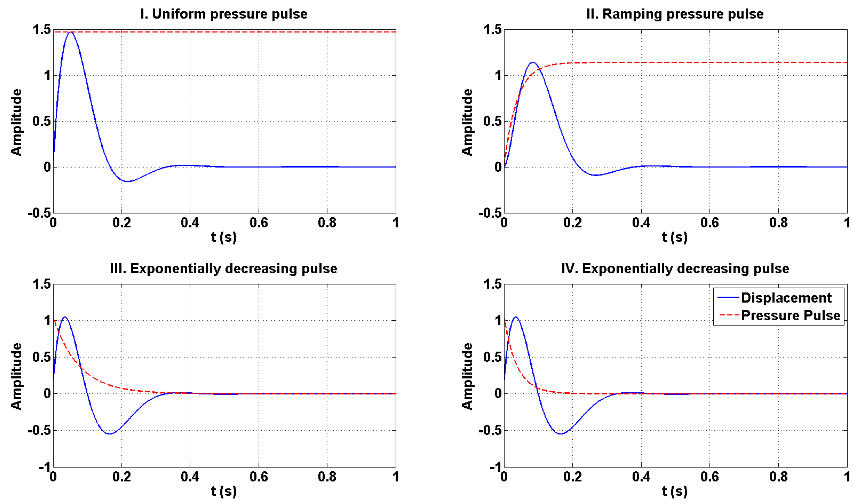

FIG. 2. Particle displacements resulting from applications of arbitrary pressure pulses. I) $p(t) = p_o$. II) $p(t) = p_o(1 - e^{-\sqrt{2}\omega\tau})$. III) $p(t) = p_o e^{-\omega\tau/\sqrt{2}}$. IV) $p(t) = p_o e^{-\sqrt{2}\omega\tau}$. The decreasing exponential shown in III most is the the pulse that is most likely to resemble that of an explosion since the application of pressure is approximately an impulse, which rapidly deteriorates or "burns out" in a short amount of time.

the waveform. The displacement given above in Equation 13 is only valid for application of a uniform pressure pulse. The displacement for any arbitrary pressure pulse that is zero inside the cavity can be obtained via convolution with the displacement resulting from application of a uniform pressure pulse shown in Equation 13, such that:

$$U(\tau) = \frac{d}{d\tau}\int_0^{\tau} p(n)u(\tau - n)\,dn, \tag{14}$$

where $U(\tau)$ is the motion resulting from an arbitrary pressure pulse $p(n)$, and $u(\tau)$ is the displacement resulting from application of a uniform pressure pulse (Equation 13).

Equations 12 and 13 yields some very important theoretical predictions regarding the nature of a an explosive source. Particle displacement should increase with larger charge sizes since the cavity radius is proportional to the amount of explosive that is used; in the presence of a higher rigidity medium however a decrease in particle displacement should be expected. The frequency of the travelling waveform, which can be expressed as:

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{2}}{3\pi}\frac{v}{a}, \tag{15}$$

predicts that a decrease in the dominant frequency should be observed when charge size increases. However, since the p-wave velocity increases with rigidity, waveforms travelling in a high-rigidity medium should be richer in higher frequency content.

MODELLING AN ADEQUATE PRESSURE PULSE FOR DYNAMITE

Figure 2 shows particle displacements from the SHCM that result from application of various pressure sources. The pressure pulses are shown in red while the resulting displacements are shown in blue. An exact form of a pressure pulse for an explosion is difficult to

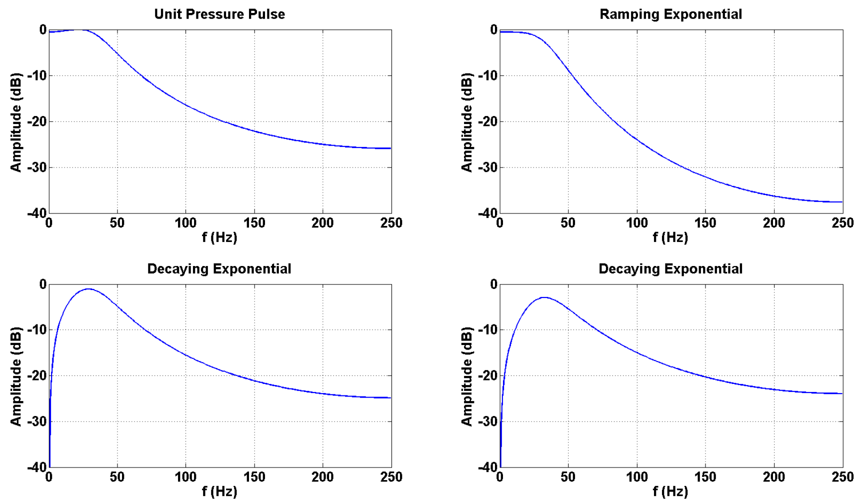

FIG. 3. A series of frequency spectra that result from the pressure pulses shown in Figure 2. The choice of pressure pulse for this model alters the frequency spectra significantly in both the high and low frequencies.

determine and can only represent an approximation. Ideally, a pressure pulse for a dynamite should start at something that is initially high, representing an impulse due to the rapid transfer of energy. This impulse should then rapidly decay over a short period of time, representing the rapid transfer of energy to its surroundings. Therefore, the pressure pulse III shown in Figure 2 is likely to be the most realistic form of pressure resulting from a dynamite explosion since it fulfils all of the previously mentioned criteria.

The choice of the pressure pulse used in the SHCM can have significant effects on the frequency spectra that result from the Fourier Transform of the displacement. Figure 3 shows the frequency spectra that result from the pressure pulses that have been used in Figure 2. One of the most noticeable differences that can be seen in these spectra is the low frequency roll-off that results from the exponentially decreasing pressure pulses. The ramping exponential and the unit pressure pulse do not appear to have any roll-off at all in the low frequency portion of the spectra however, all of the pressure functions appear to diminish at high frequencies as expected. It should therefore be expected that the pressure pulse selected for this study (the first exponentially decreasing function)) should predict low frequency roll off in all the frequency spectra.

THE ROLE OF CHARGE DEPTH IN PARTICLE DISPLACEMENT

Equation 13 has been derived assuming that the charge is located directly on the surface along with the receiver. In practice, this is not a realistic assumption since dynamite is usually buried in the subsurface when it is used for geophysical surveying. It is therefore important to understand how the burial depth of a charge affects the particle displacement in the SHCM. Figure 4 shows a charge buried at a depth z at a distance x from the receiver. Note that the center of the charge is assumed to be at zero depth in order to maintain the spherical assumptions that were used to derive the particle displacement in the SHCM. The medium in this case is still assumed to be an infinite half-space and since the waves emanating from the surface of the cavity are strictly compressional, the distance R in Equation

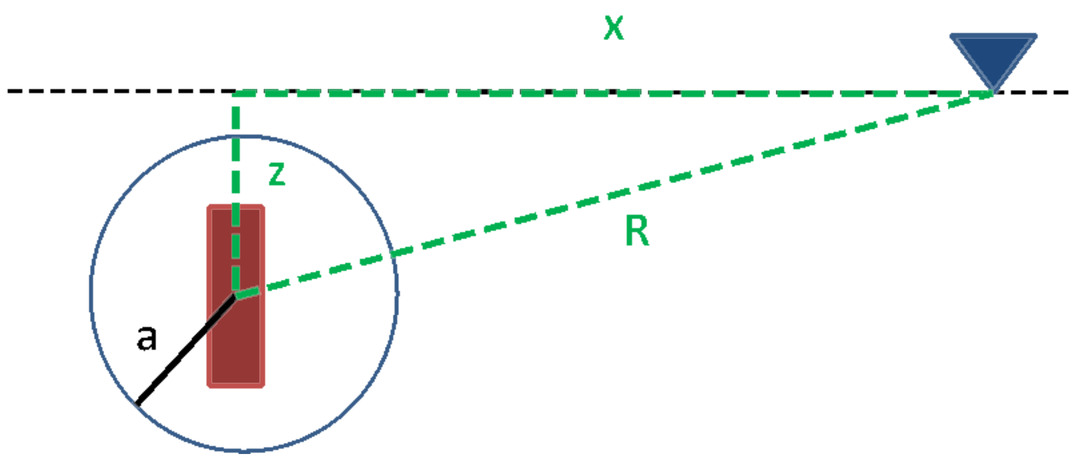

FIG. 4. Graphical depiction of a buried charge using the SHCM. The charge is buried at a depth of z to the center of the source at a distance x from the receiver.

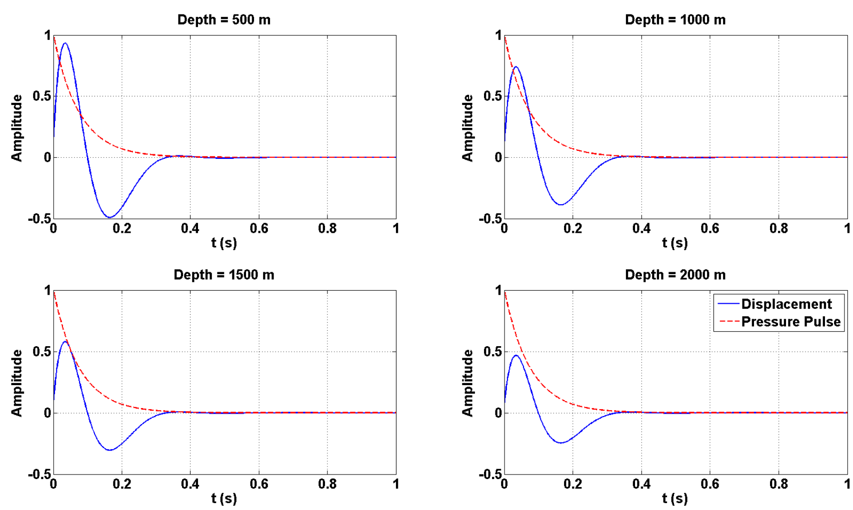

FIG. 5. Particle displacements from the SHCM that result from various burial depths z with a = 100 m, μ = 1 N/m, and x = 1000 m. The particle displacement appears to decrease with burial depth.

13 can be expressed as:

$$R = \sqrt{x^2 + z^2}, \tag{16}$$

where x is the distance from source to receiver, and z is depth at which the center of the charge is located.

Figure 5 shows a series particle displacements that result from the SHCM for varying depths along with their frequency spectra shown in Figure 6. These results show that the amplitude of the particle displacements decrease with increased burial depth; which is expected due to the solution to the wave equation shown in Equation 4. In Figure 7 shows a the maximum amplitude of the particle displacement as a function of depth that result from various offsets. Based on these observations, it appears that the SHCM predicts that the maximum amplitude falls off much more rapidly for smaller offsets than that of larger offsets. Additionally, the frequency spectra shown in Figure 6 show that the high frequency

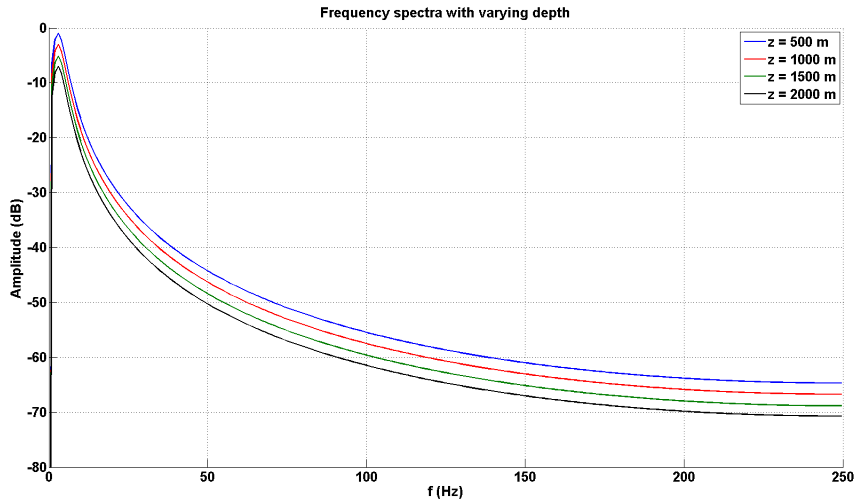

FIG. 6. Frequency spectra of the the particle displacements shown in Figure 5. The dominant frequency appears to remain constant with depth however the frequency content of the wave appears to decrease with increased depth.

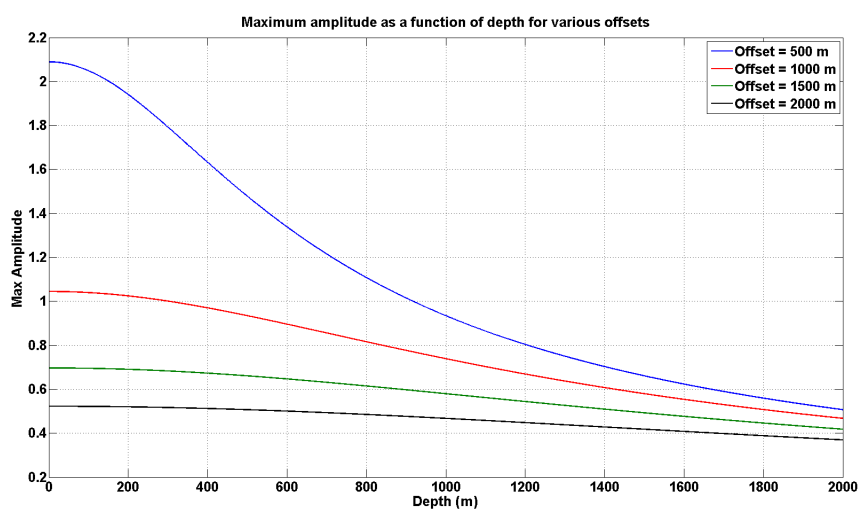

FIG. 7. The maximum amplitude of the particle displacement as a function of the burial depth for various offsets with a = 100 m, and μ = 1 N/m. It appears that the maximum amplitude decreases much more rapidly for smaller offsets.

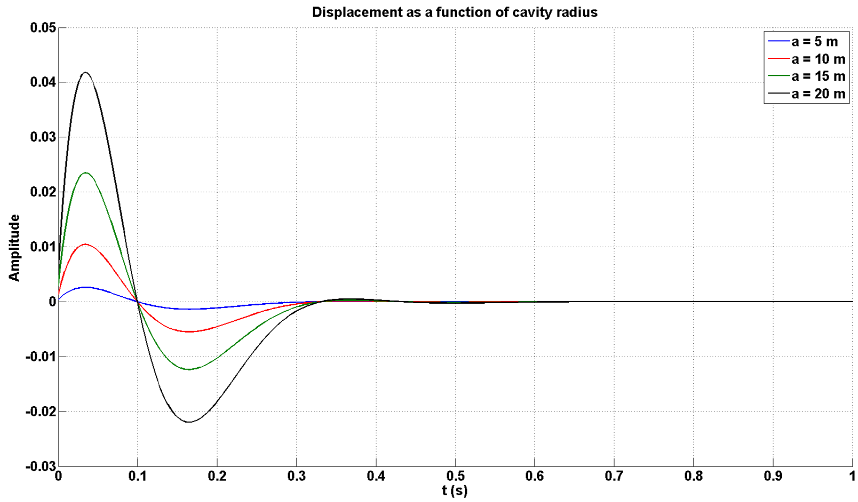

FIG. 8. Particle displacement resulting from varying cavity size with μ = 1 N/m and x = 1000 m. The maximum amplitude increases when the cavity radius increases; consequently, since the cavity radius is proportional to the charge size, the particle displacement should increase with charge size in the SHCM.

content of the waves emanating from the walls of the cavity decreases with increased depth, however, the dominant frequency does not appear to be affected by the burial depth.

THE ROLE OF THE CAVITY RADIUS

The cavity radius possibly the most important factor affecting the motion of the particles in the SHCM. Additionally, the cavity radius is assumed to be directly proportional to the amount of charge used so it provides to the most significant link to an explosive pressure source. Figure 8 shows a series of particle displacements that result from increasing cavity radius. The maximum amplitudes in each increases with larger cavity radii, as predicted in Equation 13. This would also be expected in practice since the charge size is assumed to be directly related to the cavity radius in the SHCM. Larger amounts of energy will be transferred to the subsurface when larger charge sizes are used, resulting in increased particle displacement. The increase in particle displacement with increased cavity radius in the SHCM suggests that it is a viable model for explosive pressure sources.

Shot power is an extremely important factor in deigning surveys using explosive pressure sources. Increased power will greatly increase the ability to resolve features in the subsurface however the frequency content of waves emanated from an explosion also play an important role in resolving features in the subsurface. It is therefore necessary to understand the relationship between shot power, frequency content, cavity radius, and the charge size in order to implement the SHCM in practice. Figure 9 shows calculated shot power from the SHCM as a function of the cavity radius. The power increases with charge size which agrees with observations from conventional explosive pressure sources, where the shot power increases with the charge size (Sharpe, 1949). In order to establish a general relationship between shot power and charge size, a series of polynomials were fit to the shot power. It was found that a cubic relationship, shown in Figure 9, shows that the relationship between shot power and and cavity radius is cubic.

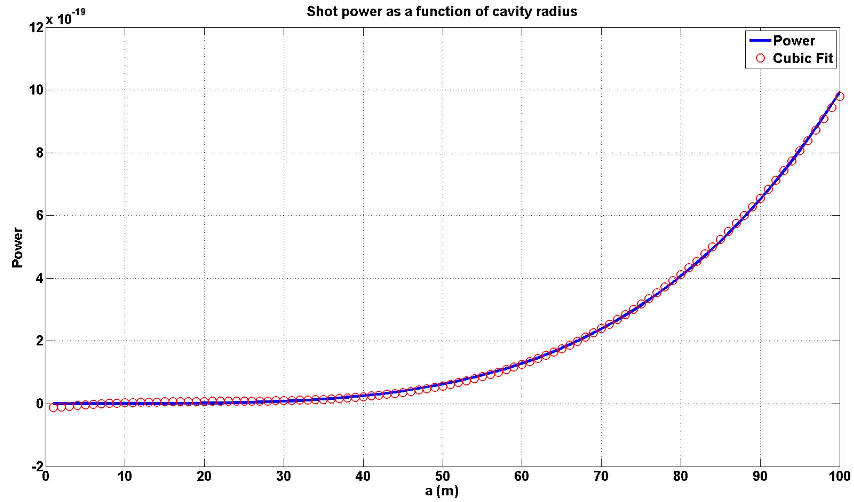

FIG. 9. Plot of the shot power as a function of cavity radius (shown in blue) produced with μ = 1 N/m and x = 1000 m. A cubic polynomial has been fit to the shot power showing that there appears to be a cubic relationship between shot power and cavity radius.

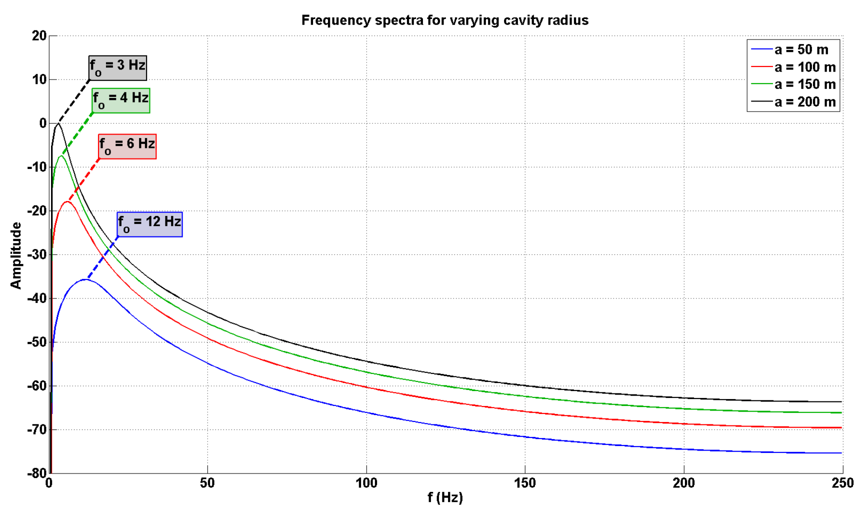

FIG. 10. The frequency spectra resulting from particle displacements shown in Figure 8. The SHCM appears to predict that the dominant frequency decreases with with increased charge size, however, the high frequency content of the travelling waves increases when larger charges are used.

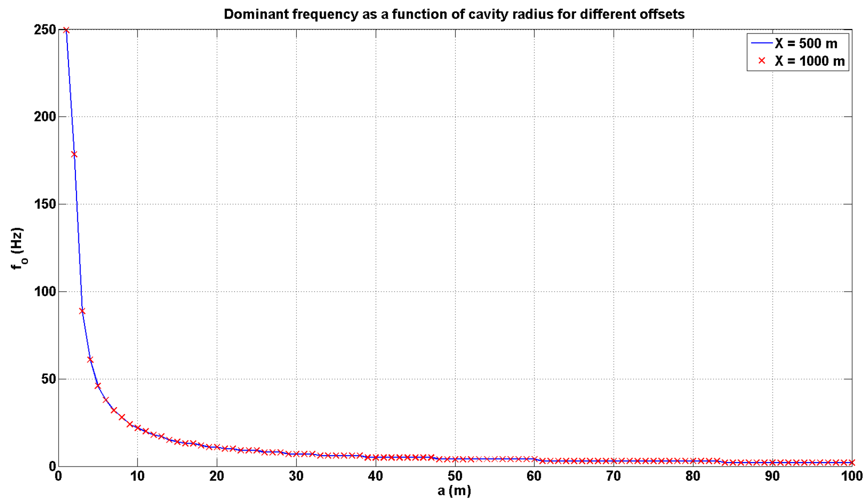

FIG. 11. Dominant frequency as a function of the cavity radius produced for various offsets with μ = 1 N/m. The dominant frequency appears to decrease rapidly for smaller cavity sizes and also seems to be independent of the offset in the SHCM.

Figure 10 shows the frequency content of the particle displacements in Figure 8. These yield some interesting results since the dominant frequency decreases with increased charge size, which is observed in practice (Sharpe, 1949), however, the frequency content of the wave diminishes with smaller charge sizes. High frequency content is extremely important in seismic data processing, so the larger dominant frequencies resulting smaller charge sizes may not necessarily be more effective in data processing than larger charge sizes, assuming that the SHCM accurately predicts the nature of an explosive source. Figure 11 shows the dominant frequency as a function of the cavity size for two separate offsets. A rapid decrease for cavity radii below 10 m is observed here and the dominant frequency resulting from the SHCM appears to be offset independent due to the similarity between the two functions.

THE EFFECT OF RIGIDITY ON THE FREQUENCY SPECTRA

The Sharpe displacement in Equation 13 also predicts that the rigidity of the medium, μ, should have a significant effect on the particle displacement and thus the frequency spectra. One of the major goals of this study is to implement the SCHM in such a way that it can accurately predict the behaviour of charges deployed in the field. It is therefore important to understand how the changing rigidity can affect the frequency spectra predicted by the SHCM in order to account for frequency variations encountered in practice. Equation 13 suggests that an overall decrease in particle displacement should be observed in the SCHM due to the inverse relationship between rigidity and displacement. However, it is not entirely clear from this equation alone whether or not the dominant frequency or overall form of the frequency spectra will be affected by a change in rigidity. Figure 12 shows the frequency spectra that result for varying values of rigidity in the SHCM model. As predicted by Equation 13 a uniform decrease in amplitude is observed across the entire spectrum and the overall form does not appear to deviate significantly from that of Figure 2 and 3. One of the more interesting results in this case is the lack of change in the dominant frequency with varying μ-values, which suggests that the dominant frequency should remain independent

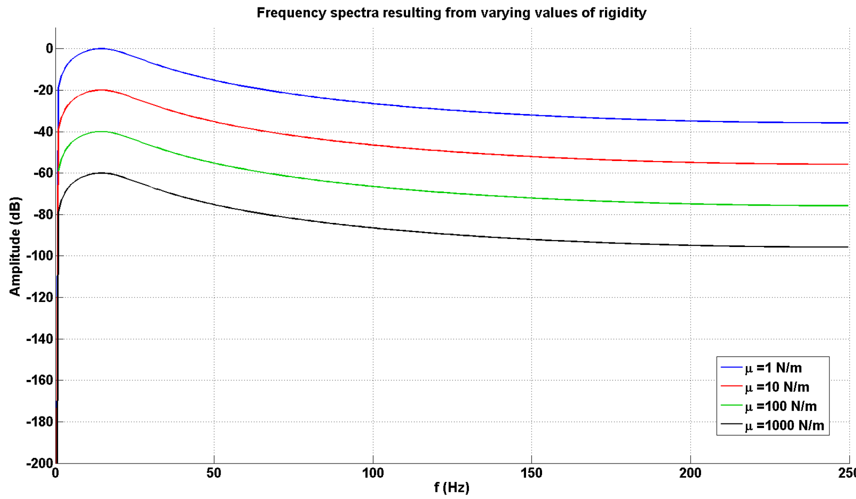

FIG. 12. Frequency spectra that result for varying values of medium rigidity. Increased rigidity appears to uniformly decrease the amplitude spectra over all frequencies. However, the dominant frequency and the low-frequency roll-off do not appear to be affected by changing rigidity.

of the rigidity in the SHCM.

CONCLUSIONS

The theoretical predictions set forth by the SHCM for travelling compressional waves appear to coincide with some common assumptions regarding dynamite explosions. Power is known to increase with charge size for explosive sources, which is predicted by the SHCM as shown in Figures 8 and 9. Since power and displacement both appear to increase with cavity radius, and the cavity radius is assumed to be directly proportional to the charge size, it can be concluded that the SHCM may be able to accurately predict and explain the nature of p-waves emanating from explosive sources.

The depth of the source appears to have a significant affect on the particle displacement in the SHCM. Observation of Figures 5 and 7 reveals that this model is greatly influenced by both the depth and offset since the maximum amplitude of the displacements vary so greatly for smaller offsets and shallow depths. Dynamite is generally buried close to the surface so the SHCM may be exceptionally prone to inaccuracies in its predictions at close offsets since particle displacement at shallow depths show the greatest variation at small offsets. Therefore, it is important that caution is used when using the SHCM for shallow burial depths and offsets smaller than 1000 m.

Shot power, as predicted by the SHCM, appears to be related to cavity radius via a cubic relationship as shown in Figure 9. This relationship suggests that it may be possible to establish a link between charge size and cavity radius if power spectra are obtained for varying charge sizes. A common assumption in geophysical surveys is that it is preferable to use smaller charge sizes for data processing since they have a larger dominant frequency, despite a loss in power that results from small charges. This appears to also be the case with the predictions set forth by the SHCM, however, this model also predicts that the high frequency content of the travelling waves diminishes when smaller charge sizes are used as seen in Figure 10. If the SHCM is able to accurately predict the nature of compressional waves emitted from an explosion, this could lead to improvements in survey design by implementing larger charge sizes to improve resolution and shot power while also maintaining a reasonably large dominant frequency.

There are some significant limitations to the SHCM that are directly related to the assumptions made in its derivation, which may prove to be unreasonable in practice. The biggest assumption made in the SHCM is that the charge is located in a perfectly homogeneous medium with equal Lamé parameters. Homogeneity is an extremely idealized assumption which is not necessarily valid in practice due to variations in the subsurface. Since this assumption does not match the subsurface conditions that are usually observed in practice, the SHCM may be limited for modelling explosive sources in more realistic media. Additionally, for charges that are buried significantly close to the surface, the SHCM may be exceptionally unreliable due to the assumptions made regarding the Lamé parameters being equal, this is not a valid assumption for unconsolidated materials so the derivation of the particle displacement shown in Equation 13 would not be viable in this case.

In summary, the most important observations that result from the SHCM are as follows: (1) The pressure pulse used significantly affects the form of the frequency spectra predicted by the SHCM. (2) Dominant frequency decreases with increased charge sizes however, there is a significant loss in high frequency content with small charges. (3) The frequency spectra that result from exponentially decreasing pressure sources, which in this case has assumed to be the best model for a dynamite explosion, should result in a low-frequency roll-off in the frequency spectra. (4) Larger amplitude responses are expected for bigger charge sizes and thus should produce the most power in practice. (5) Increased rigidity of the medium decreases the overall amplitude response resulting from explosive pressure sources however, the dominant frequency appears to be independent of such changes.

Future work on this subject includes tying these observations to dynamite data collected in the Priddis 2011 and Hussar 2012 experiments conducted by CREWES in order to see if a relationship between cavity radius and charge size can be reliably established. Additionally, the frequency content from these data will also be analysed in order to see if it match the frequency predictions of the Sharpe Model.

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