A hybrid tomography method for crosswell seismic inversion

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SUMMARY

The waveform tomography, or full waveform inversion (FWI), is in general better than the ray tomography method; however a reliable initial model is usually required to ensure the success of seismic inversion. In this work we designed a cascade-like hybrid tomography technology to solve the crosswell seismic inversion problem. The new method is a combination of several widely used tomography technologies. We start from the Radon-transform based back projection (BP) method, which produces a smooth initial velocity model. Next, the Linear Iterative Reconstruction (LIR) method is adopted to update this initial model, then it is further improved by the nonlinear Gradient-based Eikonal Traveltime Tomography (GETT) method. The velocity model reconstructed from the previous multi ray tomography methods is sufficiently reliable to serve as the initial model for the computational intensive waveform tomography method, from which the accurate velocity model is obtained. The numerical example shows that this hybrid tomography method has great potential in reconstructing accurate acoustic velocity or other high-resolution reservoir characterization for crosswell seismic inversion. With this hybrid tomography method, the inversion result of three cascade ray tomography methods is used as the initial model for FWI that provide a perfect initial velocity model, which leads to a significant reduction for the computation time and the iteration number. It is noticeable that this hybrid tomography method is able to obtain an accurate velocity even when the recorded seismic data is in poor coverage at spatial and the signal-to-noise ratio is low.

INTRODUCTION

Seismic tomography is typically solved as an inverse problem, in which the misfit function measuring the difference between a set of observational data and a set of synthetics is minimized. Depends on the recorded data being used, seismic tomography can be classified into the waveform tomography (also known as diffraction tomography or full waveform inversion, FWI) and the traveltime tomography (or ray tomography).

It is widely accepted that the waveform tomography, or FWI, is in general better than the ray tomography because of the dynamic and kinematic information in the wavefield (Lehmann, 2007). However, FWI is usually ill-posed, computationally intensive and sensitive to the initial model (Virieux and Operto, 2009). Hence a reliable and accurate initial model is critical for the success of FWI. In fact, choosing a good initial velocity model is a commonly used regularity strategy when the seismic inversion problem is solved. A good initial velocity model will not only prevent the optimization procedure from converging to the local minima, but also significantly speed up the iteration, which in return will reduce the computational cost and deliver the final results in a timely manner.

Building the reliable initial model is one of the most topical issues for successful application of FWI (Sirgue, 2006; Prieux et al, 2010). A natural idea is to find an approximated initial velocity model through the traveltime tomography method, for instance, the Back Projection (BP) method, the Linear Iterative Reconstruction (LIR) method and the Gradient-based Eikonal Traveltime Tomography (GETT) method, which are much faster than FWI. In fact any of these three methods can be used as the initial model feeder for the FWI. A simple comparison shows that BP is the fastest one and does not need any initial guess, however it delivers the least accurate result. On the other hand, the nonlinear GETT is the most expensive ray tomography method and needs an initial model to start, however it delivers the best result that is accurate enough to be used as the initial model for FWI. The linear LIR, which also requires an initial model, standing somewhere between BP and GETT method.

Given that the Eikonal equation is the high-frequency approximation of the wave equation, it is reasonable to use the result of GETT as the initial model of FWI. However, this nonlinear tomography method itself needs to be solved as a model-based inverse problem, and the gradient-based optimization algorithm is employed in this work, so again an initial velocity model is required. Then many linear LIR methods, which have been widely used in industry to solve the large-scale sparse matrix system, are used to produce the initial velocity model for GETT in our hybrid method. In principle any random initial guess can be used for the linear iterative method, however it turns out that the result by the BP method is a good choice.

Given the huge computational efforts (CPU time, memory requirement, etc.) needed by FWI, even a small reduction of iterative steps due to the use of a more accurate initial model could lead to considerable saving in computing resources especially for the low signal-to-noise ratio observed data. We therefore propose the new hybrid method, in which four widely used methods: BP, LIR, GETT and FWI are implemented in a sequence, where the cheaper method is used as the initial model feeder for the more expensive method. More specifically, we first implement BP as the initial model feeder for LIR, which is then implemented as the initial model feeder for GETT, and finally we implement FWI which takes the result of GETT as its' initial velocity model. By carefully assessing the tradeoff between efficiency and accuracy, and adjusting the parameters such the iteration numbers and the model size, a balanced and cost-effective hybrid method can be obtained.

METHODOLOGY

The workflow of the hybrid tomography technology is shown in Figure 1. As mentioned before, it is composed of multi technologies to reconstruct the velocity model from crosswell seismic data. Based on this workflow, we give an introduction and the implementation to each of the four components of the hybrid method.

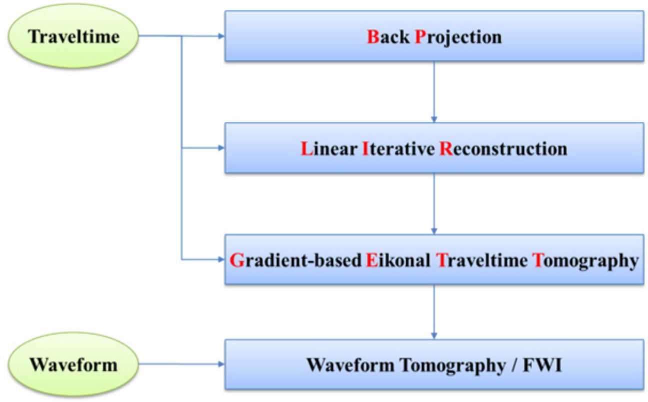

Figure 1: Workflow of the hybrid tomography technique.

Back Projection tomography

The back projection (BP) is the dual transform of the Radon transform, which has been used as the theory basis for computerized tomography (CT). BP is the simplest concept and the easiest method in terms of implementation. In fact, the back projection result is not the original parameters distribution but a smoothed-out version of it. Nowadays, BP has been replaced by the improved filtered back-projection (FBP) method, which has been widely implemented in commercial scanners (Feeman, 2010). In brief, BP is an operation that computes the average value of the projected values (or Radon transforms) using the following formula:

$$s(x,y)=\frac{1}{\pi}\int_0^{\pi} t(x\cos\theta+y\sin\theta,\theta)d\theta \tag{1}$$

where s is the slowness, which is the reciprocity of velocity, t is the traveltime of the seismic ray that passes through the point (x,y), θ is the angle of the seismic ray. The image created by Equation (1) however has a quite blurred appearance due to the summing of all the values of the points traversed for every ray.

The advantage of BP reconstruction is the efficiency. It has been used in the early seismic experiments (Wong, 1993), which rarely been used today in the seismic industry because the seismic data acquisition is limited by the spatial range. In addition to its' efficiency, BP method doesn't need initial model, which makes it a perfect starter of our hybrid tomography method, especially when a prior information of the velocity model is not available. It is a convenient way to create a starting model for other more sophisticated tomography methods.

Linear Iterative Reconstruction tomography

The Linear Iterative Reconstruction (LIR) method refers to all iterative algorithms that solve the large-scale sparse linear algebra matrix to reconstruct the image. The Kaczmarz's method (Gordon, 1970) was the first that has been employed to explore the first application of the iterative reconstruction technique, which is also called ART (algebraic reconstruction techniques) in the literature. In fact, the linear tomography problem can be expressed as

$$\mathbf{G}\cdot\mathbf{s}=\mathbf{T} \tag{2}$$

Where the matrix $\mathbf{G}$ is composed of entries g_{ij}, which is the length of *i-th* ray that crosses the *j-th* cell. The vectors $\mathbf{s}$ and $\mathbf{T}$ represent the slowness and observed traveltime respectively. In general, the matrix $\mathbf{G}$ is a large-scale sparse matrix, which is usually solved by the iterative reconstruction methods. As a matter of fact, the LIR is the family of iterative algebraic methods for solving this tomography problem including ART, SIRT, LSQR and CG, to name a few. The widely used simultaneous iterative reconstruction technique (SIRT) can be expressed as

$$s^{n+1}=s^{n}+\lambda_{n}\frac{1}{N}\sum_{i=1}^{N}\frac{t_{i}-\left\langle g_{i},s^{n,i}\right\rangle}{\left\|g_{i}\right\|^{2}}g_{i} \tag{3}$$

Where n is the iteration number, and λ is a parameter chosen for the iterative step length control. Apparently this iterative method requires an initial model. Fortunately, LIR method is not very sensitive to the initial model. Hence a zero initial guess works well in most cases. However it has been observed that even a slight improvement of the initial guess will improve the inversion result significantly, especially when the problem is ill-posed. It turned out that the result from the back projection method is a good choice of the initial model for SIRT method.

It is obvious that the main problem of the LIR method is that the linear matrix $\mathbf{G}$ is underdetermined, in which the number of unknowns is greater than the number constraints. For example, it is very difficult to obtain reasonably accurate solutions for the 10,000 unknowns from just 1,000 equations, as shown in our example later. So, the model reduction technique is adopted to resolve this issue. Using the result of BP method, the LIR method can produce a fairly good result but still is not good enough to be used as the initial model for FWI. A nonlinear optimization based ray tomography is therefore considered in our hybrid method to further improve the result.

Gradient-based Eikonal Traveltime tomography

The nonlinear Gradient-based Eikonal Traveltime Tomography (GETT) method reconstructs the velocity through solving the inverse problem constrained by the Eikonal equation. The misfit function of the inverse problem is defined as the difference between the observed data and the simulated traveltime from the Eikonal equation

$$E_{EIKONAL}(\mathbf{m})=\frac{1}{2}\left(\mathbf{t}_{obs}-\mathbf{t}_{cal}(\mathbf{m})\right)^{T}\left(\mathbf{t}_{obs}-\mathbf{t}_{cal}(\mathbf{m})\right) \tag{4}$$

where $\mathbf{t}_{obs}$ is the observed first-arrival traveltime, $\mathbf{t}_{cal}$ is the predicted traveltime based on the current velocity model $\mathbf{m}$. The first-arrival traveltime is computed with the Eikonal solver. A variety of numerical methods have been developed in the past few decades, for instance, the fast marching method (Sethian and Popovici, 1999), the fast sweeping method (Zhao, 2004) and the semi-Lagrangian schemes for Hamilton-Jacobi equation (Falcone, 2002), to name a few. To minimize the misfit function in Eq. (4), one needs to calculate the gradient of this misfit function with respect to the velocity model

$\mathbf{m}$, which can be done using the adjoint-state technique (Huang, 2012; Taillandier, 2009; Plessix, 2006).

In this work, the fast sweeping algorithm is used to solve the Eikonal equation, while the gradient is calculated using the adjoint-state method. The limited memory BFGS (L-BFGS) is used to solve the nonlinear optimization problem. It is obvious that the result of the previous linear iterative reconstruction method is a good initial velocity model for this gradient-based nonlinear method. It is expected, due to the advantage of the gradient-based inversion in optimization direction, this result is better than that of the LIR, thus is a sufficiently reliable initial model for FWI. Moreover, it is significantly cheaper than FWI, and the traveltime tomography is robust and accurate enough to provide a good initial velocity model for FWI, especially when the signal-to-noise ratio of observed data is low.

Waveform tomography

Waveform tomography, or FWI, uses all the waveform information of the seismic data. It is an unconstrained optimization problem, in which we minimize the least squares difference between the observed data and the synthetic seismogram

$$E_{FWI}(\mathbf{m}) = \frac{1}{2}\left(\mathbf{d}_{obs} - \mathbf{d}_{cal}(\mathbf{m})\right)^T \left(\mathbf{d}_{obs} - \mathbf{d}_{cal}(\mathbf{m})\right) \tag{5}$$

where $\mathbf{d}_{obs}$ is the recorded seismic data, $\mathbf{d}_{cal}(\mathbf{m})$ is the predicted seismic data at the designed shot-receivers position.

Using the second-order Taylor-Lagrange expansion of the misfit function (5) at the vicinity of the initial model $\mathbf{m}_0$, one can obtain the derivative with respect to the velocity model $\mathbf{m}$, then the perturbation model vector at the initial model $\mathbf{m}_0$ can be expressed as (Virieux, 2009),

$$\Delta\mathbf{m} = -\left[\frac{\partial^2 E(\mathbf{m}_0)}{\partial \mathbf{m}^2}\right]^{-1} \frac{\partial E(\mathbf{m}_0)}{\partial \mathbf{m}} \tag{6}$$

The perturbation model is searched in the opposite direction of the steepest ascent of the misfit function at the initial model $\mathbf{m}_0$. The second derivative of the misfit function is the Hessian that defines the curvatures of the objective function. Then, the updated model $\mathbf{m}$ can be written as the sum of the initial model $\mathbf{m}_0$ plus this perturbation.

It is worthy to point out that the algorithm given in Eq. (6) can also be adapted to minimize the misfit function given in Eq. (4). Moreover, both the FWI and the GETT can be solved by the L-BFGS method (Liu and Nocedal, 1989).

In summary, a reliable initial velocity model is not only helpful to improve the convergence of iteration, but also critical to prevent the iteration from converging to local minima. The initial model for FWI will be provided by the nonlinear gradient-based Eikonal Traveltime tomography method, as it is the most accurate ray tomography method. The initial model for the nonlinear GETT method is provided by the linear iterative reconstruction (LIR) method such as ART and SIRT, which takes the result of BP as its' initial model. Here we mention that, any of the three traveltime tomography methods shown in Figure 1 can be used directly as the initial model feeder. However, the cascade-like hybrid method shown in Figure 1 is proved very effective and efficient.

EXAMPLES

A 2D model on 100 meter by 100 meter is designed to test our hybrid tomography technique. With the spatial sample interval of 1 meter, the model size is 10201. In our experiment, the observations of 11 shots (marked with red star) and 101 receivers (marked with blue circle) have been made, as shown in Figure 2 (a).

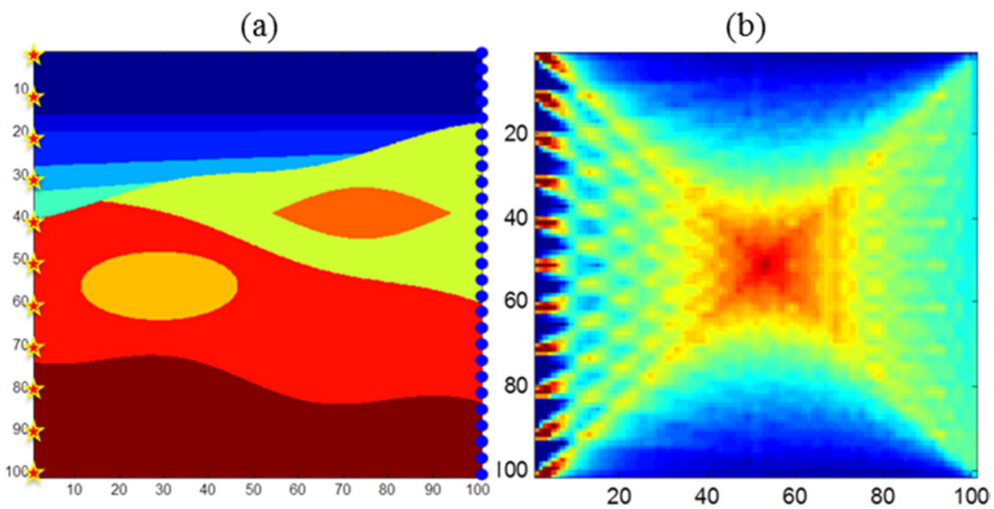

Figure 2: The data acquisition design for the 2D model. (a) the true velocity model; (b) the ray density of the data.

From Figure 2 (b), we can see that the ray density is uneven as there are more seismic rays crossing in the middle part of the domain. It is obvious that the blue region corresponding to low ray density is difficult to recover with the ray tomography method. Finite difference method is used to model the seismic recording. One shot recording for this model is shown in Figure 3 (a).

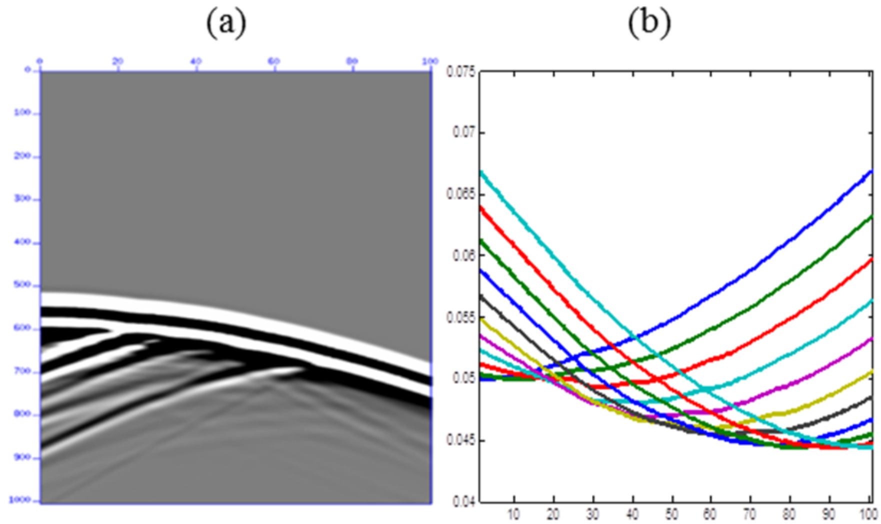

Figure 3: (a) the observed waveform of 1 shot, (b) First arrival traveltime of 11 shots

With these 11-shot recordings, the 101 travel times of each shot recording have been picked. The first arrival traveltime curves of the 11 shots are shown in Figure 3(b). Given that there is only 11x101=1111 observed traveltime, it is very difficult to recover all of the 10201 parameters in the velocity model. This problem is severely underdetermined, so the model reduction technique has been used to resolve this issue in the hybrid method.

Using the 1111 traveltime, the back projection method is used to obtain a velocity model, which is shown in Figure 4(a). Clearly, the velocity model is blurred at the region where the ray density is low, while a much better result is obtained at the region where the ray density is high, as shown in Figure 2(b). After several iterations of the LIR method based on the initial velocity from the BP method, the rough outline of the velocity model appears in the inversion result, as shown in Figure 4(b). This result is then used as the initial model for the gradient-based Eikonal Traveltime tomography method. Updated by the nonlinear optimization method, a more accurate velocity model is obtained and displayed in Figure 4(c). It is clear, as one can see from Figure 4(a)-(c), that the accuracy of the results are somewhat affected by the ray density. Finally, the result of GETT is used as the initial model for FWI to recover the velocity. Clearly, the number of iterations required by FWI to produce a very satisfying velocity model has been reduced significantly. Apparently such improvement in accuracy and efficiency is due to the use of the excellent initial model by the traveltime tomography methods. The blurring phenomenon in the initial model is successfully removed because the detail waveform information has been incorporated by FWI. The inversion result by the new hybrid tomography technology is shown in Figure 4(d), which clearly is very close to the true velocity model.

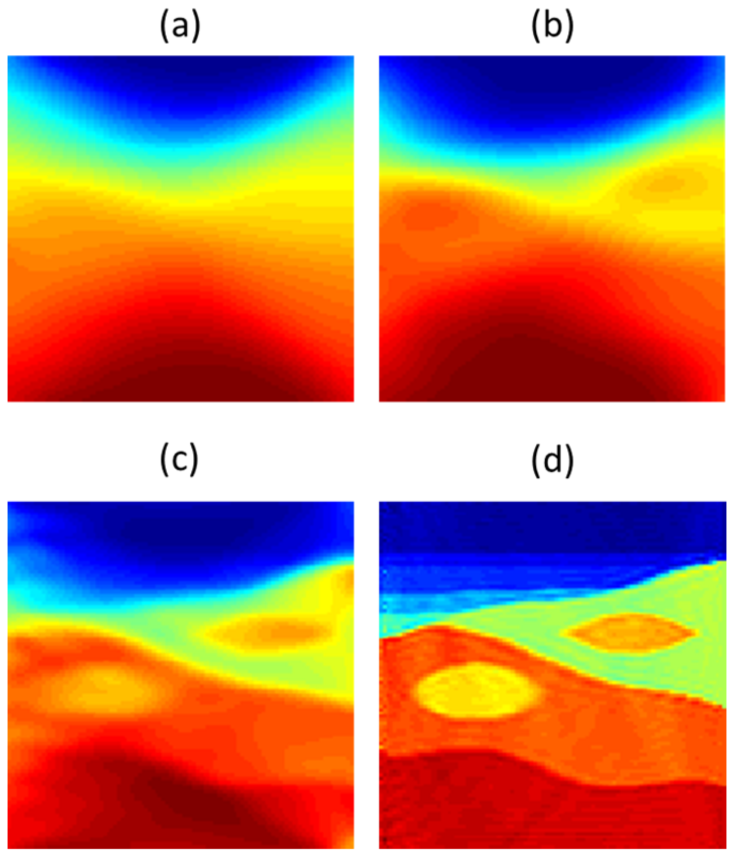

Figure 4: A step by step illustration of the inversion results by the hybrid method. (a) Result by BP method; (b) Result by LIR based on BP; (c) Result by GETT based on LIR; (d) Result by FWI based on GETT.

Apparently, as can be seen from Figure 4(a)-(d), the recovered velocity models are getting more and more accurate step by step. It is very clear from the Figure 5 that our hybrid tomography method can save many calculation times when compared with a random guessed initial velocity model. The computation time will be significantly decreased when this hybrid tomography method is used to achieve the same value for the object function.

Figure 6 shows the square error between the true model and the recovered velocity. It is clear the hybrid is very efficient to get a velocity model that close to the true model. Only several steps can achieve a satisfied model error with this hybrid tomography method, which is very valuable for the full waveform inversion problem.

It is clear that all the velocity inversion result of the ray tomography method can be used as the initial model for FWI. Figure 5 and Figure 6 show the comparsion of different combination of the ray tomography methods with FWI. Considering the cascade relation from BP, SIRT, GEET to FWI, every method has its advantage to inversion the velocity model, the combination of more methods will save more computation times and achieve the satisfied model error in limited steps.

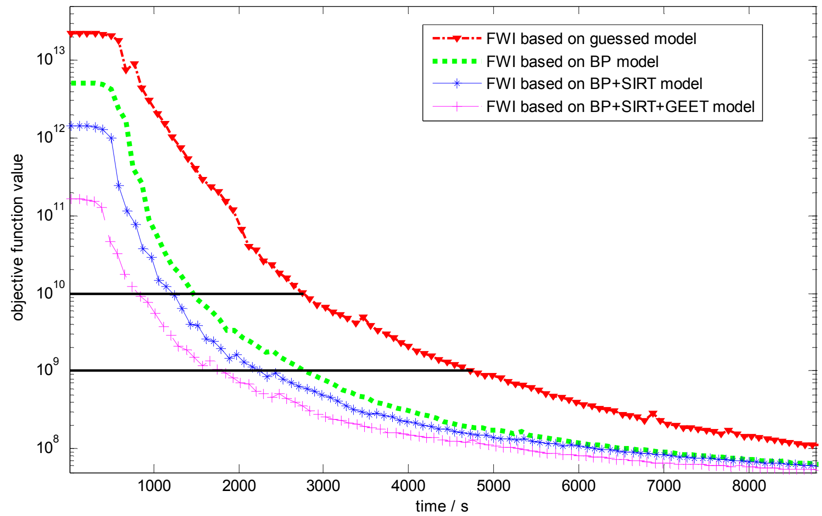

Figure 5: Comparsion of the objective function value with the calculation time for different hybrid method

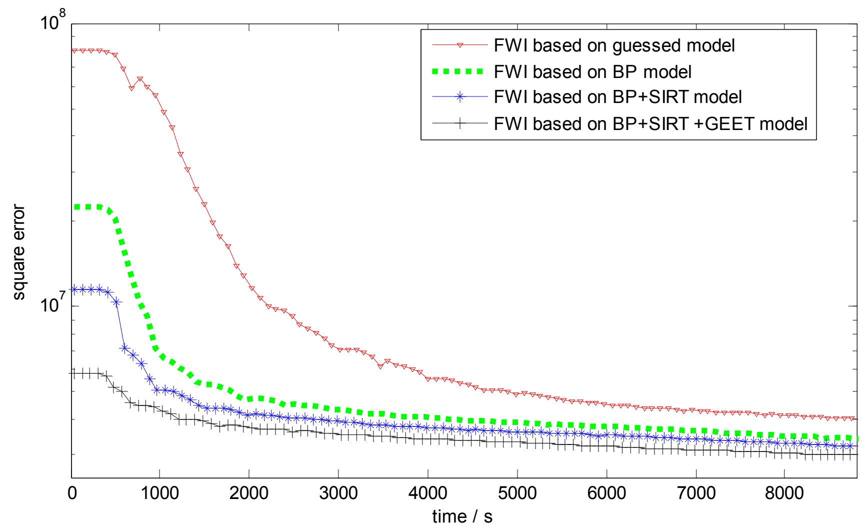

Figure 6: Comparsion of the square error value between the true model and the inversion result with the calculation time for different hybrid method

CONCLUSIONS

A hybrid seismic tomography technology is designed to solve the crosswell seismic inversion problem, based on the analysis and comparison of different waveform tomography and traveltime tomography technologies. By carefully exploiting the tradeoff between accuracy and efficiency of each component, an integrated workflow of optimal performance is obtained. With the combination of these methods, the cheaper method that delivers less accurate result is used as the initial model feeder for the method that is computationally more expensive to obtain more accurate result. Numerical results demonstrated that this hybrid tomography method is cost-effective and accurate.

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